

STATIC AND DYNAMIC QUANTITY-SETTING GAMES: AN IN-CLASS EXPERIMENT

This paper illustrates the results of a case study on teaching economics issues employing an experiment where students were made to play quantity-setting games employing the Stackelberg and Cournot theory of oligopoly. A strictly theoretical approach to the study of the oligopolistic market structure is replaced by a discovery-learning method. The goal of the in-class experiment is both to illustrate to the students the economics theory through a learning by doing approach and to allow the instructors to discover how students act when they have to develop their own strategies, placing them in a role similar to that of firms aiming to maximize the profits. The main finding shows how students converged toward the Nash-equilibrium quantity. Several firms or groups of students, who were producing high output level at the beginning of the game later on reduced their output since they realized that their profit could increase by just producing less. At the end of the experiment, students have emphasized that they have really learned what is like to interact in a market structure where firms can influence the market variables but not absolutely control them.

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1. Introduction

Oligopoly is a typical topic of all microeconomics courses usually taught at the Economics faculties and, at the same time, a market structure very suitable for experimental economics and economic experiments (Davis and Holt, 1993). It occurs when just a few firms, often assumed to produce an identical product, share a large portion of the industry. Because there are only few firms, each one when decides about its actions has to take into account the other firms' reactions (Smith, 2000). This means that if a firm changes the price or the output level, the profits of its rivals will be influenced as well (Simon, 1997). Then, the rivals may also react by changing their price or output levels. Since firms may strategically interact in different way and be unpredictable, there is not a single model of oligopoly (Schotter, 2009; Perloff, 2008; Holt, 2007; Varian, 1998) Here, I focus on the Stackelberg and Cournot theory of duopoly.

Students in microeconomics are used to approach the topic of the oligopolistic market structure in a strictly theoretical, old fashion, yet effective manner. Since this

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year, at the University Matej Bel, it was decided to give to the students the possibility to study the oligopoly theory in a learning by doing style. In order to provide students in our microeconomics course with a clearer understanding of the oligopoly theory, it has been used an in-class experiment that reproduces in much the same strategy facing firms in an oligopolistic industry. Classroom games and experiments provide instructors with an alternative teaching mechanism (Lacombe and Ryan, 2003). A possible way is to run an in-class experiment after discussing with students the textbooks' parts on oligopoly and game theory (Meister, 1999). Students act as the players in the in-class experiment, placing them in a role similar to that of a manager employed by an oligopolistic firm (Seiver, 1995). At the end of the experiment, students reported that they have really learned what it is like to strategically interact in an oligopolistic structure in which firms have some influence on the market variables but not absolute control over them, and, that the attempt to produce more can actually shrink the profit. The instructors discovered a more concentration and active participation from the student during the in-class experiment than during the theoretical class. Students seemed really engaged in developing their own winning strategy in order to get a better final grade for the microeconomics course, so that extra credit points were found a good incentive for paying more attention. Students have also reported that it is an exciting challenge to elaborate strategies in order to gain the highest profits. Finally, it must be said that few students felt irritated because of their incapacity to control their rival's reactions, but soon it was explained to them that science can also play a strange game.

The main finding of the experiment is that at the beginning of the game most of the groups of students were producing more than the quantity that should have been theoretically produced in equilibrium. They were producing too much thinking to do so was the optimal choice. Then, output levels come close to the Nash-equilibrium quantity as the game advanced. Several groups who were producing high output level at the beginning of the game later on reduced their output since they realized that their profit and, thus, extra credit points could increase by just producing less. This is proof that students were effectively able to discover in practice what they should learn in theory. In which way the theoretical lecture that preceded the in-class experiment influenced the students' learning process could also be tested as extension for further case studies on economics education.

The paper is structured as follows: Section 2 reports the theoretical framework. Section 3 describes the rules of the game. Section 4 shows the results. Section 5 concludes.

2. The Oligopoly Theory à la Stackelberg and à la Cournot

The main features of an oligopolistic market are: (1) a few numbers of firms, (2) interdependence among the firms, (3) relatively high barriers to the entry of new firms, (4) firms often are assumed to produce an identical product but they can also produce differentiated ones, (5) economies of scale (Uramova, 2001). According to the Stackelberg theory of oligopoly, the Nash equilibrium of the one-shot game, in

which there are two firms and the strategies available to each firm are the different quantities it might be produced, assumes that firms set quantities sequentially in order to maximize their profit (Gibbons, 1992). Let q_1 and q_2 denote the quantities produced by firms 1 and 2. Let firm 1 behave as a leader and decide its own quantity q_1 as first-mover. Let $P(q_1 + q_2) = a - b(Q)$ a generic linear inverse market demand function or, in other words, the market price when the industry quantity is $Q=(q_1 + q_2)$. Yet, let's assume a constant marginal cost equals c , the same for both firms. Then, in order to solve for the Nash equilibrium of this game, we first need to take into consideration the follower's profit maximization problem from the leader's viewpoint. Indeed, the follower's profit (π_2) can be written as a function of the leader's quantity q_1

$$\pi_2(q_1, q_2) = q_2 [P(q_1 + q_2) - c] \quad (1)$$

The first-order condition for solving the follower's maximization problem implies

$$\max_{q_2} \pi_2(q_1, q_2) = \max_{q_2} q_2 [a - bq_1 - bq_2 - c] \quad (2)$$

which yields

$$q_2 = f(q_1) = \frac{a - bq_1 - c}{2b} \quad (3)$$

This equation is also known as follower's reaction function since it computes the follower's best reaction to any given output choice by the leader (Carmichael, 2005).

Since the leader is aware to influence the follower's output choice and it can predict the quantity choice q_2 , the remaining leader's profit maximization problem just has to take into account the follower's reaction, and it can be stated as

$$\pi_1(q_1, q_2) = q_1 [P(q_1 + q_2) - c] \quad (4)$$

so that $q_2 = f(q_1)$

The first-order condition for solving the leader's maximization problem in the case of our linear (inverse) market demand function implies

$$\max_{q_1} \pi_1(q_1, f(q_1)) = \max_{q_1} q_1 [a - bq_1 - bf(q_1) - c] \quad (5)$$

which yields

$$q_1 = \frac{a - c}{2b} \quad (6)$$

$$\text{and } q_2 = \frac{a - c}{4b} \quad (7)$$

as Nash equilibrium of the Stackelberg duopoly game.

In the in-class experiment, the Nash equilibrium of the one-shot Stackelberg game predicts that the leader produces 6 units while the follower produces 3 units of output. This is a dynamic equilibrium because each player maximizes its own profit given its rival's output choice and no group has an incentive to deviate from its predicted choice (Carmichael, 2005). The common market price is 4 and the total industry output equals to 9 units. Given the total costs, the leader earns a profit of 18 and the follower a profit of 9.

The Nash equilibrium of the one-shot Cournot game assumes that firms set quantities simultaneously and in order to maximize profit each firm has to make assumption about the other firm's output choice (Gibbons, 1992). In this static model of duopoly, the profit maximization problem facing both duopolistic firms is a symmetric one because the production technology is the same for both firms and so the marginal cost. Hence, it can be stated as

$$\max_{q_1} \pi_1(q_1, q_2) = \max_{q_1} q_1 [a - bq_1 - bq_2 - c] \quad (8)$$

$$\max_{q_2} \pi_2(q_1, q_2) = \max_{q_2} q_2 [a - bq_1 - bq_2 - c] \quad (9)$$

which yields the two duopolistic firms' (symmetric) reaction functions as in equation (3). Solving the system

$$\begin{cases} q_1 = \frac{a - bq_2 - c}{2b} \\ q_2 = \frac{a - bq_1 - c}{2b} \end{cases} \quad (10)$$

yields

$$q_1 = \frac{a - c}{3b} = q_2 \quad (11)$$

as Nash equilibrium of the Cournot duopoly game.

In the in-class experiment, the Nash equilibrium of the one-shot Cournot game predicts that each player produces 4 units of output. Also this is an equilibrium, such that each player's output choice is the best response to any arbitrary output choice of

the other player (Holt, 2007; Hemenway, Moore, and Whitney, 1987). This simultaneous-move equilibrium yields a common market price of 5 and a total industry output of 8 units. The profit for each group is equal to 16.

Note that in the Nash equilibrium of the one-shot Stackelberg game the total industry production is greater than the total industry production in the Nash equilibrium of the one-shot Cournot game and the market price is lower in the former than in the latter. In terms of social welfare, because of the greater industry production and the lower price, consumers surplus must be higher in the Stackelberg game while producers surplus is expected to be higher in the Cournot game. It is also true that producers surplus must be lower in the Stackelberg game while consumers surplus is expected to be lower in the Cournot game. The fact that consumers are better off implies that producers are worse off in the Stackelberg than in the Cournot game (Connolly and Munro, 1999). Noteworthy, in the Stackelberg game, the leader could choose its Cournot quantity, in which case the follower's best reaction would have been its own Cournot quantity too. Thus, in the Stackelberg game, the leader's profit must be higher than that in the Cournot game, but because of the market price is lower in the Stackelberg game then the total industry profits are lower. So, the fact that the leader is better off implies that the follower is worse off in the Stackelberg than in the Cournot game. The latter point highlights a particular feature of the game theory, that sometimes having more information could actually make a player worse off (Gibbons, 1992). Indeed, in the Stackelberg game, the follower knows that the leader knows the follower's best reaction to any given choice of the leader.

If the theoretical Nash equilibria are realized in every round of the whole game played during the in-class experiment, each group and group's member too will earn 5 extra-credit points and increase its final course grade by 5 percent (100 points are available from the written test).

3. Description of the Experiment

In order to run the experiment, students in the class are divided into eight small groups of three people (firms) and then randomly matched across four duopolistic industries. In each round, each group is paired with the same other group (i.e. fixed matching). The game has a finite number of rounds but students do not know how many rounds the game will last. This prevents them from having weird behavior due to the fact that the game is in its last round (Raguseo and Horehájová, 2009). Moreover, this is a good approximation of the reality because normally firms do not know ex-ante when the market for their products will dry up (Meister, 1999). In order not to allow students to predict when the game will be over, it was indicated on the in-class experiment worksheet that several more rounds were really going to be played. For every round, each group has to decide its profit maximizing output level. When decisions are taken, the market price is unknown because it depends on the total output produced in the industry, that is the sum of the quantity produced by both duopolists. In the first two rounds, a sequential-move quantity-setting duopoly game is played between each pair of groups, which alternatively play the role of the

Stackelberg quantity leader. This has been included in order to test the theoretical knowledge learned by students from the previous lecture. Moreover, such an approach has been thought for allowing students to practically understand the reaction mechanism behind the concept of best-response function. In the remaining rounds, a simultaneous-move quantity-setting game à la Cournot is played between groups, where each group, when deciding, has to make assumptions about the rival's decision.

Before to run the in-class experiment, I distribute one worksheet to each group of students (firm) in the duopolistic industry. The inverse market demand function and the cost function are known in advance by the students. In the cost function there are only variable costs. The production technology is the same for all firms and so is the marginal cost. Students have information about the capacity constraint, but they do not know in advance the market price. The common market price, at which it is possible to sell all the units produced in any round, is determined by the total industry production. Therefore, it depends not only on the decision of a single group, but also on the decision of the other group. So, what makes the experiment interesting from the instructor's viewpoint is to discover how students interact when they have to decide their own strategy. And, since they have to make assumptions about the rivals' strategy in a repeated number of rounds, they should be able not only to strategically think in an introspective way but also to learn from the past rounds. Then, for every round, each group reports its quantity, the other group's quantity, the common market price, its profit and the other group's profit. The use of such information has also been thought as a further stimulus to rationally think about strategic interaction between firms in a duopolistic market. Since students can see how their and their rival's profit (and mostly extra-credit) changes at any round of the game, they can also better judge if their strategy is a winning one. To calculate if they are producing the profit maximizing level of output, students are allowed to use calculators. Moreover, students are not explicitly forbidden to tacitly collude although exchange of information between paired groups in any manner is not allowed.

To stimulate students to maximize profit, each student earns extra credit depending on the whole group's average profit yielded over the game. Extra credits are points that count towards the final assessment in the course. All students belonging to the same group are awarded the same number of extra credit points. For this experiment, each duopolistic industry is endowed with 10 extra credit points. The higher the average profit earned by a group, the more extra credit points are earned by each member of the group. Consequently, students would have an incentive to maximize their group's profit because the more extra credit points they earn, the more easily they will pass the exam of microeconomics. Note that the final course grade is fundamentally based on a written test (100 credit points), then the extra credit points (up to 10) from the in-class experiment are added (see annex 1 for the rules of the game).

4. Outcome of the Experiment

This experiment, which was conducted during one of the classes of microeconomics, lasted ten rounds. In the first two rounds, when a sequential-move game was played, the Nash equilibrium was reached in two of the four two-firm industries. In the following rounds, when a simultaneous-move game was played, only one “perfect” industry reached the Nash equilibrium already at the first round and then the profit maximizing equilibrium quantity was repeatedly produced for all the remaining rounds. The firms in the other industries were producing too much and, being the common price determined by the sum of quantities produced, earning too low profits. For instance, in the second round the average industry quantity was 5,4 and the average profit equals 5,1 (see annex 2 for the detailed outcome of the experiment).

In the third round, several groups kept production close to the same level as the previous round.

In the fourth round, the average quantity decreased to 4,7 which is not surprising following two rounds with low profit. All industries but one, notably reduced their output level. Due to the increased common price the average profit per each group notably rose. In the sole industry where the total quantity and price remain unchanged, the group that increased the output earned a higher profit cheating on the group that reduced the output.

In the fifth round, in the “latter” industry, the consequence of the free-rider behaviour of the group that increased the quantity can be figured out. The group that reduced the output in the previous round, now sharply increases its output, from 5 to 8 as a sign of punishment against the rival group. Due to the very high production level the resulting market price is zero (it can’t be negative) and no profits are earned by either group.

In the fifth and sixth round, all other groups kept their production levels very close to their previous round levels moving toward the profit maximizing equilibrium quantity.

In the sixth round, apart from the “perfect” industry, the profit maximizing equilibrium quantity is also produced in the “latter” industry, which is under consideration.

In the seventh round, the consequence of the punishment seems to yield a profitable result in the “latter” industry where a total amount of 6 units of output is produced, which corresponds to the monopolist profit maximization output level. Each group produces 3 units and earns a profit equals 18 each. Anyway, this joint-profit maximization output level is not a Nash equilibrium, since each group is tempted to produce more. This can be seen observing what happened in a “new” industry. In the seventh round, as well as it was in the sixth round, in this “new” industry, the two groups of students acting as players were able to coordinate on the monopolist

output level below the duopoly theoretical prediction. Holt (2007) argues that the reason has to be found in the fixed nature of the matching, whereby if matching were random the players of the experiment couldn't coordinate. Even though generally accepted, the previous argumentation doesn't take into account the incentive to cheat on the rival through an output expansion. Perhaps, this can appear clearer observing the behaviour of the "new" and "latter" duopolistic industries in the eighth round.

In the eighth round, the groups of students playing in both these industries seek to cheat on the rival, at the same time, increasing the output produced. The resulting raise in total industry output decreased the common price and, as a consequence, the individual firm's profit shrank.

In the ninth round, the profit maximizing equilibrium quantity of 4 units per group is produced in all but one industry. This is the Nash equilibrium of this simultaneous-move game, meaning that now no group has interest to modify its own output decision being aware of its rival's decision.

As proof, we could see that during the last round, unbeknownst to the students, the same strategy as the previous round had been played. At the very end of the game, equilibrium was reached in three of the four duopolistic industries.

5. Concluding Remarks

The following class, students were asked to write an anonymous report about their experience with the economic experiment. Students generally reported that they had been enthusiastic about the new experience and the innovative way to teach economics. They emphasized:

It was an original, interesting and helpful way for a better understanding of the oligopolistic market structure to link the theory with a more practical example. It was stimulating and challenging for students to seek to develop strategies for attempting to maximize their profit and extra credit points. Playing the game helped students to have a more concrete idea of what in reality appears to be the maximization-profit problem facing a duopolist. The possibility of gaining some extra credit points, proportionally to the average profit earned by each group over the game, has been detected as a good motivation to be more careful when taking into consideration the potential rival's reaction. It also stimulated both a competitive atmosphere between groups and a cooperative spirit within the same group. Students also considered the calculations of revenue, costs and profits' values as a good exercise for approaching microeconomics concepts. At the beginning of the game, some students had some troubles with understanding the rules and filling in the worksheet but as far as the rounds progressed they did better and better. A few students felt particularly irritated when their incapacity to control their rival's reactions yielded very small profit.

In my opinion, the use of in-class experiments is an easy, funny, smart and a very efficient way to teach students about oligopoly. Students participating to the in-class experiment were generally enthusiastic about it. The in-class experiment described here has also been developed in order to provide to the economics educators a simple method with which efficiently copy with one of the most difficult topics in microeconomics. Indeed, the interpretation of the final exam results also showed that students on average performed better on oligopoly questions than on other microeconomics questions. As suggestion for further research, this duopoly game could be extended conducting the experiment before the theoretical lecture has taken place or randomly matching the pairs of groups at each round.

Annex 1

In-class experiment worksheet

Industry: _____ group: _____
Members' name: _____ Date: _____

Inverse market demand function: $P = 13 - (q_1 + q_2)$													
Cost function: _____ $TC = q$ $MC = 1$													
Capacity constraint: _____ 10													
Determination of Price as a function of Total Quantity													
$Q = q_1 + q_2$	1	2	3	4	5	6	7	8	9	10	11	12	13+
Price	12	11	10	9	8	7	6	5	4	3	2	1	0
Round	Group A quantity			Group B quantity			Market price			Group A profit			Group B profit
1	L			f									
2	F			L									
3													
4													
5													
6													
7													
8													
9													
10													
11													
12													
13													
14													
15													
16													

Rules of the game

Students in the class are divided into small groups. I distribute one in-class experiment worksheet to each group in the class. Please, write the name of each group's member on the top of the worksheet. You are going to experiment what we theoretically discussed yesterday about oligopoly and game theory! Your choice is about your own quantity to produce each round. Remember, that the market price depends on the total industry output. Thus, your profit will depend not only on your output, but also on your rival's output. In the first round, a sequential-move quantity-setting duopoly game is played, where group A is the Stackelberg quantity leader (L) and group B is the follower (f). In the second round, once again a sequential-move game is played, where group A plays the role of the follower (f) while now group B is the Stackelberg quantity leader (L). In the remaining rounds, a

simultaneous-move quantity-setting game á la Cournot is played between matched groups. You do not know how many rounds the game will last. You have information on the inverse market demand and the cost functions. Note that there are not fixed costs. Also you have a capacity constraint of 10 units of output to be produced. Then, for every round, each group has to reports its quantity, the other group's quantity, the common market price of the good, its profit and the other group's profit. In order to calculate the profit maximizing level of output you are allowed to use calculators.

Annex 2

Outcome of the experiment

Round	"LATTER" INDUSTRY					"NEW" INDUSTRY					"RESIDUAL" INDUSTRY					"PERFECT" INDUSTRY				
	q1	q2	P(q1+q2)	□□	□□	q1	q2	P(q1+q2)	□□	□□	q1	q2	P(q1+q2)	□□	□□	q1	q2	P(q1+q2)	□□	□□
1	5	5	3	10	10	4	2	7	24	12	6	3	4	18	9	6	3	4	18	9
2	5	7	1	0	0	5	8	0	-	-	3	6	4	9	18	3	6	4	9	18
3	6	5	2	6	5	7	5	1	0	0	6	7	0	-	-	4	4	5	16	16
4	5	6	2	5	6	5	4	4	15	12	5	5	3	10	10	4	4	5	16	16
5	8	6	0	-	-	4	2	7	24	12	4	5	4	12	15	4	4	5	16	16
6	4	4	5	16	16	3	3	7	18	18	4	3	6	20	15	4	4	5	16	16
7	3	3	7	18	18	3	3	7	18	18	4	4	5	16	16	4	4	5	16	16
8	4	5	4	12	15	4	4	5	16	16	4	6	3	8	12	4	4	5	16	16
9	6	6	1	0	0	4	4	5	16	16	4	4	5	16	16	4	4	5	16	16
10	-	-	-	-	-	4	4	5	16	16	4	4	5	16	16	4	4	5	16	16

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