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Wei-Bin Zhang[1]

WEALTH AND INCOME DISTRIBUTION AMONG HETEROGENEOUS HOUSEHOLDS IN A NEOCLASSICAL GROWTH MODEL WITH ONE CAPITAL AND MULTIPLE CONSUMER GOODS

This paper brings heterogeneous households and heterogeneous sectors in the traditional Solow-Uzawa neoclassical growth model. It is influenced by the neoclassical growth theory and the post-Keynesian theory of growth and distribution. The model deals with not only the economic structure, but also income and wealth distributions between heterogeneous households. We show the transitional paths of the variables and determine the economic equilibrium for a two-group and two-sector economy. We also conduct out comparative dynamic analysis with regard to human capital, propensity to save, and population size. We demonstrate, for instance, that rises in the skilled and unskilled groups' the propensities to save have the same impacts on the national and sectoral production levels, input factor distributions, rate of interest, wage rate, and national wealth; but the skilled (unskilled) group's wealth and consumption level are reduced (increased) and the wealth and consumption gaps between the two groups are diminished (enlarged) as a long-term consequence of a rise in the unskilled (skilled) group's propensity to save.

JEL: O41; E25

1. Introduction

This study deals with two important issues in economic growth through generalizing the traditional neoclassical growth theory. The one issue is related to economic structural change. It is well known that the neoclassical economic growth theory with the Solow one-sector model as the core model is very poor at explaining economic structural change. Moreover, the Solow model does not have a proper micro economic mechanism for determining saving and consumption of heterogeneous goods and services. The other issue is related to wealth and income distribution. The neoclassical growth theory is almost silent on this important issue. A main reason for the lack of formal modeling in distributional

[1] *Prof. Wei-Bin Zhang, Ritsumeikan Asia Pacific University, home phone: 0977-73-9787; office phone: 0977-78-1020; fax: 0977-78-1123; E-mail: wbz1@apu.ac.jp.*

issues is that the neoclassical growth theory still lacks a proper analytical framework for effectively dealing with growth issues with heterogeneous households and physical capital accumulation. The Ramsey-type growth models of capital accumulation which are the core model in the neoclassical growth theory with micro foundation cannot effectively analyze distributional issues.

Modern economic systems are characterized of increasing complexity of economic interactions and rapid economic structural changes. The importance of economic structural change has long time ago been recognized by, for instance, Kaldor (1963), Kuznets (1971), and Maddison (1980). It is commonly observed among many economies that the shares of output and labor first shifted from agriculture to industry, and later from industry to service. It was also commonly considered that productivity growth in the service sector was lower than in the other sectors. The so-called cost disease of services proposed by Baumol (1967) foresaw that services prices would increase more rapidly than the prices of commodities. In recent years developed economies, like the European Union, Japan and the US, have predominated by the service sector. Unlike in the Uzawa two-sector model which is most well-known for dealing with economic structural change in the neoclassical growth theory, services in reality cannot be properly analyzed by a single homogeneous sector. There is great heterogeneity in growth rates among service activities. For instance, As Jorgenson and Timmer (2011: 1) observe, “Personal, finance, and business services have low productivity growth and increasing shares in employment and GDP. By contrast, shares of distribution services are constant, and productivity growth is rapid. We find that the labour share in value-added is declining, while the use of ICT capital and skilled labour is increasing in all sectors and regions.” Their empirical study also confirms that the traditional trichotomy among agriculture, manufacturing, and services is not very relevant for describing modern economies. They ask for greater emphasis to be focused on individual service sectors for revealing the complexity of economic growth and structural change. The purpose of this study is to bring multiple consumer good and service sectors in the Uzawa sector, generalizing the single consumer good sector to any number consumer good and service sectors. It should be noted that there are many studies on various aspects of multiple sector economies (see, for instance, Stiglitz, 1967; Mino, 1996; Obstfeld and Rogoff, 1996; Drugeon and Venditti, 2001; Temple, 2005; Ngai and Pissarides, 2007; Restuccia *et al*. 2008; Kapur, 2012). This study deviates from the traditional literature by using Zhang’s approach to household decision on consumption and saving (Zhang, 1993).

Not only economic structure, but also wealth and income distribution and their interactions with economic growth are concerns of this study. This paper brings heterogeneous households into the neoclassical growth theory. Our approach to including heterogeneous households is influenced by the post-Keynesian theory of growth and distribution (e.g., Pasinetti, 1974; and Salvadori, 1991). Most of the Post-Keynesian growth models with heterogeneous households is developed within one-sector production sector frameworks. It should be noted that Stiglitz (1967) developed a growth model with two sectors and two classes, by integrating the post-Keynesian theory and Uzawa two-sector model. Nevertheless, there are few further studies along the research line. This study proposes an economic structural model similar to the Stigliz model but in an alternative approach to household behavior. Another model which affects this study is the growth model by Zhang (1996). In the neoclassical one-sector growth model, the population is classified into heterogeneous households. This study is to integrate Zhang's growth model and Uzawa's two sector growth model. The rest of the paper is organized as follows. Section 2 defines the growth model with heterogeneous households and multiple sectors. Section 3 deals with dynamic properties of the model and simulates the motion of the economy. Section 4 carries out comparative dynamic analysis with regard to human capital, propensities to save, and population sizes. Section 5 makes concluding remarks.

2. The Multi-Sector Model with Heterogeneous Groups

The economy consists of $m + 1$ distinct production sectors. There are one capital good sector and m consumption goods sectors. Capital depreciates at a constant exponential rate δ_k. There are n groups of households, indexed by v, $v = 1, \cdots, n$, each type with a fixed number of the population, denoted by $\overline{N}_v$. As far as economic structure is concerned, the case of $m = 0$ and $v = 1$ corresponds to the Solow model, and the case of $m = 1$ and $v = 1$ corresponds to the Uzawa model. The aggregated labor force is given by

$$N = \sum_{v=1}^{n} h_v \overline{N}_v ,$$

where h_v are the fixed level of human capital of group v. The production functions are taken on the following Cobb-Douglas form

$$F_j(t) = A_j K_j^{\alpha_j}(t) N_j^{\beta_j}(t), \ \ \alpha_j + \beta_j = 1, \ \ \alpha_j, \beta_j > 0, \ \ j = i, 1, \cdots, m$$

where F_j are the output of sector j, $K_j(t)$ and $N_j(t)$ are respectively the capital and labor employed by sector j, and the subscripts i and q ($q = 1, \cdots, m$) denote the capital good sector and the q consumption good sector. We have

$$f_j(k_j(t)) = A_j k_j^{\alpha_j}(t), \ \ A_j > 0, \ \ 0 < \alpha_j < 1, \ \ j = i, 1, \cdots, m, \tag{1}$$

where

$$f_j(t) \equiv \frac{F_j(t)}{N_j(t)}, \ \ k_j(t) \equiv \frac{K_j(t)}{N_j(t)}.$$

Markets are competitive. We assume that the capital good serves as a medium of exchange and is taken as numeraire. The price of consumption good q is denoted by $p_q(t)$. The rate of interest $r(t)$ and wage rates $w_v(t)$ are determined by markets. The marginal conditions are given by

$$r(t) + \delta_k = \alpha_i A_i k_i^{-\beta_i}(t) = \alpha_q A_q p_q k_q^{-\beta_q}(t),$$

$$w_v(t) = h_v \beta_i A_i k_i^{\alpha_i}(t) = h_v \beta_q A_q p_q(t) k_q^{\alpha_q}(t), \quad q = 1, \cdots, m. \tag{2}$$

As labor and capital are fully employed, we have

$$n_i(t)k_i(t) + \sum_{q=1}^{m} n_q(t)k_q(t) = k(t), \;\; n_i(t) + \sum_{q=1}^{m} n_q(t) = 1 \tag{3}$$

where

$$k(t) \equiv \frac{K(t)}{N}, \;\; n_j(t) \equiv \frac{N_j(t)}{N}, \;\; j = i, 1, ..., m.$$

Consumers make decisions on choice of consumption levels of services and commodities as well as on how much to save. Following Zhang (1996), we describe behavior of the consumers as follows

$$Max\, U_v(t) = s_v^{\lambda}(t) \prod_{q=1}^{m} c_{vq}^{\xi_{vq}}(t), \;\; \lambda_v, \xi_{vq} > 0, \;\; \lambda_v + \sum_{q=1}^{m} \xi_{vq} = 1,$$

$$\text{s.t.: } s_v(t) + \sum_{q=1}^{m} p_q(t)c_{vq}(t) = \hat{y}_v(t) \equiv r(t)\bar{k}_v(t) + w_v(t) + \bar{k}_v(t), \tag{4}$$

where $s_v(t)$ and $c_{vq}(t)$ are respectively the level of savings and the consumption level of consumption good q, λ_v and ξ_{vq} are respectively the representative household's propensity to save and to consume consumption good q, and $\bar{k}_v(t)$ and $\hat{y}_v(t)$ are respectively the wealth and the disposable income of the representative household of group v. Maximizing U_v subject to the budget constraints yields

$$s_v(t) = \lambda_v \hat{y}_v(t), \;\; p_q(t)c_{vq}(t) = \xi_{vq} \hat{y}_v(t), \;\; v = 1, \cdots, n, \;\; q = 1, \cdots, m. \tag{5}$$

According to the definition of $s_v(t)$, the representative household's wealth changes according to the following differential equation

$$\dot{\bar{k}}_v(t) = s_v(t) - \bar{k}_v(t).$$

Substituting $s_v(t)$ in (5) into the equations yields

$$\dot{\bar{k}}_v(t) = \lambda_v \, \hat{y}_v(t) - \bar{k}_v(t). \tag{6}$$

The output of the consumer goods sector is consumed by the households. That is

$$\sum_{v=1}^{n} c_{vq}(t)\bar{N}_v = F_q(t), \;\; q = 1, \cdots, m. \tag{7}$$

As output of the capital good sector is equal to the depreciation of capital stock and the net savings, we have

$$S(t) - K(t) + \delta_k \, K(t) = F_i(t), \tag{8}$$

where

$$S(t) = \sum_{v=1}^{n} s_v(t)\bar{N}_v, \;\; K(t) = \sum_{v=1}^{n} \bar{k}_v(t)\bar{N}_v.$$

We have thus completed the model.

3. The dynamic system

This section studies dynamic properties of the multiple sectors group model. The system has many variables and contains many equations. But it is now shown that the whole system is described by n differential equations. First, we introduce a $(n - 1)-$dimensional vector $\{k_v(t)\} = \{k_2(t), \ldots, k_n(t)\}$.

Lemma 1

The dynamics of the economic system is given by the following n differential equations with $k_i(t)$ and $\{\bar{k}_v(t)\}$ as the variables

$$\dot{k}_i = \bar{\varphi}_1(k_i, \{\bar{k}_v\}),$$

$$\dot{\bar{k}}_v = \bar{\varphi}_v(k_i, \{\bar{k}_v\}), \;\; v = 2, \cdots, n, \tag{9}$$

in which $\bar{\varphi}_v$, $v = 1, 2,, \cdots, n$, are functions of $k_i(t)$ and $\{\bar{k}_v(t)\}$. Moreover, all the other variables are determined as functions of $k_i(t)$ and $\{\bar{k}_v(t)\}$ at any point of time by the following procedure: $k(t)$ by (A10) $\rightarrow$ $k_1(t)$ by (A9) $\rightarrow$ $\hat{y}_v(t)$ by (A7) $\rightarrow$ $k_q(t) = \overline{\alpha}_q\, k_i(t)$ $\rightarrow$ $n_q(t)$ by (A6) $\rightarrow$ $c_{vq}(t)$ and $s_v(t)$ by (5) $\rightarrow$ $n_i(t)$ by (A3) $\rightarrow$ $p_q(t)$ by (A2) $\rightarrow$ $r(t)$ and $w_v(t)$ by (2) $\rightarrow$ $f_j(t) = A_j\, k_j^{\alpha_j}(t), \;\; j = i, 1, \cdots, m \rightarrow$ $N_j(t) = n_j(t) N \rightarrow K_j(t) = k_j(t) N_j(t) \rightarrow F_j(t) = f_j(t) N_j(t)$.

We have explicitly expressed the dynamics in terms of $k_i(t)$ and $\{\bar{k}_v(t)\}$, it is straightforward to analyze dynamic properties of the system. Nevertheless, as it takes a large space to explicitly interpret behavior, we simulate the model to illustrate motion of the system. For simplicity, we are concerned with the two-sector two-group model, i.e., $m = 1$ and $n = 2$. Following Lemma 1, we can plot the motion of the economic system. We specify the production functions, the population structure, and preferences as follows

$$\begin{pmatrix} A_i \\ A_1 \end{pmatrix} = \begin{pmatrix} 2.2 \\ 1.8 \end{pmatrix}, \; \begin{pmatrix} \alpha_i \\ \alpha_1 \end{pmatrix} = \begin{pmatrix} 0.33 \\ 0.3 \end{pmatrix}, \; \begin{pmatrix} \overline{N}_1 \\ \overline{N}_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 18 \end{pmatrix}, \; \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}, \; \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0.85 \\ 0.7 \end{pmatrix}, \; \delta_k = 0.05 \cdot \quad (10)$$

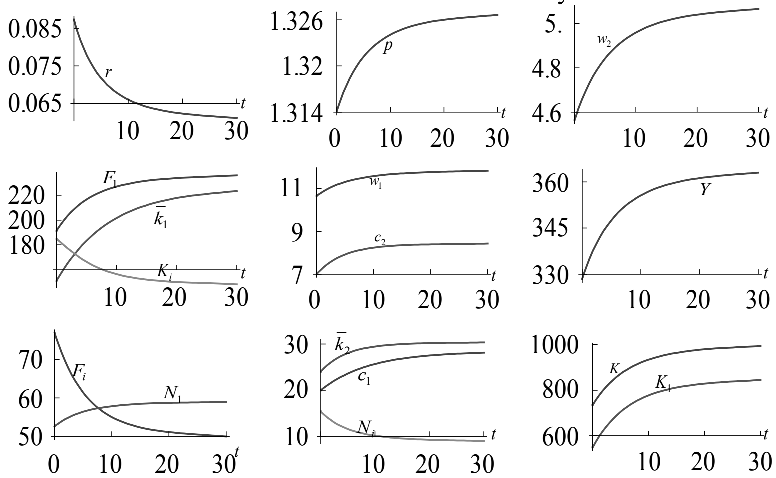
Group 1 has higher propensity to save and higher human capital level than group 2. The population size of group 1 is lower than group 2. For convenience of interpretation, we call group 1 and group 2 respectively skilled and unskilled groups. We specify initial conditions as follows

$$k_i(0) = 12, \;\; k_2(0) = 24.$$

The national economy is plotted in Figure 1.

Figure 1

The Motion of the Economic System



From Figure 1, the rate of interest falls in association with rising in the national wealth. The two groups' levels of consumption and wealth per household are all increased. Figure 1 demonstrates that all the variables move towards stationary states. This hints the existence of equilibrium. Our simulation identifies the equilibrium values of these variables as follows

$$r = 0.061,\ p = 1.33,\ K = 996.7,\ Y = 373.4,\ k_{1i} = 16.52,\ k_1 = 14.37,$$
$$N_i = 8.98,\ N_1 = 59.02,\ F_i = 49.83,\ F_1 = 236.4,\ w_1 = 11.83,\ w_2 = 5.07, \quad (17)$$
$$\bar{k}_1 = 225.24,\ \bar{k}_2 = 30.34,\ c_1 = 28.35,\ c_2 = 8.43.$$

Although the ratio of the wage rate gap is about 2.3, the ratio of the per capita wealth between the two groups is 7.4. The ratio of the consumption levels is about 3.4. In the long term, the labor force is concentrate in the consumer good sector and the sector is the main contributor to the national output. The eigenvalues are $\{-0.26, 0\}$.

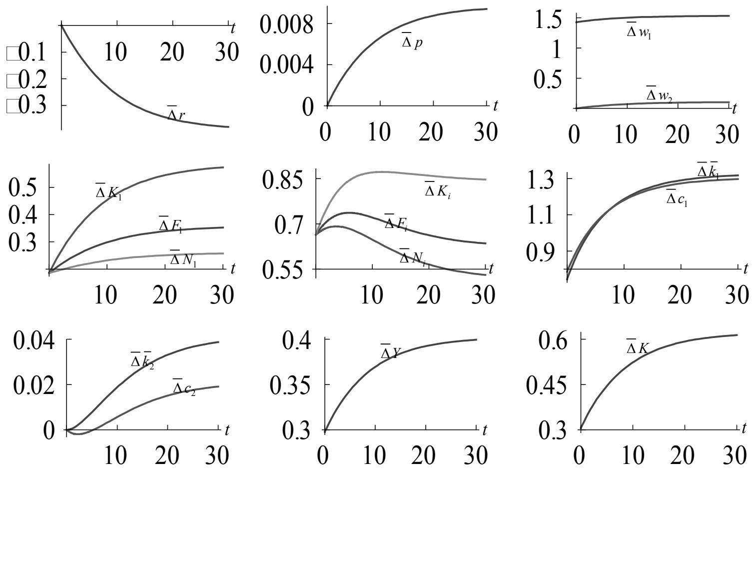
4. Comparative Dynamic Analysis in Some Parameters by Simulation

The previous section shows that the economic system has a unique equilibrium. This guarantees that we can make comparative dynamic analysis. This section examines effects of exogenous changes in some parameters on the economic system. As the system contains so many variables which tangle nonlinearly with each other over time, it is actually not easy to accurately interpret how all these variables interact over time.

Improvements in group 1's human capital

Figure 2

An Improvement in Group 1's Human Capital

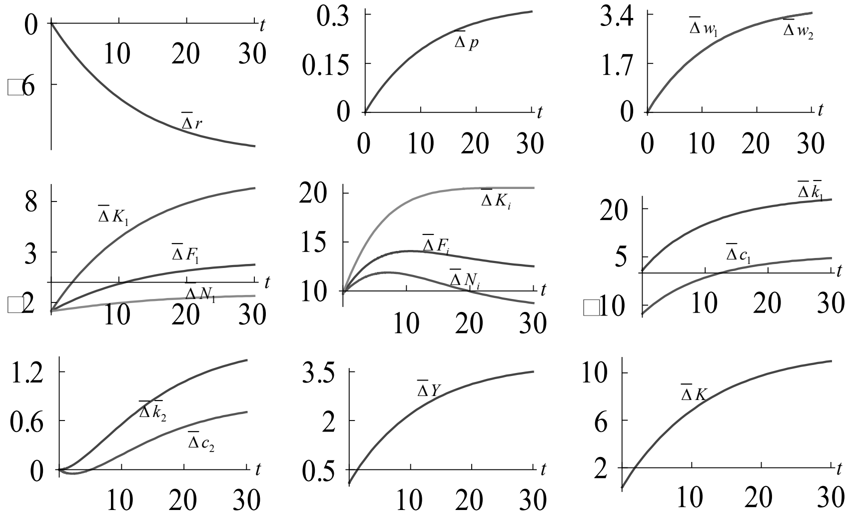


We first examine the case that skilled group's human capital is enhanced as follows: $h_1: 7 \Rightarrow 7.1$. The simulation results are plotted in Figure 2. In the plots a variable $\overline{\Delta} x(t)$ means the change rate of the variable, $x(t)$, in percentage due to changes in some parameter value. The rate of interest is lowered, the other variables are augmented. The increase rates of skilled group's wage rate, consumption level and wealth are much higher than the corresponding variables for unskilled group's.

Augments in two groups' propensities to save

Figure 3

An Augment in Group 1's Propensity to Save

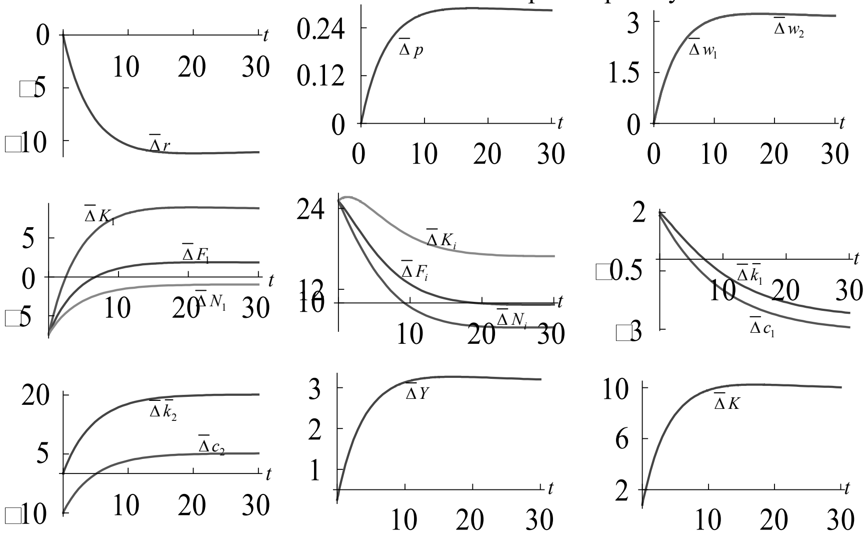


We now allow skilled group's propensity to save to be increased as follows: $\lambda_1: 0.85 \Rightarrow 0.87$. The simulation results are plotted in Figure 3. The enhanced propensity augments the national wealth and reduces the rate of interest. As more capital is desired, the capital good sector's output levels ad two input factors are increased. The price of consumer goods becomes higher. The two groups' wage rates are increased in the same rate. It should be noted that although they are increased in the same rate, skilled group's wage rate rises more than unskilled group's because skilled group's wage rate is higher than unskilled group's. This occurs because the rise in the wage rates is due to the change in the capital input which augments the productivity of labor. Some of the consumer good sector's labor force is shifted to the capital good sector. Initially the level of consumer good production falls, but subsequently rises. This occurs as initially both the labor and capital inputs lower. But subsequently as the economy accumulates more capital, the sector's capital is increased and but higher than the reduction in the labor input. The net result increases the output level of the consumer good sector. The skilled group's consumption level is lowered initially but subsequently enhanced. This occurs as initially the household saves more from the disposable income and reduces consumption. But subsequently the increased disposable income "covers" the reduced consumption. We conclude that in the long term a rise in skilled group's propensity to save benefits the economy as a whole as well as each household in every group. It should be noted that the change leads to enlarged gaps in per capita wealth and consumption level as well as wage rate. Hence, society becomes more unequal

The impacts of the following rise in unskilled group's propensity to save: $\lambda_2 :\ 0.7 \Rightarrow 0.73$ are illustrated in Figure 4. Comparing Figure 3 and 4, we see that as far as the change directions in the variables are concerned, the effects in the production level, input factor distributions, rate of interest, wage rate, national output, and national wealth are the same when the skilled group and unskilled group augment the propensities to save. The main difference is the long term impact on the per capita wealth and consumption level. The skilled group's wealth and consumption are reduced as the unskilled group saves more from the disposable income. As the unskilled saves more, the national wealth is augmented and the rate of interested is reduced. As the lowered rate of interest diminishes the return from the skilled group's wealth, the net effect in the increased wage rate and lowered rate of interest reduces the skilled group's per capita wealth and consumption level. Hence, a rise in the unskilled group's propensity to save reduces the wealth and consumption gaps between the two groups, even though the skilled group economically hurts by the other group's preference change.

Figure 4

A Rise in the Unskilled Group's Propensity to Save

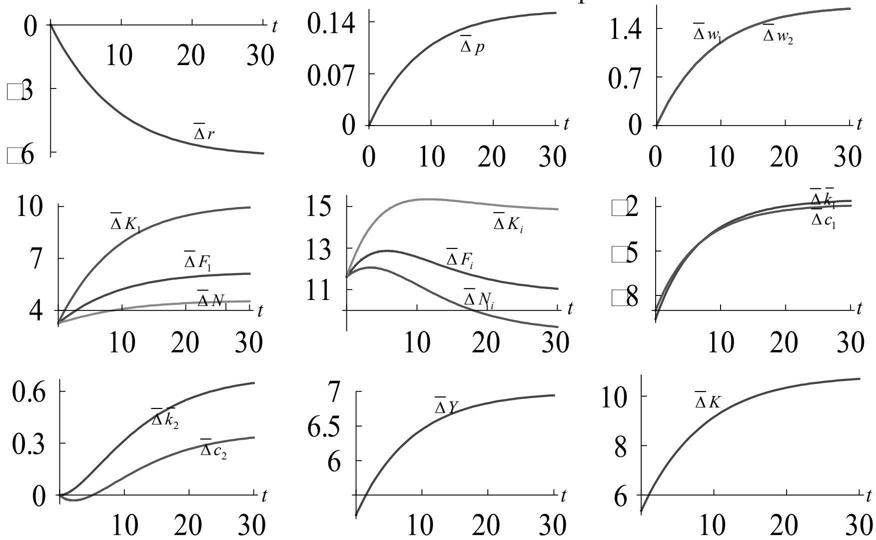


A rise in two groups' population sizes

An important issue in, for instance, the US is how new immigrants may affect the current citizens. Immigrants may have different preferences and human capital. It is important to see what happen to the economy when the immigration policy requires immigration applicants to have high skills. The Solow model, for instance, predicts that there is no impact on the current population in terms of per capita wealth and consumption level. Although our model is based on the Solow model and is characterized of constant returns to scale in production, our model would predict the same impact as the Solow model if we treat the population homogeneous. We now examine what will happen to the economy when the population of each group varies. First, we increase the skilled population in the following way: $N_2: 2 \Rightarrow 2.5.$ As the country has more skilled immigrants, the current skilled population's wealth and consumption level are lowered, even though the unskilled household's wealth and consumption level are increased. Both groups' wage rates are increased in association with falling in the rate of interest. The total levels of the two inputs are increased. The output levels and two input factors of the two sectors are increased. The national output is augmented in association with rising in the price of consumer good.

Figure 5

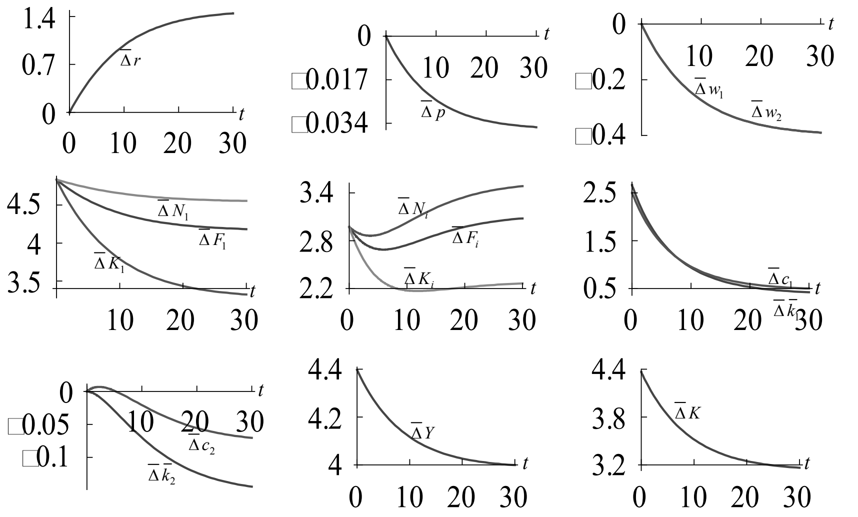
A Rise in the Skilled Population



We now simulate the motion of the economy when the unskilled population changes as follows: $N_2: 18 \Rightarrow 19.$ As far as change directions in the variables are concerned, the change in the unskilled population has the same impact on the national wealth, national output, the per capita wealth and consumption levels, and the output and two input factors of the two sectors as change in the skilled population. Nevertheless, the effects on the rate of interest, the price of consumer good, and wage rates are the opposite in changes of the two groups' population sizes.

Figure 6

A Rise in the Unskilled Population Size



5. Conclusions

This study brought heterogeneous households and heterogeneous sectors in the traditional Solow-Uzawa neoclassical growth models. We succeeded in formally modeling the complicated economic issues with Zhang's utility function. We illustrated the motion of a two-group-two sector economy and determined the economic equilibrium. We also carry out comparative dynamic analysis with regard to human capital, population and propensity to save. As our model is influenced by the core models in the neoclassical and post-Keynesian approaches and there are different forms of extensions and generalizations of these models, we may broaden the validity of our model on the basis of these extensions and generalizations. We may also further study the model by using other forms of production or utility functions.

Appendix: Proving Lemma 1

We now show that the motion of the system is determined by n differential equations. First, from the equations (2), we get

$$k_q = \overline{\alpha}_q k_i , \tag{A1}$$

where

$$\overline{\alpha}_q \equiv \frac{\beta_i \alpha_q}{\beta_q \alpha_i}, \quad q = 1, \cdots, m.$$

By $k_q = \overline{\alpha}_q k_i$ and $\beta_i f_i = \beta_q p_q f_q$, we solve

$$p_q = \frac{\beta_i A_i}{\beta_q \overline{\alpha}^{\alpha_q} A_q} k_i^{\alpha_i - \alpha_q}. \tag{A2}$$

According to the definitions of S, K, s_v, and $\bar{k}_v$, we have

$$S - \delta K = \sum_{v=1}^{n} s_v \overline{N}_v - \delta N k,$$

where $\delta \equiv 1 - \delta_k$. From this equation, (8), and $s_v = \lambda_v \hat{y}_v$, we obtain

$$n_i = \frac{\sum_v \lambda_v \bar{n}_v \hat{y}_v - \delta k}{f_i}, \tag{A3}$$

where $\bar{n}_v \equiv \overline{N}_v / N$. By $p_{vq} c_{vq} = \xi_{vq} \hat{y}_v$ and the equations (A2), we have

$$c_{vq} = \frac{\xi_{vq} \hat{y}_v f_q^{'}}{f_i^{'}}. \tag{A4}$$

From (A4) and (7), we have

$$\sum_{v=1}^{n} c_{vq} \bar{n}_v = n_q f_q. \tag{A5}$$

Solve n_q from (A4) and (A5)

$$n_q = \frac{\alpha_q}{k_q f_i^{'}} \sum_{v=1}^{n} \xi_{vq} \bar{n}_v \hat{y}_v, \quad q = 1, \cdots, m. \tag{A6}$$

From the definitions of $\hat{y}_v$ and (2)

$$\hat{y}_v = \varphi_v\left(k_i, \bar{k}_v\right) \equiv \varphi_0\left(k_i\right)\bar{k}_v + h_v \beta_i k_i^{\alpha_i}, \tag{A7}$$

in which

$$\varphi_0\left(k_i\right) \equiv \alpha_i A_i k_i^{-\beta_i} + \delta.$$

According to the definitions of $\bar{n}_v$, $\bar{k}_v$, and k, we have

$$\sum_{v=1}^{n} \bar{n}_v \bar{k}_v = k. \tag{A8}$$

From (A8), we solve

$$\bar{k}_1 = \left(k - \sum_{v=2}^{n} \bar{n}_v \bar{k}_v \right) \frac{1}{\bar{n}_1}. \tag{A9}$$

Substituting (A3), (A6) and (A7) into (3), we obtain

$$k = \varphi\left(k_i, \{\bar{k}_v\}\right) \equiv \frac{h_1 \bar{n}_1 \beta_i k_i^{\alpha_i} - \varphi_0 \sum_{v=2}^{n} \bar{n}_v \bar{k}_v + \left(\bar{n}_1 / \bar{\alpha}_1\right) \sum_{v=2}^{n} \bar{\alpha}_v \varphi_v}{\left(\delta + f_i / k_i\right) \bar{n}_1 / \bar{\alpha}_1 - \varphi_0}, \tag{A10}$$

where we use (A9) and

$$\bar{\alpha}_v \equiv \left(\lambda_v + \frac{1}{\alpha_i} \sum_{q=1}^{m} \alpha_q \xi_{vq} \right) \bar{n}_v.$$

From (A10) we see that k can be expressed as a function of k_i and $\{\bar{k}_v\}$ at any point of time. From the equations (6) and (A7), we obtain

$$\dot{\bar{k}}_v = \bar{\varphi}_v\left(k_i, \{\bar{k}_v\}\right) \equiv \lambda_v \varphi_0 \bar{k}_v + \lambda_v h_v \beta_i k_i^{\alpha_i} - \bar{k}_v, \quad v = 2, \cdots, n. \tag{A11}$$

The above equation contains only k_i and $\{\bar{k}_v\}$.

Taking derivatives of (A9) with respect to t and then substituting

$$\dot{\bar{k}}_1 = \left(\lambda_1 \varphi_0 - 1\right) \bar{k}_1 + h_1 \beta_i k_i^{\alpha_i},$$

into the resulted equation, we obtain

$$\dot{k}_i = \bar{\varphi}_1\left(k_i, \{\bar{k}_v\}\right) \equiv \left(\frac{\partial \varphi}{\partial k_i}\right)^{-1} \left[\left(\lambda_1 \varphi_0 - 1\right) \left(\varphi - \sum_{v=2}^{n} \bar{n}_v \bar{k}_v \right) + \bar{n}_1 h_1 \beta_i k_i^{\alpha_i} - \sum_{v=2}^{n} \left(\frac{\partial \varphi}{\partial \bar{k}_v} - \bar{n}_v \right) \bar{n}_v \bar{\varphi}_v \right]. \tag{A12}$$

The above equation contains k_i and $\{\bar{k}_v\}$. The n differential equations (A11) and (A12) contain n variables k_i and $\{\bar{k}_v\}$. Summarizing the above analysis, we proved the lemma.

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