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PENSION CAPITAL INVESTMENT IN THE CONTEXT OF A PRIVATE PENSION FUND

An analysis of the pension legislation has been performed, particularly in the field of restrictions on asset allocation into various funds, and a model for real investment profile has been proposed that secures minimal risk at the given profitability and satisfies legal requirements, as well as immunizes the sub-portfolio of risk-free securities from changes in the market interest rate. The numerical experiment has been carried out with and without regard to transaction expenses. It has been estimated that frequent renegotiation of the portfolio leads to a decrease in profits from investing as a result of transaction expenses.

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Introduction

The development of market relations in Russia has promoted creation of various financial institutions differing from each other in status, specificity of functioning, social and economic importance. Banks, insurance and investment companies, non-state (i.e. private) pension funds, etc. are assigned to the most widespread financial institutions of Russia. The given research views the non-state pension fund (NPF) in detail as one of the most important social financial institutions involved in the pension fund scheme of Russia, along with the Pension Fund of the Russian Federation and managing companies.

The program for further development of the country's pension system accepted by the Russian Federation Government implies necessity of a substantially enhancing the non-state pension funds' role and responsibility for providing the Russian citizens with a decent pension. As opposed to the Pension Fund of Russia, whose main task lies in ensuring the minimal level of the citizens' pension, the most important goal of the non-state pension fund activity is set to be the one of securing pension capital of the citizens and providing

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the pensioner with a maximum possible level of the income loss replacement after retirement (Baskakov, 2007, p. 15-24; Solov'ev, 2011, p. 1-6).

A non-state pension fund is a special organizational-legal form of a noncommercial organization for social security whose activity includes operating as a compulsory pension insurance carrier in accordance with the Federal Law “On the compulsory pension insurance in the Russian Federation” and agreements on compulsory pension insurance. The NFP activity as a compulsory pension insurer includes accumulating pension capital funds, organizing investment of pension capital funds, accounting the pension capital funds of the insured parties, assigning and paying out the funded component of the retirement pension to the insured parties, etc.

The scope of this work covers the NFP activity in the *compulsory pension benefits*. However, it must be mentioned that the NPF also involves two large strands like non-state pension benefits and professional pension insurance.

Pension capital is formed at the expense of:

- assets, paid off ahead of schedule from the Pension Fund of the Russian Federation to the NPF upon the application of the insured party and not yet transferred to the managing company;
- assets transferred to the managing company trust management by the fund;
- assets received by the fund from the managing companies for payment to the insured parties or their legal successors and not yet transferred for payout reserve formation, for paying out the old-age retirement pension, for carrying out the urgent pension payments, one-time payments, and payments to legal successors;
- payments transferred to the NPF by the preceding insurer due to conclusion of a compulsory pension insurance contract between the insured party and the NPF;
- assets received by the NPF from managing companies for further transfer to the Pension Fund of the Russian Federation or another fund and not yet transferred to the Pension Fund of the Russian Federation or other funds;
- parts of property intended for the fund’s statutory activity in case of forwarding this property to cover the negative result from investing the pension assets by the decision of the NPF council.

Any financial institution that carries out asset investment activity endeavors to maximize the profit from investment. Non-state pension funds are also oriented at receiving high yield when investing the pension capital funds, though they are legally limited by the following principles³:

- providing safeguard of the assets mentioned;
- providing profitability, diversification and liquidity of investment portfolios;

³ Federal Statute of 07 May 1998 №75-FZ “On private pension funds”.

- defining the investment strategy based on the objective criteria that can be evaluated numerically;
- reliability of securities;
- disclosure of the pension capital fund investment process for the organs of state and public monitoring and supervision, etc.
- professional management of investment process.

Analysis of pension legislation

Since 01 January 2002, the Pension System of the Russian Federation has been functioning within the framework of distributional-accumulative principle, remaining within the limits of laws.⁴ These laws estimate the circle of participants in the Russian pension system, the bases of state regulation of the compulsory pension insurance in the Russian Federation, grounds for emerging and the realization order of the retirement pension liability of the Russian Federation citizens, along with the legal foundation of relations in formation, and investment of pension capital funds intended for financing the funded component of the retirement pension.

At present, pension capital may be allocated in the following asset classes:

1. state securities of the Russian Federation;
2. state securities of the Russian Federation constituent entities;
3. bonds of Russian emitters;
4. stocks of Russian emitters created in a form of open joint-stock companies;
5. shares (stocks, fractions) of index investment funds that allocate funds into state securities of foreign countries, as well as bonds and stocks of other foreign emitters;
6. mortgage-backed securities issued in accordance with the legislation of the Russian Federation on mortgage-backed securities;
7. cash assets in Roubles on accounts in credit organizations;

⁴ Federal Statute of 07 May 1998 №75-FZ “On private pension funds”; The Russian Federation Government Decree of 30 June 2003 N 379 “On extra restrictions on the pension capital investment into various asset classes and determination of the maximum fraction of separate asset classes in the investment portfolio in accordance with Articles 26 and 28 of the Federal Statute “On capital investment for financing the funded component of the retirement pension in the Russian Federation” and Article 36.15 of the Federal Statute “On non-state pension funds”; The decree of 31 August 2002 N 652 “On adoption of regulations for investing the funds of insurance contributions into financing the payments on the funded component of the retirement pension that has been received by the Pension Fund of the Russian Federation within the financial year”; The Federal Statute of 24 July 2002 N 111-FZ “On investing the capital to form the funded component of the retirement pension in the Russian Federation”.

8. deposits in the currency of the Russian Federation and foreign currency in credit organizations;
9. foreign currency on accounts in credit organizations;
10. securities of international financial organizations eligible for allocating and (or) public trading in the Russian Federation in accordance with the Russian Federation legislation of security market.

In order to realize the declared methods, special attention has been given to selecting restrictions when forming the investment portfolio of pension capital (see Table 1 and 2).

Table 1

Portfolio restrictions on asset types

Assets	Restrictions, in % from the assets (not more)
State securities of the Russian Federation	No restrictions
State securities of the Russian Federation constituent entities and municipal bonds	40
Bonds of Russian emitters except the state securities of the Russian Federation and securities of the Russian Federation constituent entities and bonds whose liability is guaranteed by the Russian Federation, as well as other bonds, whose emitter has been assigned with the rating of a long-term credit capacity on liability in the currency of the Russian Federation or foreign currency by one of the international rating agencies, in particular: Fitch-Ratings, Standard & Poor's and Moody's Investors Service that have been accredited in the order set up by a federal executive body of the financial market area at a level not lower than the sovereign rating of the Russian Federation on liability in the currency of the Russian Federation or foreign currency.	80
Stocks of Russian emitters that are open joint-stock companies	65
Mortgage-backed securities issued in accordance with the legislation on mortgage-backed securities	40
Deposits and cash assets on bank accounts	80
Shares (stocks, fractions) of index investment funds that allocate funds into securities of foreign emitters (funds allocated into the securities of foreign emitters)	20
Bonds of international financial organizations	20

Minimal restrictions in fractions of some assets or the other in the investment portfolio of pension capital are not statutory. The bonds issued by the Federal Government also lack maximum fractions. Moreover, some restrictions on securities of one emitter or a group of linked emitters in the investment portfolio have been assigned – their number need not exceed 10% (with exception of state securities of the Russian Federation, securities whose payments are guaranteed by the Russian Federation, as well as mortgage-backed securities). This restriction is aimed at reducing the risk of investments into securities of a single company.

Table 2

Portfolio restrictions according to the portfolio structure

Assets	Restrictions, in % from the assets (not more) or other
Securities of one issuer or a group of linked issuers (with exception of state securities of the Russian Federation, state securities whose payments are guaranteed by the Russian Federation, as well as mortgage-backed securities and securities whose issuer is assigned with the rating of a long-term credit capacity on liability in the currency of the Russian Federation or foreign currency by one of the international rating agencies accredited in the order set up by a federal executive body of the financial market area at a level not lower than the sovereign rating of the Russian Federation on liability in the currency of the Russian Federation or foreign currency)	10
Deposits in a credit organization and securities issued by this credit organization	25
Securities emitted by affiliated parties of the managing company and specialized depository	10
Deposits in credit organizations affiliated with the managing company	20
Stocks of one issuer	10
Bonds of one issuer (with exception of state securities of the Russian Federation, state securities whose payments are guaranteed by the Russian Federation, as well as mortgage-backed securities and securities whose issuer is assigned with the rating of a long-term credit capacity on liability in the currency of the Russian Federation or foreign currency by one of the international rating agencies accredited in the order set up by a federal executive body of the financial market area at a level not lower than the sovereign rating of the Russian Federation on liability in the currency of the Russian Federation or foreign currency)	40
Securities of one issuer (with exception of state securities of the Russian Federation, state securities whose payments are guaranteed by the Russian Federation, as well as mortgage-backed securities and securities whose issuer is assigned with the rating of a long-term credit capacity on liability in the currency of the Russian Federation or foreign currency by one of the international rating agencies accredited in the order set up by a federal executive body of the financial market area at a level not lower than the sovereign rating of the Russian Federation on liability in the currency of the Russian Federation or foreign currency)	Not exceeding 50% of total amount of the given issuer's float securities
Cash assets in roubles and foreign currency on bank accounts in credit organizations, deposits in the currency of the Russian Federation and foreign currency in credit organizations	80
Securities of foreign issuers	20

Another restriction common to all securities is qualified as restriction on the maximum fraction of one issuer's securities – not more than 50% of the stocks of this issuer's float securities, with exception of state securities of the Russian Federation, state securities whose payments are guaranteed by the Russian Federation, as well as mortgage-backed

securities issued in accordance with the Russian Federation legislation on mortgage-backed securities and meeting the requirements specified by the Russian Federation Government.

The condition for acquiring securities, apart from the requirements listed above, is circulation on the organized market of securities. An exception is state securities issued for institutional investors and securities at primary distribution subject to the criteria set up by the authorized federal executive body by agreement with the federal executive body of the financial market area.⁵

Model description

The given article considers an investment portfolio that includes $k + N$ assets $A = \{A_1, A_2, \dots, A_{k+N}\}$, where the first k of securities implies risk-free assets included in the portfolio by the investor (this study suggests it to be bonds) with profitability of $r = (r_1, r_2, \dots, r_k)$; $A_i, i = k + 1, k + N$ – risky assets (Markowitz, 1952, p 77-91; Mitsel, Rekundal, 2011, p. 2-6). All assets are grouped in accordance with their M naming (bonds, stocks, etc.). The bond sub-portfolio $A_s, s = \overline{1, k}$ is chosen in accordance with the conditions for the immunization strategy proposed by F. Reddington (Redington, 1952, p. 16-27); its profitability is a determined value, asset profitability $A_i, i = \overline{k + 1, k + N}$ is determined by the formula:

$$\rho_i(t) = \frac{S_i(t+1) - S_i(t) + C_i(t+1)}{S_i(t)}, \rho_i(t) > -1,$$

where $S_i(t)$ и $S_i(t+1)$ is the market cost of the asset $A_i \in A, i = \overline{k + 1, k + N}$ in discrete moments of time t and $t + 1$; $C_i(t+1)$ is the value of the net cash flow linked with the asset A_i within the interval between t and $t + 1$: dividends, coupon payments, etc. The asset profitability $\rho_i(t)$ is a random value with the parameters of: $M(\rho_i) = \mu_i, D(\rho_i) = V_{ii} = \sigma_i^2, \text{cov}(\rho_i, \rho_j) = V_{ij}$.

At the $t = t_0$ point of time, the investor forms an unfolded portfolio

$$X(t_0) \in \widehat{X} = \left\{ X = (x_1, \dots, x_k, x_{k+1}, \dots, x_{k+N-1}, x_{k+N}) : \sum_{i=1}^{k+N} x_i = 1 \right\},$$

⁵ <http://www.pfrf.ru> (retrieved: 25.05.14); <http://www.minfin.ru> (retrieved: 25.05.14); <http://www.ipension.ru> (retrieved: 25.05.14); <http://www.actuaries.ru> (retrieved: 25.05.14).

where x_i , $i = \overline{1, k + N}$ indicates which fraction of the investor's capital is allocated in the asset $A_i \in A$, $i = \overline{1, k + N}$, while $x' = (x_1, \dots, x_k)$, $x'' = (x_{k+1}, \dots, x_{k+N})$ are correspondingly the risk-free and risky fractions of the investment portfolio. Being the whole total of portfolios, the \widehat{X} set that can be formed from $k + N$ assets is called an accessible set.

In addition, values $y_p = \sum_{j=k_{p-1}+1}^{k_p} x_j$, where $k_0 = 0$, $p = \overline{1, M}$ must also be calculated, that

will define the cumulative fraction of financial assets in the naming of p in the X portfolio.

According to the approach by Markowitz, any portfolio $X \in \widehat{X}$ is characterized by two indices – the expected value m (that indicates the expected profitability of the X portfolio's risky fraction) and the σ^2 variance (that characterizes the level of the risk connected with the X portfolio, or, otherwise, the extent of the profitability value range around the expected level). The investor, when forming a portfolio of securities, aims at reducing its variance and increasing its expected profitability.

This paper considers a problem of forming an optimal investment portfolio $X^* \in \widehat{X}$ that includes an $x' = (x_1, \dots, x_k)$ immunized (risk-free) sub-portfolio of bonds and an $x'' = (x_{k+1}, \dots, x_{N+k})$ sub-portfolio of risky assets with an m given value of expected profitability an σ^2 minimal risk with consideration of statutory restrictions on the capital volume of pension capital investments into each asset or asset class. In such problem statement profile, the criterion for optimality at the $t = t_0$ point of time may be recorded as follows:

$$(x''(t_0))^T \cdot V(t_0) \cdot x''(t_0) \rightarrow \min_x \quad (1)$$

with restrictions of the following part:

$$\left\{ \begin{array}{l} r^T(t_0)x'(t_0) + \mu^T(t_0)x''(t_0) \geq m \\ 0 \leq x(t_0) \leq b \\ y(t_0) \leq c \\ D(t_0)^T x'(t_0) = T \\ e_1^T x'(t_0) + e_2^T x''(t_0) = 1 \\ x'_i, x''_j \geq 0 \end{array} \right. \quad (2)$$

where b, c are column vectors that define the maximum possible investment values, and, correspondingly, $k + N$ and M dimensions; $D^T = (D_1, D_2, \dots, D_k)$ is the bond duration

$$D_s = \frac{\sum_{z=1}^n (t_z - t_0) \frac{C_z^s}{(1+r^*)^{365(t_z-t_0)}}}{S_s},$$

vector for the initial $t = t_0$ moment of time, and thus $D_s = \frac{\sum_{z=1}^n (t_z - t_0) \frac{C_z^s}{(1+r^*)^{365(t_z-t_0)}}}{S_s}$, where C_z^s indicates all payments on the bond of $A_s, s = \overline{1, k}$, r is the market interest rate, S_s is the market price, n is the number of payouts on the bond; T is the investment horizon; e_1, e_2 are vectors of the k and N dimensions, correspondingly, that comprise units.

In economic terms, the formula (1) expresses the risk of a portfolio, which should be minimized. In the formula (2) the first inequality expresses the portfolio yield, which shall not be less than a given value of m . The second and third restrictions (2) associated with restrictions on the volume of investments in securities. Fourth ratio (2) associated with immunization subportfolios bonds, the fifth ratio is the proportion of the normalization of investment.

The selected model enables making calculations to find a balance between the profitability and safety of pension capital both for investors and insured parties.

The investment portfolio reorganization

At the $t = t_1$ point of time, the investor faces the problem of reorganizing the $X(t_0)$ investment portfolio, in the event of the portfolio's expected yield income (e.g., coupon yield or redemption of a partial par value) or in the event of the portfolio duration deviation from the investment horizon. In order for the bond sub-portfolio to be immunized (protected) from interest rate changes after the $t = t_1$ point of time, it is necessary that the portfolio duration at the $t = t_1$ point of time be aligned with its $(T - t_1)$ investment horizon. Thus, the $X(t_0)$ investment portfolio must be balanced anew in accord with the model (note that the income received within the t_0 to t_1 point of time must also be reinvested). In order to solve this problem, it is necessary to find an optimal solution for the following problem:

$$\begin{aligned}
 & (x''(t_1))^T \cdot V(t_1) \cdot x''(t_1) \rightarrow \min_x \\
 & \left\{ \begin{array}{l}
 r^T(t_1)x'(t_1) + \mu^T(t_1)x''(t_1) \geq m \\
 0 \leq x(t_1) \leq b \\
 y(t_1) \leq c \\
 D(t_1)^T x' = T - t_1 \\
 e_1^T x'(t_1) + e_2^T x''(t_1) = 1 \\
 x'_i, x''_j \geq 0
 \end{array} \right. \quad (3)
 \end{aligned}$$

The process of the investment portfolio reorganization may be repeated in the $t = t_2$ point of time, when the next payment from the $X(t_1)$ portfolio is received. If, at any moment of time, a sub-portfolio with the required duration can not be formed, the existing sub-portfolio must be sold.

Transaction cost account at portfolio formation

Let, for the $t = t_0$ point of time, the investor form an $X(t_0)$ investment portfolio with a cost of $W(t_0)$ for the T period of time. Let us denote the value of transaction costs for acquisition of securities as C_b and that for security sale – as C_a . Then, in order to form a portfolio with a $V(t_0)$ cost, the investor will need a sum of $W(t_0) \cdot (1 + \tilde{N}_b)$. The structure of the $X(t_0)$ portfolio for the $t = t_0$ point of time is determined from the optimal control problem (2).

The investment portfolio reorganization with consideration of transaction costs

Let there be the first payment from the $X(t_0)$ portfolio at the t_1 point of time, and the portfolio duration deviate from the $T - t_1$ investment horizon. The conditions for carrying out the investment portfolio reorganization are following:

- 1) portfolio reorganization requires transaction costs from the investor;

2) the prices of financial instruments (stocks and bonds) included in the investment portfolio have changed to the values of $S_j(t_1)$ ($j = \overline{1, k + N}$); and the bond durations have changed to the values of $D_i(t_1)$ ($i = \overline{1, k}$).

In order for a portfolio to be formed, whose bond sub-portfolio duration equals $T - t_1$ years, the following system must be solved:

$$\begin{cases} (x''(t_1))^T \cdot V(t_1) \cdot x''(t_1) \rightarrow \min_x \\ r^T(t_1)x'(t_1) + \mu^T(t_1)x''(t_1) \geq m \\ 0 \leq x(t_1) \leq b \\ y(t_1) \leq c \\ D(t_1)^T x' = T - t_1 \\ e_1^T x'(t_1) + e_2^T x''(t_1) = 1 \end{cases} \quad (4)$$

Let $x_1^1, x_2^1 \dots x_k^1, x_{k+1}^1 \dots x_{k+N}^1$ be the solution for this system. For portfolio reorganization, some bonds must be sold, and some must be bought. Accordingly, the $W(t_1)$ value share will be directed at transaction costs connected with the portfolio reorganization. Let us use C for the transaction cost value, and q_i, z_i for the sums of money spent on buying securities and received when selling securities, correspondingly.

To minimize transaction costs, a linear programming problem must be solved:

$$\begin{cases} \min C \\ C = C_b \sum_{i=1}^{N+k} q_i + C_a \sum_{i=1}^{N+k} z_i \\ \frac{V(t_0)_i}{S_i^0} S_i^1 + q_i - z_i = x_i^1 \cdot (W(t_1) - C), i = \overline{1, N + k} \end{cases} \quad (5)$$

where S_i^0 and S_i^1 are prices for securities of the i type at the $t = t_0$ and $t = t_1$ points of time, correspondingly. Let us denote $q_1^1, q_2^1, \dots, q_k^1, \dots, q_{N+k}^1, z_1^1, z_2^1, \dots, z_k^1, \dots, z_{N+k}^1, C^1$ as the problem solution (3). Then, at the $t = t_1$ point of time, an $X(t_1)$ investment portfolio with a $\overline{W}(t_1) = W(t_1) - C^1$ cost will be formed.

Numerical simulation

Let us consider the problem of managing a pension capital investment portfolio that consists of 2 stock types and 4 bond types traded at the Moscow Central Stock Exchange: JSC “Nomos-Bank”, JSC “Aeroflot”, bond of the Tomsk, Volgograd, Sverdlovsk Regions and bonds of Evraz Holding Finance. The basis for calculations is taken from the retrospective data on the profitability of the selected securities.⁶ [14-16]. Let us assume that the investor must form an investment portfolio for a period of 1 year and meanwhile obey the conditions of strategy immunization lying as the basis for the portfolio formation model. The period before reconsidering the formed investment portfolio is taken to equal 1 month.

The following input data have been taken for simulation (see Tables 3, 4).

Table 3

Prices of securities selected for the investment portfolio

Point of time	0	1	2	3	4	5	6	7	8	9	10	11
Tomsk Region 7	205.84	206.32	206.32	206.28	204.80	204.76	203.86	203.72	203.48	201.18	201.10	200.00
Volgograd Region 9	695.80	695.10	697.97	697.90	699.30	699.37	400.04	400.56	399.80	399.96	399.08	399.20
Tver Region 9	624.24	624.36	624.00	624.00	622.26	621.90	619.08	614.58	614.82	610.38	608.34	607.68
Evraz Holding Finance5	962.80	979.30	982.80	982.60	983.20	979.60	982.90	982.80	988.50	988.60	970.50	962.40
Nomos-Bank	824.40	810.10	845.00	843.70	864.80	908.80	880.00	831.00	933.70	949.00	965.00	960.00
Aeroflot	53.55	53.00	52.99	53.90	52.30	56.70	57.36	48.82	54.44	57.40	60.42	83.70

Table 4

Profitability and durations of bonds

Points of time	Tomsk Region 7		Volgograd Region 9		Tver Region 9		Evraz Holding Finance 5	
	profitability %	duration days	profitability %	duration days	profitability %	duration days	profitability %	duration days
0	7.72	306.00	8.28	343.00	8.81	388.00	9.57	1110.00
1	7.80	286.00	8.43	310.00	8.90	356.00	9.55	1071.00
2	8.03	265.00	7.92	280.00	8.32	324.00	9.32	1028.00
3	8.06	232.00	8.82	261.00	8.35	291.00	9.18	1016.00
4	6.43	197.00	7.45	238.00	7.65	258.00	9.22	992.00
5	6.92	178.00	7.77	204.00	13.08	235.00	9.71	985.00
6	7.41	146.00	8.84	335.00	8.42	211.00	9.27	968.00
7	6.42	116.00	6.45	303.00	8.83	188.00	9.41	930.00
8	6.86	86.00	8.06	266.00	8.18	159.00	9.04	898.00
9	7.61	57.00	7.36	240.00	8.49	131.00	10.15	872.00
10	7.47	20.00	7.45	210.00	7.7	103.00	9.95	848.00
11	5.48	redemption	7.79	184.00	7.12	76.00	10.31	812.00

The results of simulation are given in Tables 5, 6.

⁶ <http://www.cbonds.ru> (retrieved 28.05.14); <http://www.finam.ru> (retrieved: 28.05.14); <http://www.mixec.ru> (retrieved: 28.05.14).

Table 5

The investment portfolio structure (fractions):

Point of time	0	1	2	3	4	5
Tomsk Region 7	0.25	0.25	0.25	0.25	0.25	0.25
Volgograd Region 9	0.25	0.25	0.25	0.25	0.25	0.25
Tver Region 9	0.25	0.25	0.25	0.25	0.25	0.25
Evrast Holding Finance5	0.10	0.09	0.10	0.08	0.07	0.06
Nomos-Bank	0.05	0.06	0.05	0.07	0.08	0.09
Aeroflot	0.10	0.10	0.10	0.10	0.10	0.10

At the $t = t_5$ time point, it is not possible to reorganize the functioning investment portfolio from the initial list of securities since the condition of equality between the bond sub-portfolio duration and the remaining investment period is not observed. As a result, the initial portfolio is not to be sold.

Table 6

Transaction costs for purchase and sale of securities at each step

0	1	2	3	4	5
5000	121.7726	0	118.2279	160.2675	5475.4990

Conclusion

The given paper has considered the procedure of forming and reorganizing the investment portfolio of pension capital in a context of the NPF activity. In the course of research, the active legislation of the Russian Federation has been analyzed with a purpose of further assignment of restrictions on the structure and composition of the investment portfolio at the time of its formation. We obtain the optimization model, which includes two subportfolios – risky and risk-free. The model is a further development of the Markowitz model. The task of portfolio management considers transaction costs collected by a stock exchange for carrying out transactions of security purchase and sale. As a result of the numerical simulation, it has been estimated that the investment cost of the portfolio obtained at the period ending will be less than that obtained in the absence of commission. Thus, with the presence of transaction costs, the investor faces a problem of determining the frequency of reconsidering the formed portfolio.

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