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GROWTH, RESEARCH, AND FREE TRADE WITH KNOWLEDGE AS GLOBAL PUBLIC CAPITAL

The purpose of this study is to explain dynamics of global growth and trade patterns with wealth and utilizing knowledge as basic determinants. It builds a multi-country growth model with economic structure and research. Global economy composes of any number of countries and each country has one production sector and one university. Knowledge is through learning by doing and research. Knowledge is global public good and is applied by countries with different utilization efficiencies. The production sector is the same as in the one-sector growth Solow model (Solow, 1956), while capital mobility and trade patterns are determined like in the Oniki-Uzawa model (Oniki and Uzawa, 1956). We use a utility function proposed by Zhang (1993) to determines saving and consumption. The movement of the system is given by differential equations. We simulate the model. Our comparative analysis provides some insights into the complexity of international trade with endogenous wealth and knowledge.
JEL: F11; O30

1. Introduction

Inequalities in income and wealth among different groups, regions, and nations and dynamics of inequalities are crucial issues for economics. We study effects of trade upon income distribution among nations in a globalizing world economy. Many studies demonstrate that productivity differences explain much of the variation in incomes across countries, and technology plays a key role in determining productivity (Krugman and Venables, 1995; Manasse and Turrini, 2001; Agénor, 2004; Aghion et al. 2009; Gersbach et al. 2013). The pattern of worldwide technical change is determined largely by international technology diffusion. Moreover, a few rich countries account for most of the world's creation of new technology. Obviously we need proper analytical frameworks for analyzing global economic interactions with microeconomic foundations. As developed, industrializing and developing economies are well connected with trades, it is important to examine how changes in preference or technology in one country can affect the country as well as other countries in a well-connected world economy. For instance, an underdeveloped economy with large population, like India or China, may affect different economies as its technology is improved or population is increased. We are also concerned with how trade patterns may be affected as

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technologies are further improved or propensities to save are reduced in developed economies like the US or Japan. It is well known that only a few formal economic theories properly address inequalities in income and wealth among nations with microeconomic foundation. The purpose of this study is to study inequalities among nations with endogenous wealth accumulation and knowledge dynamics. We emphasize effects of free trade and national governments' research policies on global growth and inequalities among nations.

As far as modelling production and trade patterns is concerned, we follow the neoclassical growth trade model, particularly the Oniki-Uzawa model (Oniki and Uzawa, 1965). As reviewed by Findlay (1984), the pure theory of international trade developed before the 1960s failed to properly analyze the connection between economic growth and international trade. The classical Ricardian theory of comparative advantage and the Heckscher-Ohlin theory did not consider labor and capital stocks (or land) as endogenously changeable variables. In the last three or four decades, trade theory has made some systematic treatment of capital accumulation or technological changes in the context of international economics. A dynamic model with endogenous capital accumulation and capital movements was initially developed by Oniki and Uzawa and others (e.g., Oniki and Uzawa, 1965; Johnson, 1971), in terms of the two-country, two-good, two-factor model of trade. The model has been extended and generalized to analyze the interdependence between trade patterns and economic growth (e.g., Jones and Kenen, 1984; Ethier and Svensson, 1986; Bhagwati, 1991; Wong, 1995; Sorger, 2002; Vellutini; 2003). Zhang (2008) provides an extensive review on the literature. Irrespective of analytical difficulties involved in analyzing two-country, dynamic-optimization models with capital accumulation, many efforts have been made to examine the impact of saving, technology, and various policies upon trade patterns within this framework (e.g., Frenkel and Razin, 1987; Jensen, 1994; Valdés, 1999; Nishimura and Shimomura, 2002). As far as capital accumulation and trade pattern determination are concerned, our study follows the Oniki-Uzawa framework (Ikeda and Ono, 1992), even though this study deviates from the traditional approach in modelling behavior of households.

In our model economic dynamics is fueled by public investment in research, learning by doing and physical investment. Our model is to integrate the basic economic growth mechanisms in the neoclassical growth theory with capital accumulation and growth theory with endogenous knowledge. We consider knowledge as an international public good in the sense that all countries access knowledge and the utilization of knowledge by one country does not affect that by others. Trade economists have recently developed different trade models endogenous knowledge (e.g., Chari and Hopenhayn, 1991; Martin and Ottaviano, 2001; Brecher *et al*. 2002; Nocco, 2005; Hinloopen, 2014). These studies attempted to formalize trade patterns with endogenous technological change and monopolistic competition. They often link trade theory with increasing-returns growth theory. These approaches deal with dynamic interdependence between trade patterns, R&D efforts, and various economic policies. Although these studies explore the relationship between trade policy and long-run growth either with knowledge or with capital, but in most of them not with both capital and knowledge in a unified framework. This paper examines interactions among physical accumulation, knowledge dynamics, trade, within a compact analytical framework.

A common limitation in most of trade models with endogenous capital and/or knowledge is that they deal with the world economy with only two national economies (e.g., Grossman and

Helpman, 1991; Wong, 1995; Jensen and Wong, 1998; Obstfeld and Rogoff, 1998). The world consists of different countries and different countries have different preferences, technologies and resources. Conclusions made from analyzing a two-country model might provide limited or even misleading insights into the complexity of multi-country economy. It is necessary to deal with a world economy with any number of national economies. It is well known that dynamic-optimization models with capital accumulation are associated with analytical difficulties. To avoid these difficulties, this study applies an alternative approach to consumer behavior. The model in this study is a further development of the studies by Zhang. Zhang (1992) proposed a multi-country model with capital accumulation and knowledge creation. The study does not consider research and does not simulate behavior of the model. Zhang Zhang (1993) develops an endogenous growth model with wealth accumulation and knowledge dynamics. This study does not simulate the model and is limited a national economy. Zhang (2012) considers different sources of knowledge dynamics for a national economy without interregional or international trade and simulates the model so that one can observe the dynamic behavior of the economic system. This study synthesizes the basic ideas in these study for a multi-country global economy with wealth accumulation and knowledge dynamics. This study models the behavior of households in an alternative way for a multi-country economy with free trade and assumes that knowledge creation is through learning by doing and research. This paper is organized as follows. Section 2 defines the multi-country model with capital accumulation and knowledge creation. Section 3 shows that the dynamics of the world economy with J countries can be described by $(J+1)$-dimensional differential equations. As mathematical analysis of the system is too complicated, we demonstrate some of the dynamic properties by simulation when the world economy consists of three countries. Section 4 carries out comparative dynamic analysis examine respectively effects of changes in each country's knowledge utilization efficiency and creativity, research policy, the propensity to save, and the population. Section 9 concludes the study. The analytical results in section 3 are confirmed in Appendix A1. Section A2 examines the case when all the countries have the same preference. We show that the motion of the global economy can be expressed by a two-dimensional differential equations system and we can explicitly determine the dynamic properties of the global economy.

2. The multi-country trade model with capital and knowledge

We consider a global economy with any number of countries. Each country has one production sector and one university. We use subscript indexes i and r to denote the production sector and the university, respectively. The university is financially supported by the government through taxing the production sector. The governments fix the tax rates and obtain tax incomes from the private sectors. The research sector employs labor and capital in such a way that the research output is maximized with the government's tax income. Knowledge growth is through learning by doing by the production sector and research activities by the university. In describing the production sector, we follow the neoclassical trade framework. Follows the Oniki-Uzawa trade model and its various extensions with one capital goods, we assume that the countries produce a homogenous commodity. Most aspects of production sectors in our model are similar to the neo-classical one-sector growth model (Burmeister and Dobell, 1970).

There is only one (durable) good in the global economy under consideration. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use knowledge, capital and labor. Exchanges take place in perfectly competitive markets. Production sectors sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. All savings volunteered by households are absorbed by firms. We require savings and investment to be equal at any point of time. The system consists of multiple countries, indexed by $j = 1, ..., J$. Each country has a fixed labor force, N_j, ($j = 1, ..., J$). Let prices be measured in terms of the commodity and the price of the commodity be unity. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the j th country. In the free trade system, the interest rate is identical throughout the world economy, i.e., $r(t) = r_j(t)$. For convenience, we term the people working in the production sector as workers and the people working in the university as scientists. The population is classified into workers and scientists. Let $N_{qj}(t)$ and $K_{qj}(t)$ stand for the labor force and capital stocks employed by sector q, $q = i, r$, in country j.

Behavior of producers

First, we describe behavior of the production sections. We assume that there are three factors, physical capital, labor, and knowledge at each point of time t. We use $F_j(t)$ to stand for the output level of the production sector by country j. The production functions are given by

$$F_j(t) = A_j Z^{m_j}(t) K_{ij}^{\alpha_j}(t) N_{ij}^{\beta_j}(t), \ \ A_j > 0, \ \ \alpha_j + \beta_j = 1, \ \ \alpha_j, \beta_j > 0, \ \ j = 1, \cdots, J,$$

in which $Z(t)\ (> 0)$ is the knowledge stock at time t. Here, we call m_j country j's knowledge utilization efficiency parameter. If we interpret $Z^{m_j/\beta}(t) N_j(t)$ as country j's human capital or qualified labor force (e.g., Iacopetta, 2011), we see that the production function is a neoclassical one and homogeneous of degree one with the inputs. Many studies show that basic research positively affects applied research (Nelson and Winter, 1982; Jaffe, 1989; Nelson, 2002; Adams, 1990; Acs et al. 1992; Mansfield, 1998). We may also interpret the above formation as follows. We consider that a company's applied knowledge is a nonlinear function of general knowledge. We consider that the total productivity factor is proportionally related to $Z^{m_j}(t)$. We thus have the above production function with knowledge as public capital.

Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest, $r(t)$, and wage rates, $w_j(t)$, are determined by markets. Hence, for any individual firm $r(t)$ and $w_j(t)$ are given at each point of time. The production sector chooses the two variables $K_{ij}(t)$ and $N_{ij}(t)$ to maximize its profit. The marginal conditions are given by

$$r(t) + \delta_{kj} = \bar{\tau}_j \, A_j \, \alpha_j \, Z^{m_j}(t) k_j^{-\beta_j}(t), \;\; w_j(t) = \bar{\tau}_j \, A_j \, \beta_j \, Z^{m_j}(t) k_j^{\alpha_j}(t), \tag{1}$$

where δ_{kj} are the depreciation rate of physical capital in country j and

$$k_j(t) \equiv \frac{K_{ij}(t)}{N_{ij}(t)}, \;\; \bar{\tau}_j \equiv 1 - \tau_j \, ,$$

in which τ_j is country $j's$ tax rate on its production sector.

Behavior of consumers

Each household gets income from wealth ownership and wages. Consumers make decisions on consumption levels of goods as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang in the early 1990s (Zhang, 1993). Let $\bar{k}_j(t)$ stand for the per capita wealth in country j . Each consumer of country j obtains income

$$y_j(t) = r(t)\bar{k}_j(t) + w_j(t), \;\; j = 1, \cdots, J \, , \tag{2}$$

from the interest payment $r(t)\bar{k}_j(t)$ and the wage payment $w_j(t)$. We call $y_j(t)$ the current income in the sense that it comes from consumers' wages and consumers' current earnings from ownership of wealth. The disposable income that consumers are using for consuming, saving, or transferring are not necessarily equal to the current income because consumers can sell wealth to pay, for instance, the current consumption if the current income is not sufficient for buying food and touring the country. The maximum value of wealth that a consumer can sell to purchase goods equal to $\bar{k}_j(t)$. We define the disposable income as follows

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \tag{3}$$

The disposable income is used for saving and consumption. It should be noted that the value, $\bar{k}_j(t)$, (i.e., $p(t)\bar{k}_j(t)$ with $p(t) = 1$), in the above equation is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we consider $\bar{k}_j(t)$ as the amount of the income that the consumer obtains at time t by selling all of his wealth. Hence, at time t the consumer has the total amount of income equaling $\hat{y}_j(t)$ to distribute between consuming and saving.

At each point of time, a consumer distributes the total available budget between savings, $s_j(t)$, and consumption of goods, $c_j(t)$. The budget constraint is

$$c_j(t) + s_j(t) = \hat{y}_j(t) = r\,\bar{k}_j(t) + w_j(t) + \bar{k}_j(t). \tag{4}$$

At each point of time, consumers have two variables to decide. Equation (4) means that consumption and savings exhaust the consumer's disposable income.

We assume that utility levels that the consumers obtain are dependent on the consumption level of commodity, $c_j(t)$, and the savings, $s_j(t)$. The utility level of the consumer in country j, $U_j(t)$, is specified as follows

$$U_j(t) = c_j^{\xi_{0j}}(t) s_j^{\lambda_{0j}}(t), \ \ \xi_{0j}, \lambda_{0j} > 0, \tag{5}$$

where ξ_{0j} and λ_{0j} are respectively household j 's propensities to consume and to hold wealth. Here, for simplicity, we specify the utility function with the Cobb-Douglas from. In this study we fix the preference structure. It is quite reasonable to assume that one's attitude towards the future is dependent on factors such as capital gains, the stock of durables owned by oneself, income distribution and demographic factors.

Maximizing U_j subject to the budget constraints (4) yields

$$c_j(t) = \xi_j\, \hat{y}_j(t), \ \ s_j(t) = \lambda_j\, \hat{y}_j(t), \tag{6}$$

in which

$$\xi_j \equiv \frac{\xi_{0j}}{\xi_{0j} + \lambda_{0j}}, \ \ \lambda_j \equiv \frac{\lambda_{0j}}{\xi_{0j} + \lambda_{0j}}.$$

According to the definitions of $s_j(t)$, the wealth accumulation of the representative household in country j is given by

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{7}$$

This equation states that the change in wealth is the saving minus dissaving.

Knowledge creation

Bresnahan and Trajtenberg (1995) argued that technologies have a treelike structure, with a few prime movers located at the top and all other technologies radiating out from them. They characterize general purpose technologies by pervasiveness (which means that such a technology can be used in many downstream sectors), technological dynamism (which means that it can support continuous innovational efforts and learning), and innovational complementarities (which exist because productivity of R&D in downstream sectors increases as a consequence of innovation in the general purpose technology, and vice versa). Like capital, a refined classification of knowledge and technologies tend to lead new conceptions and modelling strategies. This study uses knowledge in a highly aggregated sense. We assume a conventional production function of knowledge in which labor, capital, and knowledge are combined to create new knowledge in a deterministic way. This is an approximate description of the idea that devoting more resources to research yields more rapidly new knowledge. There does not appear to have certain evidence for supporting any form of how increases in the stock of knowledge affect the creation of new knowledge. For simplicity, we consider that research is carried out only by the universities. It is more realistic to assume that the research sector consists of two sub-sectors: a private research sector and a government research sector as in, for instance, Park (1998). In Park's model, the government may create knowledge useful for defense, space, and environment and the private sector for industrial, agricultural, and consumption goods (see also Kline and Rosenberg, 1987; Porter, 1990, 1998; Jaffe et al. 1993; Anselin et al. 1997, 2000; Fujita and Thisse, 2002; Henderson, 2003). Some overlapping knowledge, like mathematical and scientific knowledge, may be tailored for research as particular activities. We propose the following equation for knowledge growth

$$\dot{Z}(t) = \sum_{j=1}^{J} \left\{ \frac{\tau_{ij} F_j(t)}{Z^{\varepsilon_{ij}}(t)} + \tau_{rj} Z^{\varepsilon_{rj}}(t) K_{rj}^{\alpha_{rj}}(t) N_{rj}^{\beta_{rj}}(t) \right\} - \delta_z Z(t), \tag{8}$$

in which $\delta_z \ (\geq 0)$ is the depreciation rate of knowledge, and ε_{qj}, τ_{qj}, α_{rj} and β_{rj} are parameters. We require τ_{qj}, α_{rj}, and β_{rj} to be non-negative. To interpret equation (8), first let us consider a special case that knowledge accumulation is through learning by doing. The parameters τ_{ij} and δ_z are non-negative. We interpret $\tau_{ij} F(t) / Z^{\varepsilon_{ij}}(t)$ as the contribution to knowledge accumulation through learning by doing by country $j's$ production sector. To see how learning by doing occurs, assume that knowledge is a function of country $j's$ total industrial output during some period

$$Z(t) = a_1 \left\{ \int_0^t F_j(\theta) d\theta \right\}^{a_2} + a_3$$

in which a_1, a_2 and a_3 are positive parameters. The above equation implies that the knowledge accumulation through learning by doing exhibits decreasing (increasing) returns to scale in the case of $a_2 < (>) \ 1$. We interpret a_1 and a_3 as the measurements of the efficiency of learning by doing by the production sector. Taking the derivatives of the equation yields

$$\dot{Z}(t) = \frac{\tau_{ij}\, F_j(t)}{Z^{\varepsilon_{ij}}(t)}$$

in which $\tau_{ij} \equiv a_1\, a_2$ and $\varepsilon_{ij} \equiv 1 - a_2$. The term, $\tau_{rj}\, Z^{\varepsilon_{rj}}(t) K_{rj}^{\alpha_{rj}}(t) N_{rj}^{\beta_{rj}}(t)$, is the contribution to knowledge growth by country $j's$ university. It means that knowledge production of the university is positively related to the capital stocks, $K_{rj}(t)$, employed by the university and the number of scientists $N_{rj}(t)$. To interpret the parameter, ε_{rj}, we notice that on the one hand, as the knowledge stock is increased, the university may more effectively utilize traditional knowledge to discover new theorems, but on the other hand, a large stock of knowledge may make discovery of new knowledge difficult. This implies that ε_{rj} may be either positive or negative. It is reasonable to assume that the more equipments, books, and buildings, and scientists in the university employs, the more productive it becomes. That is, α_{rj} and β_{rj}, are positive. We do not require that the creation function for knowledge have constant returns to scale in capital and labor. It is possible that doubling the number of computers and scientists increases three times of the knowledge creation than before – the university's knowledge creation exhibits increasing returns to scale in scientist and capital. It is also possible for the university to have decreasing returns to scale. We thus should allow three possibilities - increasing, constant, decreasing returns to scale in scientists and capital – in the university's knowledge creation.

The university maximizing its output with the research fund

The universities are financially supported by the governments. In our model, the governments collect taxes to support the universities. As tax income are used only for supporting the utilities, we have

$$\left(r(t) + \delta_{kj}\right) K_{rj}(t) + w_j(t) N_{rj}(t) = \tau_j F_j(t), \quad j = 1, \ldots, J. \tag{9}$$

The university pays the interest, $\left(r(t) + \delta_{kj}\right) K_{rj}(t)$, the scientists' wage, $w_j(t) N_{rj}(t)$, with the research fund, $\tau_j\, F_j(t)$. We determine $K_{rj}(t)$ and $N_{rj}(t)$ by assuming that country $j's$ university utilizes its financial resource, $\tau_j F_j(t)$, in such a way that its output – contribution to knowledge growth – is maximized. The behavior of the university is thus formulated by

$$Max\ \tau_{rj} Z^{\varepsilon_{rj}}(t) K_{rj}^{\alpha_{rj}}(t) N_{rj}^{\beta_{rj}}(t),$$

$$\text{s.t.: } \left(r(t) + \delta_{kj}\right) K_{rj}(t) + w_j(t) N_{rj}(t) = \tau_j F_j(t).$$

Country $j's$ university allocates the financial resource as follows

$$K_{rj}(t) = \frac{\overline{\alpha}_j \tau_j F_j(t)}{r(t) + \delta_{kj}}, \quad N_{rj}(t) = \frac{\overline{\beta}_j \tau_j F_j(t)}{w_j(t)}, \tag{10}$$

where

$$\overline{\alpha}_j \equiv \frac{\alpha_{rj}}{\alpha_{rj} + \beta_{jr}}, \quad \overline{\beta}_j \equiv \frac{\beta_{rj}}{\alpha_{rj} + \beta_{rj}}.$$

If the other conditions remain the same, an increase in the tax rate or output enables the university to utilize more equipments and to employ more people. An increase in factor price will reduce the employment level of the factor.

Full employment and the demand and supply balance

We use $K_j(t)$ and $\overline{K}_j(t)$ to stand for the capital stocks employed and the wealth owned by country j. The assumption that the labor force and capital are always fully employed in each country is represented by

$$N_{ij}(t) + N_{rj}(t) = N_j, \quad K_{ij}(t) + K_{rj}(t) = K_j(t). \tag{11}$$

We use $K(t)$ to stand for the capital stocks of the world economy. The total capital stocks employed by the world is equal to the wealth owned by the world. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) \ = \sum_{j=1}^{J} \bar{k}_j(t) N_j. \tag{12}$$

The world production is equal to the world consumption and world net savings. That is

$$C(t) + S(t) - K(t) + \sum_{j=1}^{J} \delta_{kj} K_j(t) = F(t),$$

where

$$C(t) \equiv \sum_{j=1}^{J} c_j(t) N_j, \quad S(t) \equiv \sum_{j=1}^{J} s_j(t) N_j, \quad F(t) \equiv \sum_{j=1}^{J} F_j(t).$$

We have thus built the model with trade, economic growth, capital accumulation, knowledge creation and utilization in the world economy in which the domestic markets of each country are perfectly competitive, international product and capital markets are freely mobile and labor is internationally immobile.

3. The economic dynamics of the global economy

In Appendix A2, we examine the dynamic properties when all the household in the global economy have identical preference. We can obtain analytical properties of the dynamic analysis as the world economy is controlled only by two-dimensional differential equations. We now examine the behavior of the system when the households have different preferences among countries. We first show that the dynamics of the world economy can be expressed by $(J+1)$ differential equations.

Lemma

The dynamics of the world economy is governed by the following $(J+1)$ differential equations with $Z(t)$, $k_1(t)$ and $\bar{k}_j(t)$, $j = 2, \cdots, J$, as the variables

$$\dot{Z}(t) = \Lambda\left(k_1(t), \{\bar{k}_j(t)\}, Z(t)\right),$$

$$\dot{k}_1(t) = \Lambda_1\left(k_1(t), \{\bar{k}_j(t)\}, Z(t)\right),$$

$$\dot{\bar{k}}_j(t) = \Lambda_j\left(k_1(t), \bar{k}_j(t), Z(t)\right), \quad j = 2, ..., J,$$

in which $\{\bar{k}_j(t)\} \equiv (\bar{k}_2(t), \ , \bar{k}_J(t))$, and $\Lambda(t)$ and $\Lambda_j(t)$ are unique functions of $Z(t)$, $k_1(t)$ and $\bar{k}_j(t)$ at any point of time, defined in Appendix A1. For any given positive values of $Z(t)$, $k_1(t)$ and $\bar{k}_j(t)$ at any point of time, the other variables are uniquely determined by the following procedure: $\bar{k}_1(t)$ by (A7) $\rightarrow$ $k_j(t)$, by (A1) $\rightarrow$ $r(t)$ $\rightarrow$ $w_j(t)$ by (A2) $\rightarrow$ $N_{qj}(t)$ by (A4) $\rightarrow$ $K_j(t)$ by (A5) $\rightarrow$ $K_{qj}(t)$ by (A4) $\rightarrow$ $\hat{y}_j(t)$ by (A8) $\rightarrow$ $c_j(t)$ and $s_j(t)$ by (6) $\rightarrow$ $F_j(t)$ by the definition $\rightarrow$ $U_j(t)$ by (5).

We have the dynamic equations for the world economy with any number of countries. The system is nonlinear and is of high dimension. It is difficult to generally analyze behavior of the system. For Illustration, we simulate the motion of the global economy with three countries. We specify the parameters as follows:

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 8 \end{pmatrix}, \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.8 \\ 0.7 \end{pmatrix}, \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.2 \\ 0.1 \end{pmatrix}, \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.04 \\ 0.02 \end{pmatrix}, \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.32 \\ 0.31 \end{pmatrix}, \begin{pmatrix} \alpha_{1r} \\ \alpha_{2r} \\ \alpha_{3r} \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.4 \\ 0.4 \end{pmatrix},$$

$$\begin{pmatrix} \beta_{1r} \\ \beta_{2r} \\ \beta_{3r} \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.3 \\ 0.2 \end{pmatrix}, \begin{pmatrix} \tau_{i1} \\ \tau_{i2} \\ \tau_{i3} \end{pmatrix} = \begin{pmatrix} 0.02 \\ 0.01 \\ 0.01 \end{pmatrix}, \begin{pmatrix} \tau_{r1} \\ \tau_{r2} \\ \tau_{r3} \end{pmatrix} = \begin{pmatrix} 0.08 \\ 0.06 \\ 0.03 \end{pmatrix}, \begin{pmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \varepsilon_{i3} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \end{pmatrix}, \begin{pmatrix} \varepsilon_{r1} \\ \varepsilon_{r2} \\ \varepsilon_{r3} \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.3 \\ 0.2 \end{pmatrix},$$

$$\begin{pmatrix} \xi_{01} \\ \xi_{02} \\ \xi_{0.3} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.3 \end{pmatrix}, \begin{pmatrix} \lambda_{01} \\ \lambda_{02} \\ \lambda_{03} \end{pmatrix} = \begin{pmatrix} 0.75 \\ 0.7 \\ 0.65 \end{pmatrix}, \ \delta_k = 0.05, \ \delta_Z = 0.04. \tag{13}$$

Country $1, 2$ and $3's$ populations are respectively $3, 4$ and 8. Country 3 has the largest population. Country $1, 2$ and $3's$ total productivities, A_j, are respectively $1, 0.8$ and 0.7. Country $1, 2$ and $3's$ knowledge utilization efficiency parameters, m_j, are respectively 0.4, 0.2 and 0.1. Country 1 utilizes knowledge mostly effectively; country 2 next and country 3 utilizes knowledge lest effectively. We call the three countries respectively as developed, industrializing, and underdeveloped economies (DE, IE, UE). The DE has the highest tax rate for supporting research and the UE has the lowest tax rate. We require For simplicity, we require $\delta_k = \delta_{kj}$, $j = 1, 2, 3$. We specify the values of the parameters, α_j, in the Cobb-Douglas productions approximately equal to 0.3. The DE's learning by doing and university creativity parameters, τ_{i1} and τ_{r1}, are the highest among the countries. The returns to scale parameters in learning by doing, ε_{ij}, are all positive, which implies that knowledge exhibits decreasing returns to scale in learning by doing. The depreciation rates of physical capital and knowledge are specified respectively at 0.05 and 0.04. The DE's propensity to save is 0.75 and the UE's propensity to save is 0.65. The value of the IE's propensity is between the two other countries. In Figure 1, we plot the motion of the system with the following initial conditions

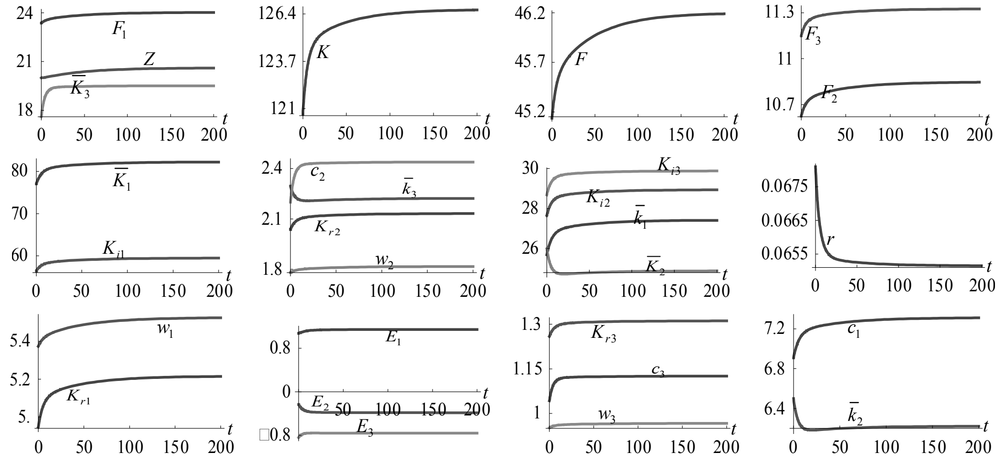
$$k_1(0) = 19.5, \ \bar{k}_2(0) = 6.5, \ \bar{k}_3(0) = 2.2, \ Z(0) = 20.$$

In Figure 1, the trade balances of the three countries are given by

$$E_j(t) = \left(\bar{K}_j(t) - K_j(t)\right) r(t), \ \ j = 1, 2, 3.$$

Figure 1

The Motion of the System



When $E_j(t)$ is positive (negative), we say that country j is in trade surplus (deficit). When $E_j(t)$ is zero, country $j's$ trade is in balance. The worker in the DE has the highest wage rate, and wealth and consumption levels, while the worker in the UE has the lowest wage rate, and wealth and consumption levels. The DE is in trade surplus, while the other two economies are in trade deficits. Due to their initial values, most of the variables in the system are increased over time till they approach their equilibrium values. From Figure 1, we see that the system is approaching its equilibrium point. Nevertheless, it is difficult to confirm whether this equilibrium point is unique. We further simplify equilibrium conditions so that we can discuss the uniqueness. First, from equations (A1) and (A2) we know that the equilibrium point is given by

$$k_j = \phi_j(k_1, Z) = \tau_{kj} Z^{\overline{m}_j} k_1^{\beta_1/\beta_j},$$

$$w_j = \overline{\phi}_j(k_1, Z) = \tau_{wj} Z^{m_{0j}} k_1^{\alpha_{wj}}, \quad j = 1, \cdots, J, \tag{14}$$

where

$$\tau_{kj} \equiv \left(\frac{\overline{\tau}_j A_j \alpha_j}{\overline{\tau}_1 A_1 \alpha_1}\right)^{1/\beta_j}, \ \overline{m}_j \equiv \frac{m_j - m_1}{\beta_j}, \ \tau_{wj} \equiv \overline{\tau}_j A_j \beta_j \tau_{kj}^{\alpha_j}, \ m_{0j} \equiv m_j + \alpha_j \overline{m}_j, \ \alpha_{wj} \equiv \frac{\beta_1 \alpha_j}{\beta_j}.$$

By equations (7), we have $s_j = \bar{k}_j$. By the definition of R and equations (1), we have

$$R(k_1, Z) = \lambda_1\left(\lambda_{u1} - \overline{\tau}_1 A_1 \alpha_1 Z^{m_1} k_1^{-\beta_1}\right), \tag{15}$$

in which $\lambda_{u1} \equiv 1/\lambda_1 - 1 + \delta_k$. From the equations for k_j in (14) and equations (A5), we have

$$K = \psi = \sum_{j=1}^{J} \frac{\tau_{kj} N_{ij} k_1^{\beta_1/\beta_j} Z^{\overline{m}_j}}{a_{ij}}. \tag{16}$$

From $s_j = \overline{k}_j$ and equations (6), we have $\hat{y}_j = \overline{k}_j / \lambda_j$. Substitute $\hat{y}_j = \overline{k}_j / \lambda_j$ into (A8)

$$\overline{k}_j = \frac{\tau_{wj} Z^{m_{0j}} k_1^{\alpha_{wj}}}{\lambda_{uj} - \overline{\tau}_1 \alpha_1 A_1 Z^{m_1} k_1^{-\beta_1}}, \quad j = 2, ..., J, \tag{17}$$

where we use equations (14) and (1) and $\lambda_{uj} \equiv 1/\lambda_j - 1 + \delta_k$. By equations (A12) we have

$$\Omega_k(k_1, Z) \equiv \sum_{j=1}^{J} \frac{\tau_{wj} n_j Z^{m_{0j}} k_1^{\alpha_{wj}}}{\lambda_{uj} - \overline{\tau}_1 \alpha_1 A_1 Z^{m_1} k_1^{-\beta_1}} - n_0 \sum_{j=1}^{J} \frac{\tau_{kj} N_{ij} k_1^{\beta_1/\beta_j} Z^{\overline{m}_j}}{a_{ij}} = 0, \tag{18}$$

in which $n_1 = 1$, we use $\Lambda = \Lambda_j = 0$, and equations (14)-(17). Substituting $\phi_j = \tau_{kj} Z^{\overline{m}_j} k_1^{\beta_1/\beta_j}$ into equation (A13) and setting the resulted equation at the equilibrium point, we have

$$\Omega_Z(k_1, Z) \equiv \sum_{j=1}^{J} \left(\overline{\tau}_{ij} \tau_{kj}^{\alpha_{ij}} Z^{x_{ij}} k_1^{\beta_1 \alpha_j / \beta_j} + \overline{\tau}_{rj} \tau_{kj}^{\alpha_{rj}} Z^{x_{rj}} k_1^{\beta_1 \alpha_{rj} / \beta_j} \right) - \delta_z = 0, \tag{19}$$

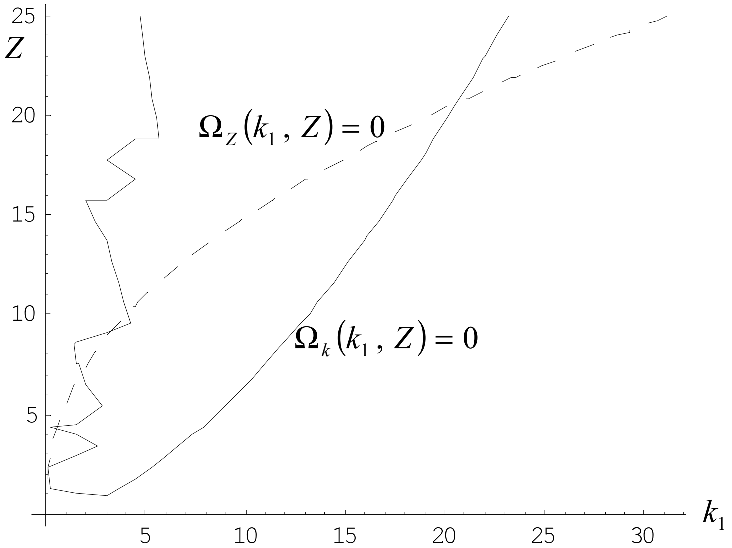
in which

$$x_{ij} \equiv m_j - \varepsilon_{ij} + \alpha_{ij} \overline{m}_j - 1, \quad x_{rj} \equiv \varepsilon_{rj} + \alpha_{rj} \overline{m}_j - 1.$$

We see that the two equations, $\Omega_k(k_1, Z) = 0$ and $\Omega_Z(k_1, Z) = 0$, contain two variables, k_1 and Z. The two equations determine equilibrium values of k_1 and Z. By equations (17), we determine $\overline{k}_j$ for $j = 2, ..., J$. Following the procedure in the lemma, we determine all the other variables at equilibrium point. We see that the main problem is to solve $\Omega_k(k_1, Z) = 0$ and $\Omega_Z(k_1, Z) = 0$, for $k_1 > 0$ and $Z > 0$. As we cannot explicitly solve the equilibrium values of k_1 and Z, we simulate the model to illustrate properties of the dynamic system. Figure 2 plots the two equations, $\Omega_k(k_1, Z) = 0$ and $\Omega_Z(k_1, Z) = 0$, for $k_1 > 0$ and $Z > 0$, under (13). The solid lines represent $\Omega_k(k_1, Z) = 0$ and the dashed line stands for $\Omega_Z(k_1, Z) = 0$.

Figure 2

Confirming the Uniqueness of Equilibrium



From Figure 2, we see that the two equations have multiple solutions. Nevertheless, it can be shown that only the following solution

$$k_1 = 20.567, \ Z = 20.610.$$

is meaningful. All the other variables are economically meaningful. For instance, we also have a solution, $k_1 = 2.195$ and $Z = 7.726$. This point is economically meaningless because at this point we have

$$\bar{k}_1 = -34.230, \ \bar{k}_2 = 22.546, \ \bar{k}_3 = 3.557.$$

As $\hat{y}_1 = \lambda_1 \bar{k}_1 < 0$, we see that the disposable income is negative, which means negative consumption in country 1.

We confirmed that the dynamic system has a unique equilibrium. We calculate the other variables at $k_1 = 20.567$ and $Z = 20.610$, as in Table 1. The global output is 46.2 and the rate of interest is about 6.5 percent. The shares of the global outputs by the DE, IE, UE are respectively 8.31, 2.78 and 1.43. The differences in labor productivity are mainly due to the differences in knowledge utilization efficiency. The table also gives the labor and capital distributions between the sectors in each country and the capital distribution among the three countries. More than half of the global capital stocks is employed by the DE. The DE uses more capital stocks in research than the IE, even though its number of scientists is less than the number in the IE. The wage rates in the DE, ID and UD are respectively 5.53, 1.82 and 0.97. We calculate the trade balances at equilibrium as follows

$$E_1 = 1.146,\ E_2 = -0.385,\ E_3 = -0.761.$$

The DE is in trade surplus and the other two economies in trade deficit.

Table 1

The Equilibrium Values of the Global Economy

Z	K	F	r	C
20.610	126.665	46.199	0.065	39.831

Country 1		Country 2		Country 3		National shares	
F_1	24.03	F_2	10.85	F_3	11.33	F_1/F	0.520
F_1/N_{i1}	8.310	F_2/N_{i2}	2.78	F_3/N_{i3}	1.43	F_2/F	0.235
K_1	64.68	K_2	30.791	K_3	31.19	F_3/F	0.245
K_{i1}	59.47	K_{i2}	28.94	K_{i3}	29.88	K_1/K	0.511
K_{r1}	5.22	K_{r2}	2.13	K_{r3}	1.31	K_2/K	0.243
N_{i1}	2.89	N_{i2}	3.90	N_{i3}	7.92	K_3/K	0.246
N_{r1}	0.11	N_{r2}	0.15	N_{r3}	0.078	$\bar{K}_1/\bar{K}$	0.649
$\bar{K}_1$	82.27	$\bar{K}_2$	24.89	$\bar{K}_3$	19.51	$\bar{K}_2/\bar{K}$	0.196
C_1	21.94	C_2	8.89	C_3	9.0	$\bar{K}_3/\bar{K}$	0.154
w_1	5.53	w_2	1.82	w_3	0.97	C_1/C	0.551
$\bar{k}_1$	27.42	$\bar{k}_2$	6.22	$\bar{k}_3$	2.44	C_2/C	0.223
$\hat{y}_1$	34.73	$\hat{y}_2$	8.44	$\hat{y}_3$	3.57	C_3/C	0.226
c_1	7.313	c_2	2.22	c_3	1.13		

It is straightforward to calculate the four eigenvalues as follows

$$\{-0.27,\ -0.21,\ -0.18,\ -0.02\}.$$

The equilibrium point is stable.

4. Comparative Dynamic Analysis

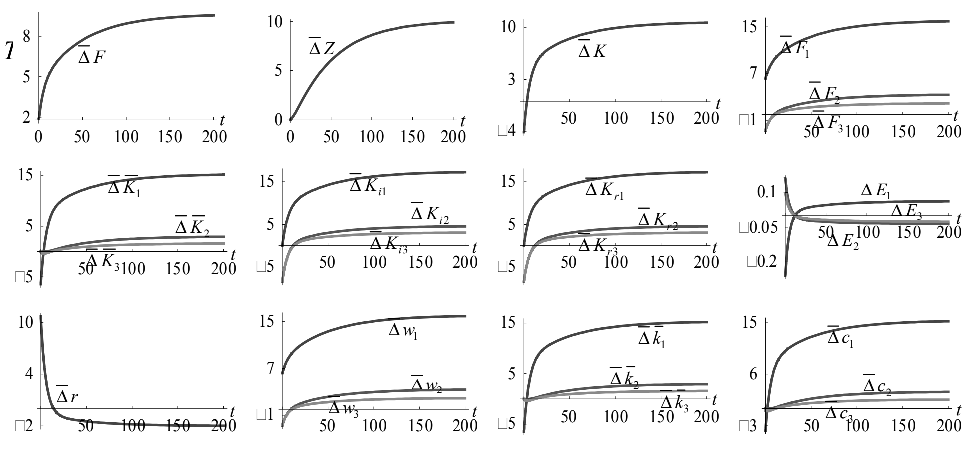
The previous section simulates the motion of the dynamic system. It is important to ask questions such as how a developing economy like India or China may affect the global economy as its technology is improved or population is enlarged; or how the global trade patterns may be affected as technologies are further improved or propensities to save are increased in developed economies like the USA or Japan. We now examine effects of changes in some parameters on the dynamic processes of the global economic system.

The DE's knowledge utilization efficiency being enhanced

First, we examine the case that the DE's knowledge utilization efficiency is increased as follows: $m_1 : 0.4 \Rightarrow 0.42$. The simulation results are plotted in Figure 3. We introduce a variable $\overline{\Delta} x_j(t)$ to stand for the change rate of the variable, $x_j(t)$, in percentage due to changes in the parameter. As the DE improves its knowledge utilization efficiency, the output, the knowledge and capital of the global economy are increased. The DE's output level rises; the other two countries' output levels fall initially and then rise. As the rate of interest rises initially and knowledge rises but not much initially, we see that the costs of production are high for the IE and UE and their productivities are not much improved, the two economies' output levels fall initially. As time passes, the world accumulates more knowledge and the rate of interest falls, the IE's and UE's output levels are increased. We see that in the long term the DE's trade balance is improved and the other two economies' trade balances slightly deteriorate. In the long term the wage rates and the levels of per capita consumptions and wealth in the three economies are all improved. Hence, we conclude that as UE improves its knowledge utilization efficiency, all the consumers in the globe benefit in the long term. We also conduct comparative dynamic analysis with regard to the UE's knowledge utilization efficiency. We observe that the effects on the global economy are similar.

Figure 3

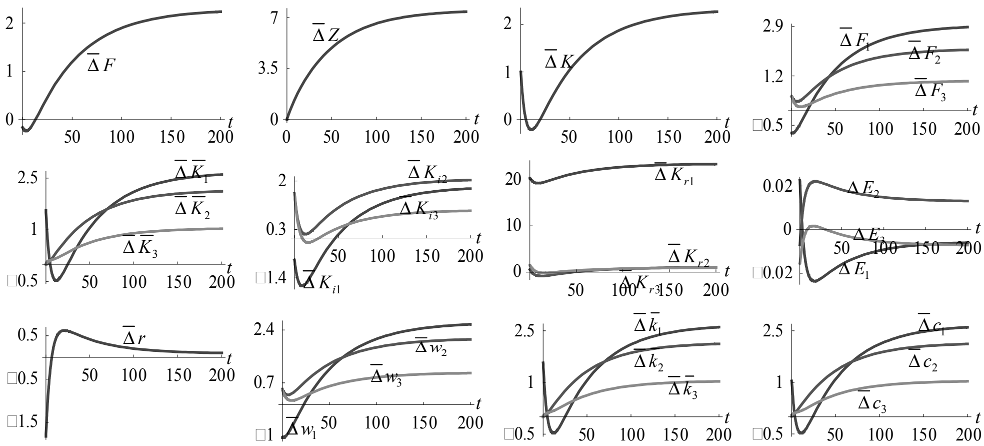
The DE's Knowledge Utilization Efficiency Being Enhanced



We now study how changes in the research policies affect the global economy. We consider that the DE increases its tax rate as follows: $\tau_1 : 0.05 \Rightarrow 0.06$. The simulation results are plotted in Figure 4. As the DE increases its tax rate to support its research activities, the knowledge, global wealth and output level are increased. The rate of interest is reduced initially and then increased in the long term. The DE's output level falls initially but rises in the long term. The other two countries' output levels are increased. As the DE's production sector has to pay more tax initially, its output level is reduced initially. As the world accumulates more knowledge and the DE applies knowledge mostly effectively, its output growth rate is higher than the other two countries' in the long term. The DE's wage rate, consumption level, and wealth level fall initially but rise in the long term. The other two countries' wage rates, consumption levels, and wealth levels are increased. The IE's trade balance is improved and the other two countries' trade balances deteriorate.

Figure 4

The DE's Tax Rate Being Enhanced

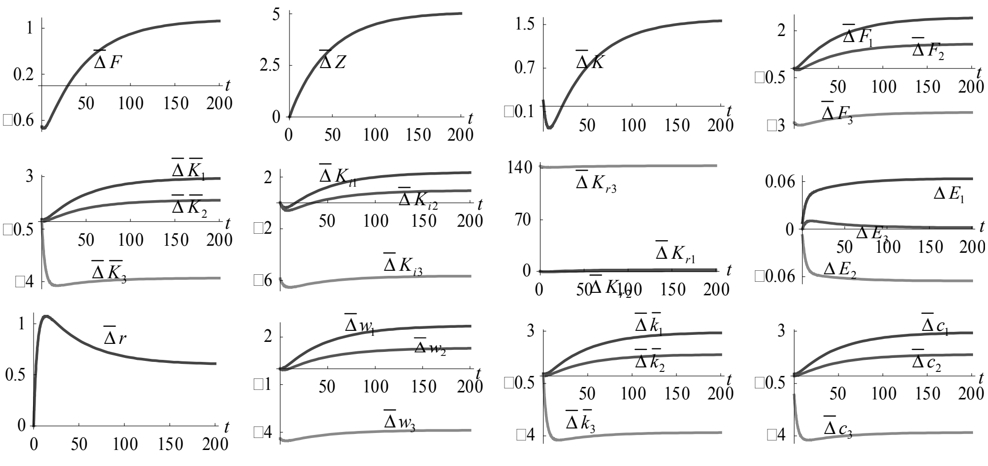


The UE's tax rate being enhanced

We study what happens to the global economy if the UE increases its tax rate as follows: $\tau_3 : 0.02 \Rightarrow 0.05$. As the UE strengthens its research policy, the knowledge stock is increased as more research is carried out by the UE. The global output and wealth are reduced initially as the UE puts more resources to research. Nevertheless, the global output and wealth are increased in the long term as a consequence of improved productivities in all the economies. The rate of interest is increased. This implies that capital costs are increased for all the economies. The UE increases its efforts in research as it increases capital and labor inputs. Nevertheless, the UE does not so effectively apply the knowledge as the other two economies. The net economic consequences on the UE are that its national output, national wealth and capital input of the production sector are all reduced. On micro level, the UE's wage rate and per capita levels of consumption and wealth are reduced. These conclusions on the UE imply that it will benefit the UE if its research resources are targeted at the fields which increase its abilities to apply the international public capital rather than to make contribution to the public capital with low abilities to using it. Moreover, the efforts in increasing the public capital will enlarge the gaps in income and wealth between the UE and the other two economies. We also see that the UE's trade balance is slightly affected, the DE's trade balance is improved, and the IE's trade balance is deteriorated. The DE and IE benefit from the increased efforts in research by the UE in terms of wage rate, wealth, national wealth, and national output. It should be remarked that according to Gersbach et al. (2013), "higher investment in basic research for a particular generation has three effects on the economy. First, basic research draws skilled labor from the production sector, thereby making skilled labor more costly and reducing consumption. Second, as basic research fosters innovation, it has a positive effect on the productivity and consumption level of the economy. And third, by increasing innovation success basic research also helps to prevent foreign entry, thereby raising innovation rents and income." (see also Arnold, 1997; Cozzi and Galli, 2009, 2011). Our conclusions from analyzing effects of encouraging research for the DE and the UE also provides insights into the complexity of research and knowledge as global public capital.

Figure 5

The UE's Tax Rate Being Enhanced

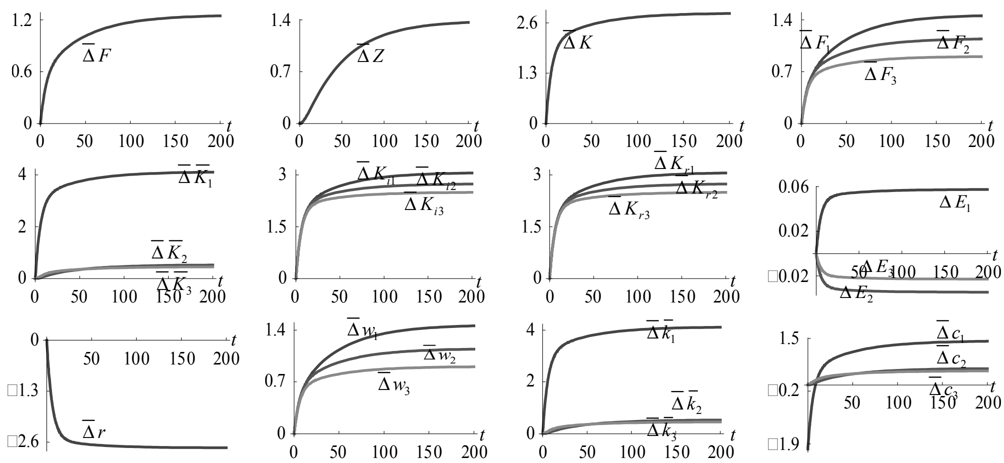


The DE's propensity to save being enhanced

We now study the effects of a rise in the DE's propensity to save as follows: $\lambda_{01} : 0.75 \Rightarrow 0.77$. The results are plotted in Figure 6. As the DE increases its propensity to save, the knowledge, global wealth, global output, and output of all the three economies are increased. The rate of interest is reduced and the wage rates of the three economies are increased. As the DE saves more out of its disposable income, its per capita consumption is reduced initially. The consumption level as a consequence of increased productivities. The wealth levels of the three economies are all increased. The consumption levels of the other two economies are increased. The DE trade balance is slightly improved and the IE's and UE's trade balances are slightly deteriorated.

Figure 6

The DE's Propensity to Save Being Enhanced



5. Conclusions

This paper proposed a multi-country growth model with capital accumulation and knowledge dynamics. Capital accumulation is through saving, while knowledge is through learning by doing and research. Knowledge is international public good and research is financially supported and conducted nationally. Production is neoclassical and is the same as in the one-sector growth Solow model. International trade is free and trade patterns are determined like in the Oniki-Uzawa model. We used a utility function, which determines saving and consumption with utility optimization without leading to a higher dimensional dynamic system like by the traditional Ramsey approach. The dynamics of J -country world economy is controlled by a $(J+1)$-dimensional differential equations system. We simulated the motion of the model for three economies and identified the existence of a unique stable equilibrium. We and carried out comparative dynamic analysis with regards to the knowledge utilization efficiency, the rates of tax for encouraging research, and the propensity to save. Our comparative analysis provides some insights into the complexity of international trade with endogenous wealth and knowledge. For instance, we show that as the UE encourages research by increasing its tax rate, the knowledge stock is increased; the global output and wealth are reduced initially and are increased in the long term; the rate of interest is increased; the UE increases its efforts in research; the net economic consequences on the UE are that its national output, national wealth and capital input of the production sector are all reduced; the UE's wage rate and per capita levels of consumption and wealth are reduced; the trade balances are slightly affected; the other economies benefit from the increased efforts in research by the UE in terms of wage rate, wealth, national wealth, and national output. These conclusions imply that it will benefit the UE if its resources are targeted at the fields which increase its abilities to apply the international public capital rather than to make contribution to the public capital with low abilities to using it. Moreover, the efforts in increasing the public capital will enlarge the gaps in income and wealth between the UE and the other economies. Our model may be extended and generalized in different directions. It is straightforward to develop the model in discrete time. It is possible to analyze behavior of the model with other forms of production or utility functions. In the contemporary literature, private research and endogenous population have been emphasized. We may also extend our model by learning from modelling some aspects of trade in the literature of gravity models (Yotov, et al. 2016).

Appendix A1: Proving the Lemma

First, from equations (1) we obtain

$$k_j = \phi_j(k_1, Z) \equiv \left(\frac{\bar{\tau}_j A_j \alpha_j Z^{m_j}}{\bar{\tau}_1 A_1 \alpha_1 Z^{m_1} k_1^{-\beta_1} + \delta_j} \right)^{1/\beta_j}, \quad j = 1, ..., J, \tag{A1}$$

where $\delta_j \equiv \delta_{k1} - \delta_{kj}$. It should be noted that $\phi_1 = k_1$. From (1) and (A1), we determine the wage rates as functions of $k_1(t)$ and $Z(t)$ as follows

$$w_j = \bar{\phi}_j(k_1, Z) \equiv \bar{\tau}_j A_j \beta_j Z^{m_j} \phi_j^{\alpha_j}(k_1, Z), \quad j = 1, \cdots, J. \tag{A2}$$

From (1) and (10), we have

$$\frac{K_{rj}}{K_{ij}} = \frac{\bar{\alpha}_j \tau_j}{\bar{\tau}_j \alpha_j}, \quad \frac{N_{rj}}{N_{ij}} = \frac{\bar{\beta}_j \tau_j}{\bar{\tau}_j \beta_j}. \tag{A3}$$

From (11) and (A3), we solve the capital and labor distribution between the production sector and the university in country j as follows

$$K_{qj} = a_{qj} K_j, \quad N_{qj} = b_{qj} N_j, \quad q = i, r, \quad j = 1, ..., J, \tag{A4}$$

where

$$a_{rj} \equiv \frac{\overline{\alpha}_j \tau_j}{\overline{\alpha}_j \tau_j + \overline{\tau}_j \alpha_j}, \ a_{ij} \equiv \frac{\overline{\tau}_j \alpha_j}{\overline{\alpha}_j \tau_j + \overline{\tau}_j \alpha_j}, \ b_{rj} \equiv \frac{\overline{\beta}_j \tau_j}{\overline{\beta}_j \tau_j + \overline{\tau}_j \beta_j}, \ b_{ij} \equiv \frac{\overline{\tau}_j \beta_j}{\overline{\beta}_j \tau_j + \overline{\tau}_j \beta_j}.$$

We conclude that the labor distribution is constant as it is determined by the tax rate and capital distribution is proportional to the total capital stocks employed by the country.

By $k_j = K_{ij} / N_{ij}$ and (A4), we have

$$K_j = \frac{N_{ij} k_j}{a_{ij}}, \ j = 1, ..., J. \tag{A5}$$

As k_j are functions of $k_1(t)$ and $Z(t)$, we see that $K_j(t)$ are also functions of $k_1(t)$ and $Z(t)$. From (A4), we also solve $K_{rj}(t)$ as functions of $k_1(t)$ and $Z(t)$. We see that the capital distribution among the countries and between sectors in each country are uniquely determined as functions of $k_1(t)$ and $Z(t)$. By $K = \sum_{j=1}^{J} K_j$, we see that K is also uniquely determined as a function of k_1 and Z. We denote this function as follows

$$K = \psi(k_1, Z).$$

Substituting $F_j = Z^{m_j} K_{ij}^{\alpha_j} N_{ij}^{\beta_j}$ into (8), we have

$$\dot{Z} = \Lambda(k_1, Z) \equiv \sum_{j=1}^{J} \left(\tau_{ij} A_j N_{ij}^{\beta_j} Z^{m_j - \varepsilon_{ij}} K_{ij}^{\alpha_j} + \tau_{rj} N_{rj}^{\beta_{rj}} Z^{\varepsilon_{rj}} K_{rj}^{\alpha_{rj}} \right) - \delta_z Z.$$

(A6)

We see that the motion of Z can be described as a unique function of k_1 and Z.

From (12), we solve

$$\bar{k}_1 = n_0 \psi(k_1, Z) - \sum_{j=2}^{J} n_j \bar{k}_j, \tag{A7}$$

in which

$$n_0 \equiv \frac{1}{N_1}, \ n_j \equiv \frac{N_j}{N_1}, \ j = 2, ..., J.$$

We see that country 1's per capita wealth, $\bar{k}_1(t)$, can be expressed as a unique function of the knowledge, country 1's capital intensity of production function and the other countries' per capita wealth, $\{\bar{k}(t)\}$, at any point of time.

From (2) and (3), we have

$$\hat{y}_j = (1 + r)\bar{k}_j + w_j. \tag{A8}$$

Substituting $s_j = \lambda_j \hat{y}_j$ and the above equations into (7), we have

$$\dot{\bar{k}}_1 = \Lambda_1(k_1, \bar{k}_1, Z) \equiv \lambda_1 w_1 - R(k_1, Z)\bar{k}_1, \tag{A9}$$

$$\dot{\bar{k}}_j = \Lambda_j(k_1, \bar{k}_j, Z) \equiv \lambda_j w_j - (1 - \lambda_j - \lambda_j r)\bar{k}_j, \quad j = 2, ..., J. \tag{A10}$$

in which $R(k_1, Z) \equiv 1 - \lambda_1 - \lambda_1 r$. Equations (A10) are the differential equations for $\bar{k}_j(t)$ in Lemma 2, $j = 2, ..., J$. Taking derivatives of (A7) with respect to t yields

$$\dot{\bar{k}}_1 = n_0 \psi_{k_1} \dot{k}_1 + n_0 \psi_Z \dot{Z} - \sum_{j=2}^{J} n_j \dot{\bar{k}}_j, \tag{A11}$$

where ψ_{k_1} and ψ_Z are the partial derivatives of $\psi(k_1, Z)$ with respect to k_1 and Z. Equaling the right-hand sizes of (A9) and (A11), we get

$$n_0 \psi_{k_1} \dot{k}_1 + n_0 \psi_Z \dot{Z} - \sum_{j=2}^{J} n_j \dot{\bar{k}}_j = \lambda_1 w_1 - R\bar{k}_1.$$

Substitute (A7) into the above equation

$$\dot{k}_1 = \Lambda_1(k_1, \{\bar{k}_j\}, Z) \equiv \left[\sum_{j=2}^{J} n_j \Lambda_j + \lambda_1 w_1 - n_0 R \psi + R \sum_{j=2}^{J} n_j \bar{k}_j - n_0 \psi_Z \Lambda\right] \frac{1}{n_0 \psi_{k_1}},$$

(A12)

where we use equations (A10) and (A6). This is the differential equation for $k_1(t)$ in Lemma 2. Substitute equations (A4), (A5), (A1) and (A12) into equation (A6), we have

$$\dot{Z} = \Lambda(k_1, Z) = \sum_{j=1}^{J} \left(\bar{\tau}_{ij} Z^{m_j - \varepsilon_{ij}} \phi_j^{\alpha_j} + \bar{\tau}_{rj} Z^{\varepsilon_{rj}} \phi_j^{\alpha_{rj}}\right) - \delta_z Z, \tag{A13}$$

where

$$\bar{\tau}_{ij} \equiv \tau_{ij} A_j N_{ij}, \quad \bar{\tau}_{rj} \equiv \tau_{rj} a_{rj}^{\alpha_{rj}} a_{ij}^{-\alpha_{rj}} N_{rj}^{\beta_{rj}} N_{ij}^{\alpha_{rj}}.$$

This is the differential equation for $Z(t)$ in the lemma. We proved the lemma.

Appendix A2: Examining the Dynamic Properties with Identical Preference

We now analyze a special case when all the households in the world have an identical preference and the depreciation rates are the same among the economies. That is

$$\xi = \xi_j , \ \ \lambda = \lambda_j , \ \ \delta_k = \delta_{kj} , \ \ \alpha = \alpha_j , \ \ j = 1, ..., J .$$

We are interested in this case because we can explicitly determine dynamic properties of the system. First, we show that all the variables in the dynamic system can be expressed as functions of $k_1(t)$ and $Z(t)$ at any point. From (1) we obtain

$$k_j(t) = M_j Z^{\overline{m}_j}(t) k_1(t), \ \ j = 1, ..., J, \tag{A14}$$

in which

$$M_j \equiv \left(\frac{A_j \overline{\tau}_j}{A_1 \overline{\tau}_1} \right)^{1/\beta}, \ \ \overline{m}_j \equiv \frac{m_j - m_1}{\beta}.$$

Country $j's$ capital intensity can be expressed as a unique function of the knowledge and country $1's$ capital intensity. The ratio between any two countries' capital intensities is related to the two countries' tax rates and the knowledge utilization efficiency. We determine the rate of interest and the wage rates as functions of $k_1(t)$ and $Z(t)$ as follows

$$r(t) = \overline{\tau}_1(t) A_1 \alpha Z^{m_1}(t) k_1^{-\beta}(t) - \delta_k ,$$
$$w_j(t) = \overline{\tau}_j A_j \beta \overline{\alpha}_j^{\alpha} Z^{\alpha \overline{m}_j + m_j}(t) k_1^{\alpha}(t), \ \ j = 1, \cdots, J. \tag{A15}$$

From (1) and (10), we have

$$\frac{K_{rj}(t)}{K_{ij}(t)} = \frac{\overline{\alpha}_j \tau_j(t)}{\overline{\tau}_j \alpha_j(t)}, \ \ \frac{N_{rj}(t)}{N_{ij}(t)} = \frac{\overline{\beta}_j \tau_j}{\overline{\tau}_j \beta_j}. \tag{A16}$$

From (11) and (A16), we solve the capital and labor distribution between the production sector and the university in country j as follows

$$K_{qj}(t) = \overline{a}_{qj} K_j(t), \ \ N_{qj} = \overline{b}_{qj} N_j , \ \ q = i, r, \ \ j = 1,..., J, \tag{A17}$$

where

$$\bar{a}_{rj} \equiv \frac{\bar{\alpha}_j \tau_j}{\bar{\alpha}_j \tau_j + \bar{\tau}_j \alpha}, \quad \bar{a}_{ij} \equiv \frac{\bar{\tau}_j \alpha_j}{\bar{\alpha}_j \tau_j + \bar{\tau}_j \alpha}, \quad \bar{b}_{rj} \equiv \frac{\bar{\beta}_j \tau_j}{\bar{\beta}_j \tau_j + \bar{\tau}_j \beta}, \quad \bar{b}_{ij} \equiv \frac{\bar{\tau}_j \beta}{\bar{\beta}_j \tau_j + \bar{\tau}_j \beta}.$$

The labor distribution is constant as it is determined by the tax rate and capital distribution is proportional to the total capital stocks employed by the country. By $k_j(t) = K_{ij}(t)/N_{ij}$ and (A17) and (A14), we have

$$K_j(t) = \overline{M}_j Z^{\overline{m}_j}(t) k_1(t), \quad j = 1, ..., J, \tag{A18}$$

where $\overline{M}_j = N_{ij} M_j / \bar{a}_{ij}$. Adding all the equations in (A18) yields

$$K(t) = k_1(t) \Lambda_0(Z(t)), \tag{A19}$$

where we use $K(t) = \sum_{j=1}^{J} K_j(t)$ and

$$\Lambda_0(Z(t)) \equiv \sum_{j=1}^{J} \overline{M}_j Z^{\overline{m}_j}(t).$$

From $F_j(t) = A_j Z^{m_j}(t) K_{ij}^{\alpha}(t) N_{ij}^{\beta}$ and (A17) and (A18), we have

$$F_j(t) = \bar{a}_{ij}^{\alpha} A_j N_{ij}^{\beta} \overline{M}_j^{\alpha} Z^{m_j + \alpha \overline{m}_j}(t) k_1^{\alpha}(t). \tag{A20}$$

Substituting (A17), (A18) and (A20) into (8), we have

$$\dot{Z}(t) = \Lambda(k_1(t), Z(t)), \tag{A21}$$

where

$$\Lambda \equiv$$

$$\sum_{j=1}^{J} \left\{ \tau_{ij} \bar{a}_{ij}^{\alpha} A_j N_{ij}^{\beta} \overline{M}_j^{\alpha}(t) Z^{m_j + \alpha \overline{m}_j - \varepsilon_{ij}}(t) k_1^{\alpha}(t) + \tau_{rj} \bar{a}_{rj}^{\alpha_{rj}} \overline{M}_j^{\alpha_{rj}} N_{rj}^{\beta_{rj}} Z^{\alpha_{rj} \overline{m}_j + \varepsilon_{rj}}(t) k_1^{\alpha_{rj}}(t) \right\} - \delta_z Z(t).$$

We see that the motion of $Z(t)$ can be described as a unique function of $k_1(t)$ and $Z(t)$.

From equations (2) and (3), we have $\hat{y}_j(t) = (1 + r(t)) \bar{k}_j(t) + w_j(t)$. Substituting $s_j(t) = \lambda \hat{y}_j(t)$ and the above equations into (7), we have

$$\dot{\bar{k}}_j(t) = \lambda w_j(t) - (1 - \lambda - \lambda r(t))\bar{k}_j(t). \tag{A22}$$

Multiplying the equation for $\bar{k}_j(t)$ by N_j and then adding the J resulted equations, we have

$$\dot{K}(t) = \lambda \beta k_1^{\alpha}(t)\sum_{j=1}^{J}\bar{\tau}_j A_j \bar{\alpha}_j^{\alpha} Z^{\alpha\bar{m}_j + m_j}(t) - \left(\bar{\lambda} - \bar{\tau}_1 A_1 \alpha \lambda Z^{m_1}(t)k_1^{-\beta}(t)\right)K(t),$$

(A23)

where we use (A15) and $K(t) = \sum_{j=1}^{J}\bar{k}_j(t)N_j$ and $\bar{\lambda} \equiv 1 - \lambda + \lambda\delta_k$.

Taking derivatives of (A19) with respect to t yields

$$\dot{K}(t) = \frac{K(t)\dot{k}_1(t)}{k_1(t)} + \left(k_1(t)\sum_{j=1}^{J}\bar{m}_1 \bar{M}_j Z^{\bar{m}_j - 1}(t)\right)\dot{Z}(t). \tag{A24}$$

Substituting (A23), (A19) and (A23) into (A24) yields

$$\dot{k}_1(t) = \tilde{\Lambda}(t) - \left(k_1(t)\sum_{j=1}^{J}\bar{m}_1 \bar{M}_j Z^{\bar{m}_j - 1}(t)\right)\frac{\Lambda(t)}{\Lambda_0(t)}, \tag{A25}$$

where

$$\tilde{\Lambda}(t) \equiv \frac{\lambda \beta k_1^{\alpha}(t)}{\Lambda_0(t)}\sum_{j=1}^{J}\bar{\tau}_j A_j \bar{\alpha}_j^{\alpha} Z^{\alpha\bar{m}_j + m_j}(t) - \left(\bar{\lambda} - \bar{\tau}_1 A_1 \alpha\lambda Z^{m_1} k_1^{-\beta}(t)\right)k_1(t).$$

Summarizing the above results, we obtain the following lemma.

Lemma A1

Assume that all the households in the world have the same preference. The motion of the two variables, $k_1(t)$ and $Z(t)$, are given by two differential equations, (A21) and (A25). For any given $k_1(t)$ and $Z(t)$, we determine $r(t)$ and $w_j(t)$, $j = 1, \cdots, J$, by (A15). The variables, $\bar{k}_j(t)$, are solved by equations (A22) as follows

$$\bar{k}_j(t) = e^{-\int(1 - \lambda - \lambda r)d\tau}\left(h_j + \lambda\int w_j(\tau)e^{\int(1 - \lambda - \lambda r)d\tau}\, d\tau\right), \quad j = 1, \cdots, J, \tag{A26}$$

where h_j are constants to be determined by initial conditions. For any given positive values of $Z(t)$, $k_1(t)$ and $\bar{k}_j(t)$ at any point of time, the other variables are uniquely determined by the following procedure: $k_j(t)$, $j = 2, \cdots, J$ by (A14) $\rightarrow$ N_{qj}, $q = i, r$, $j = 1, \cdots, J$ by (A17) $\rightarrow$ $K_j(t)$ by (A18) $\rightarrow$ $K_{qj}(t)$ by (A17) $\rightarrow$ $\hat{y}_j(t) = (1 + r(t))\bar{k}_j(t) + w_j(t)$ by (A21) $\rightarrow$ $c_j(t)$ and $s_j(t)$ by (6) $\rightarrow$ $F_j(t) = Z^{m_j}(t) K_{ij}^{\alpha}(t) N_{ij}^{\beta}(t)$.

The dynamic properties of the world economy are determined by the two differential equations. Equilibrium is determined by

$$\sum_{j=1}^{J} \left\{ \tau_{ij}\, \bar{a}_{ij}^{\alpha}\, A_j\, N_{ij}^{\beta}\, \overline{M}_j^{\alpha}\, Z^{m_j + \alpha \bar{m}_j - \varepsilon_{ij}}\, k_1^{\alpha} + \tau_{rj}\, \bar{a}_{rj}^{\alpha_{rj}}\, \overline{M}_j^{\alpha_{rj}}\, N_{rj}^{\beta_{rj}}\, Z^{\alpha_{rj} \bar{m}_j + \varepsilon_{rj}}\, k_1^{\alpha_{rj}} \right\} = \delta_z Z,$$

$$\lambda \beta k_1^{\alpha} \sum_{j=1}^{J} \bar{\tau}_j A_j \bar{\alpha}_j^{\alpha} Z^{\alpha \bar{m}_j + m_j} - \left(\bar{\lambda} - \bar{\tau}_1 A_1 \alpha \lambda Z^{m_1} k_1^{-\beta} \right) k_1 \Lambda_0 = 0. \tag{A27}$$

By the second equation in (A27), we solve

$$k_1 = \Omega_0^{1/\beta} \left(\frac{\lambda}{\bar{\lambda}} \right)^{1/\beta}, \tag{A28}$$

where

$$\Omega_0(Z) \equiv \frac{\beta}{\Lambda_0} \sum_{j=1}^{J} \bar{\tau}_j A_j \bar{\alpha}_j^{\alpha} Z^{\alpha \bar{m}_j + m_j} + \bar{\tau}_1 A_1 \alpha Z^{m_1}.$$

Substitute (A28) into the first equation in (A27)

$$\Omega(Z) \equiv \sum_{j=1}^{J} \left\{ \bar{\tau}_{ij} Z^{m_j + \alpha \bar{m}_j - \varepsilon_{ij} - 1} \Omega_0^{\alpha/\beta} + \bar{\tau}_{rj} Z^{\alpha_{rj} \bar{m}_j + \varepsilon_{rj} - 1} \Omega_0^{\alpha_{rj}/\beta} \right\} - \delta_z = 0, \tag{A29}$$

where

$$\bar{\tau}_{ij} \equiv \tau_{ij} \bar{a}_{ij}^{\alpha} A_j N_{ij}^{\beta} \overline{M}_j^{\alpha} \left(\frac{\lambda}{\bar{\lambda}} \right)^{\alpha/\beta} > 0, \quad \bar{\tau}_{rj} \equiv \tau_{rj} \bar{a}_{rj}^{\alpha_{rj}} \overline{M}_j^{\alpha_{rj}} N_{rj}^{\beta_{rj}} \left(\frac{\lambda}{\bar{\lambda}} \right)^{\alpha_{rj}/\beta} > 0.$$

From Lemma A1 and the above discussions, we have the following corollary.

Corollary A1

The number of equilibrium points is the same as the number of solutions of $\Omega(Z) = 0$, for $Z > 0$. For any solution $Z > 0$, all the other variables are uniquely determined by the following procedure: k_1 by (A28) $\rightarrow$ r and w_j, $j = 1, \cdots, J$, by (A15) $\rightarrow$ $\bar{k}_j = \lambda w_j / (1 - \lambda - \lambda r)$ $\rightarrow$ k_j, $j = 2, \cdots, J$ by (A14) $\rightarrow$ N_{qj}, $q = i, r$, $j = 1, \cdots, J$ by (A17) $\rightarrow$ K_j by (A18) $\rightarrow$ K_{qj} by (A17) $\rightarrow$ $\hat{y}_j = (1 + r)\bar{k}_j + w_j$ by (A21) $\rightarrow$ c_j and s_j by (6) $\rightarrow$ $F_j = A_j Z^{m_j} K_{ij}^{\alpha} N_{ij}^{\beta}$.

The number of equilibrium points is the same as the number of solutions of $\Omega(Z) = 0$, for $Z > 0$. As the expression is tedious, it is difficult to explicitly judge under what conditions the equation has a unique or multiple equilibrium points. To see that equation (A29) may have either a unique or multiple equilibrium points, we are concerned with a case that all the countries have identical population, identical production function, equal tax rate, and identical learning by doing and university's knowledge creation functions. In this case, the world economy is the same as a single economy. It is straightforward to show that in this case equation (A29) becomes

$$\Omega(Z) = \bar{\tau}_{0i} Z^{x_i} + \bar{\tau}_{0r} Z^{x_r} - \delta_z = 0,$$

in which we omit index j as all the countries are identical and

$$x_i \equiv \frac{m}{\beta} - \varepsilon_i - 1, \quad x_r \equiv \frac{\alpha_r m}{\beta} + \varepsilon_r - 1,$$

$$\bar{\tau}_{0i} \equiv J \bar{\tau}_i A \left(\beta \bar{\alpha}^{\alpha} \bar{\tau} + \alpha \bar{\tau}\right)^{\alpha/\beta} > 0, \quad \bar{\tau}_{0r} \equiv J \bar{\tau}_r \left(\beta \bar{\alpha}^{\alpha} \bar{\tau} + \alpha \bar{\tau}\right)^{\alpha_r/\beta} > 0.$$

It can be shown that the model in this case is a special case of the national growth model proposed by Zhang (2005: Chap. 9). The dynamic properties of the model are thoroughly examined. The properties are summarized in the following corollary.

Corollary A2

If $x_i < 0$ and $x_r < 0$ (or $x_i > 0$ and $x_r < 0$), the system has a unique stable (unstable) equilibrium point; and if $x_i < 0$ and $x_r < 0$ ($x_i > 0$ and $x_r < 0$), the system may have none, one, or two equilibrium points. When the system has two equilibrium points, the one with the higher value of Z is unstable and the other one is stable.

We interpret x_i and x_r respectively as measurements of returns to scale of the production sector and university in the dynamic system. When $x_j < (>) \, 0,$ we say that sector j displays decreasing (increasing) returns to scale in the dynamic economy. Hence, if the both sectors display decreasing (increasing) returns, the dynamic system has a unique equilibrium; if one sector displays decreasing (increasing) returns and the other sector exhibits increasing (decreasing), the system may have none, one, or two equilibrium points.

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