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BARGAINING SOLUTIONS AND PUBLIC POLICIES IN MATCHING MODELS

The aim of this paper is to show that usually the literature under or overestimate the effect of public policies on the labor market by using systematically a symmetric Nash bargaining solution to share the surplus between worker and firm. Indeed, using a matching model framework, we compare and analyze Nash, Egalitarian, Kalai-Smorodinsky and Equal-Sacrifice solutions. We show that the effects of public policies, as minimum wage or unemployment benefits, would be more or less significative depending on the bargaining solution. It appears that Nash and egalitarian solutions are less inclined to study the impacts of the introduction of the minimum wage.
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1. Introduction

The basic theoretical models neglect many important aspects of the labor market as the process of creation and destruction of jobs. Furthermore, they omit the possibility of wage negotiations and do not adequately address the issue of labor market participation. To overcome these weaknesses, several studies (Diamond, 1982; Mortensen, 1982; Pissarides, 2000) have developed the matching model as a theoretical framework. The advantage of such a model is to explicit the process of creation and destruction of jobs and the reallocations of workers and jobs. Therefore, this framework takes into account frictional unemployment and introduces the wage bargaining process.

However, the relevance of these models could be questioned since they use systemically Nash as a rule for surplus sharing. Indeed, this kind of solution could not be appropriated in some cases and leads to move away from the labor market reality. Consequently, it could

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skew the analysis and the policy decisions. Experiments due to Siegel and Fouraker (1960), Nydegger and Owen (1974) also suggest that the Nash solution is an unreasonable model of pairwise negotiations. The reason why is that players make an interpersonal comparison of utility gains such as would be the case with for example the equal-gain model of Myerson (1977) but cannot occur with the Nash solution because of the independence of irrelevant alternatives axiom.

There are other more realistic solutions that can bring more precisions in the wage bargaining process. Among them, we can include the Equal-Sacrifice, the Kalai-Smorodinsky solution (KS, thereafter) (Kalai-Smorodinsky, 1975) or the Equal-even-loss solution (Chun, 1988). The choice of solution could be decisive in the interpretation and in determining the effects of public policies on the performance of the labor market. Indeed, most theoretical work studying the effects of minimum wage on the labor market participation and on employment have used Nash surplus sharing rule.

In this perspective, Gerber and Upmann (2006) analyze a classic bargaining problem between a labor union and an employers' federation through the Nash and KS solutions. Notably, they point out the effect of the reservation wage on the employment and on the wage determination. Indeed, they conclude that a higher reservation wage leads to a lower employment level with the Nash Solution, whereas the KS-solution leads up to an ambiguity (Anbarci et al., 2002; Laroque and Salanié, 2004).

Our paper fits into this line and aims to show that usually the literature under or overestimate the effect of public policies on the labor market by using systematically a symmetric Nash bargaining solution to share the surplus between worker and firm. Indeed, using a matching model framework, we compare and analyze Nash, Egalitarian, Kalai-Smorodinsky and Equal-Sacrifice solutions. We show that the effects of public policies, as minimum wage or unemployment benefits, would be more or less significative depending on the bargaining solution. We show that Nash and egalitarian solutions are less inclined to study the impacts of the introduction of the minimum wage.

The rest of paper is organized as follows. The model is presented in Section 2. Then, the comparison of the bargaining solutions is discussed in Section 3. The effects of minimum wage and of unemployment benefits depending on the selected solution are presented in Section 4. Finally, section 5 concludes the paper.

2. Model

We consider a matching model with standard hypothesis according to Pissarides (2000), in which an economy is composed of a large exogenous number of workers and a large endogenous number of firms. Firms are supposed to be identical and offer a single job. The hypothesis of firm free-entry enables to maintain a fixed number of firms at the steady state. Agents are risk neutral and discount the future with the same rate of time preference denoted by r. The exogenous job destruction rate is s.

Frictions are present in the labor market which means that it takes time for firms with a vacant job to find a worker. Such frictions are represented by a constant-returns matching function $m(v,u)$, where u is the number of employable unemployed workers and v is the number of vacant jobs. This matching function (Pissarides, 2000) is an homogenous function of degree 1, increasing in v and u. Instantaneous matching depends on the market tightness, noted $\theta=v/u$. The probability for a firm to meet an employable worker is given by $q(\theta)=m(1,1/\theta)$. This probability is a decreasing function of θ.. A rise in the number of vacancies leads to a negative impact on the rate to fill a job due to the congestion effect. The probability for an employable worker to find a job is given by $p(\theta)=\theta q(\theta)$. This hiring probability is increasing in θ. Indeed, a rise of vacancies implies more opportunities for workers to find a job.

According to the usual Belmann's equations, the expected utility of an employed worker, denoted U_1, depends on his current wage w and on the probability, that he become unemployed (under the destruction rate s):

$$rU_1(w, \theta) = w - s(U_1 - d_1) \tag{1}$$

We denote by d_1, the expected utility of an unemployed worker, depending on the unemployment benefits b and on the probability to find a job $p(\theta)$.

$$rd_1(w, \theta) = b + p(U_1 - d_1) \tag{2}$$

Concerning the firms, we consider that the jobs are either filled or vacant. In the case of a filled job, the profit is composed of the net instantaneous income (y-w) and the future profits with respect to the destruction rate.

$$rU_2(w, \theta) = y - w - s(U_2 - d_2) \tag{3}$$

As long the job is not filled, a firm has to invest c to create a job and to look for a worker. The probability to fill the job is equal to $q(\theta)$. The value of a vacant job d_2 is given by:

$$rd_2(w, \theta) = -c + q(\theta)(U_2 - d_2) \tag{4}$$

The free-entry hypothesis implies that new jobs are created until the optimal value of a vacant job be equal to zero.

3. Wage Bargaining and Surplus Sharing

In the literature, authors usually apply the Nash solution. Why always using this solution? Some bargaining solutions are more realistic and can be used in the framework of matching models.

3.1. Nash solution

The Nash solution is the best-known solution. We respectively define by β and $1-\beta$ the bargaining strength of workers and firms. The generalized Nash optimization program is given by:

$$\text{Nash} : max\ (U_1 - d_1)^{\beta}(U_2 - d_2)^{1-\beta}$$

According to the equations (1) to (4), we deduct that:

$$U_1 - d_1 = \frac{w-b}{r+s+p(\theta)} \tag{5}$$

$$U_2 - d_2 = \frac{y-w+c}{r+s+q(\theta)} \tag{6}$$

It leads to the following negotiated wage:

$$w_N = \frac{y+c+b\Psi(\theta)\left(\frac{1-\beta}{\beta}\right)}{1+\Psi(\theta)\left(\frac{1-\beta}{\beta}\right)} \tag{7}$$

where $\Psi(\theta) = \frac{r+s+q(\theta)}{r+s+p(\theta)}$

Even if it is the most applied, it would be relevant to pursue this analysis by comparing it with the other bargaining solutions.

3.2. Egalitarian solution

In real bargaining situations, each agent makes interpersonal comparisons of its utility. One of these solutions is to apply a principle of equal gains. For any two-person bargaining problem (F,d), we define the egalitarian solution to be the unique point U in the subset F that is weakly efficient in F and satisfies the equal-gains condition:

$$U_1 - d_1 = U_2 - d_2 \tag{8}$$

By applying this solution to the matching model, we have the following result:

$$\frac{w-b}{r+s+p(\theta)} = \frac{y-w+c}{r+s+q(\theta)} \tag{9}$$

$$w_e = \frac{y + c + b\Psi(\theta)}{1 + \Psi(\theta)} \tag{10}$$

The egalitarian solution is a particular case of the Nash solution.

Proposition 1. *The wage issued from the egalitarian solution is equal to the Nash symmetric solution because the negotiation power value is equal to 0.5.*

3.3. KS solution

Amine et al. (2009) compared the Nash and the Kalai-Smorodinsky (KS) solutions in the framework of a standard matching model. They determined the value of the negotiation power in the case of the KS solution:

$$\frac{1-\beta}{\beta} = \frac{y-b+c}{y-b}$$

They proved that the power of the workers (noted β) in KS is less compared to the firms. KS enables to define the maximal utility of each agent ($U_1^{\max}, U_2^{\max}$). This utopia point is not feasible but an agent looks for reaching this point. It makes the bargaining process more realistic. The aim of the negotiation is to go away the least from this point. Each worker and each firm has a maximal payoff represented by an ideal point *I*. Concerning the firm, his ideal is to have a maximum profit resulting from the lowest wage $\tilde{w}$ payed to each worker (*i.e.* a wage equal to the unemployment benefits *b*, $\tilde{w} = b$). The ideal wage $\hat{w}$ for the worker is equal to his productivity ($\hat{w} = y$). In this case, the probability for a worker to find a job $p(\hat{\theta})$ and the probability for a firm to recruit a worker $q(\tilde{\theta})$ are supposed maximal ($p(\hat{\theta}) = 1$ and $q(\tilde{\theta}) = 1$).

The Kalai-Smorodinsky optimization program (11) gives us the following negotiated wage (12):

$$\phi^{KS} = (U_2 - d_2)(U_1^{\max} - d_1) - (U_2^{\max} - d_2)(U_1 - d_1) = 0 \tag{11}$$

$$w_{KS} = \frac{y + c + b\Psi(\theta)\left(\dfrac{y-b+c}{y-b}\right)}{1 + \Psi(\theta)\left(\dfrac{y-b+c}{y-b}\right)} \tag{12}$$

3.4. Equal Sacrifice solution

This solution (Aumann and Maschler, 1985) equalizes the sacrifice from the maximum feasible payoff net of the threat point. The interest of this bargaining solution is that the negotiation can lead to an equilibrium less than the disagreement utility. For any two-person bargaining problem (F,d), we define the equal sacrifice solution to be the unique point U in F that is weakly efficient in F and satisfies the equal-sacrifice condition:

$$U_1^{\max} - U_1 = U_2^{\max} - U_2 \tag{13}$$

$$(U_1^{\max} - d_1) - (U_1 - d_1) = (U_2^{\max} - d_2) - (U_2 - d_2) \tag{14}$$

$$\frac{\hat{w} - b}{r + s + p(\hat{\theta})} - \frac{w - b}{r + s + p(\theta)} = \frac{y - w + c}{r + s + q(\theta)} - \frac{y - \tilde{w} + c}{r + s + q(\tilde{\theta})} \tag{15}$$

$$\frac{y - b}{r + s + 1} - \frac{w - b}{r + s + p(\theta)} = \frac{y - b + c}{r + s + 1} - \frac{y - w + c}{r + s + q(\theta)} \tag{16}$$

The wage w_{es} is then deducted:

$$w_{es} = \frac{y + \frac{1 - q(\theta)}{r + s + 1} c + b\Psi(\theta)}{1 + \Psi(\theta)} \tag{17}$$

3.5. Comparison with the bargaining solutions

The Nash solution introduces the negotiation power through the parameters β and $1-\beta$.. By comparison, we can determine these values for each bargaining equilibrium. (7) and (12) let us estimate the negotiation power for the KS solution, noted β_{KS} :

$$\beta_{KS} = \frac{y - b}{2y - 2b + c} \tag{18}$$

From equations (7) and (17), the bargaining power of the equal sacrifice, noted β_{es} , is characterized by:

$$w_{es} = \frac{\Psi(\theta)\left[(y - b)(r + s + 1) + c(1 - q(\theta))\right]}{\Psi(\theta)\left[(2y - 2b + c)(r + s + 1) + c(r + s + q(\theta))\right] + c(r + s + q(\theta))} \tag{19}$$

At the end to establish the relation between the process of job creation and the negotiated wages, we calculate the first derivatives of the wage equations (7), (10), (12) and (17) with regard to the labor market tightness θ. We obtain a positive relation:

Proposition 2 *The negotiated wages issued from the bargaining solutions are increasing with the labor market tightness.*

Besides, by comparing the expressions of wages (7), (10), (12) and (17) between them, we establish the following result:

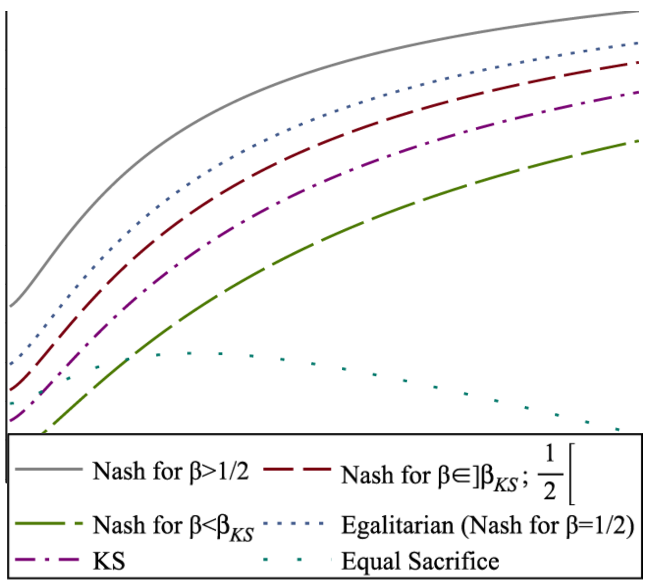
Proposition 3 *The wages determined by the four-bargaining solution are equal if and only if the negotiation power of each agent is equal (β=0.5) and the cost of a vacant job is null (c=0).*

This result is very interesting because it shows that it is not enough that the bargaining power is evenly divided between workers and firms so that the negotiated wage is the same. In addition, it is necessary that the cost of job creation is equal to zero. Already at this level of analysis, we can indeed imagine that certain solutions can discriminate or favor an agent earlier than the other one during the surplus sharing process.

The figure 1 gives the position of the bargaining curves. According to the value of the negotiation power, we deduce that the Nash curve is above the other curves for β*>0.5* and between the egalitarian and the KS curves for $\beta \in]0;0.5[$.

Figure 1

Bargaining curves



4. Public Policies Effects

In this section, we will analyze the effects of unemployment benefits and the minimum wage on wages negotiated based on the solution.

4.1. Unemployment benefits

We focus our analysis on the effect of the unemployment benefit on the bargaining equilibria. Analytically by deriving equations of negotiated wages with respect to benefits *b*, we get a positive effect for Nash, Egalitarian and Equal Sacrifice solutions. Concerning KS solution, the negotiated wage is increasing with the unemployment benefits if:

$$(y-b+c)(y-c+b+\Psi(y+c-b))<0 \tag{20}$$

To measure the effect magnitude on negotiated wage, we have to distinguish several cases corresponding to different values of the negotiation power. We denote by $\hat{\beta}$ the negotiated power for which the effect of the unemployment benefits is the same in the Nash and KS solutions:

$$\hat{\beta}=\frac{(y-b)(2\Psi c+(1+\Psi)(y-b))+\Psi c^2}{2\Psi((y-b)^2+c(c+2y-2b))+2(y-b)^2-c^2} \tag{21}$$

Comparing the effect magnitude on wages depending on the value of beta is done in our model by simulation. For this exercise, we retain the Cobb-Douglas matching function: $M(V,U)=V^{1/2}/U^{1/2}$, which gives $q(\theta)=\theta^{1/2}$ and the standard parameters values: $c = 0.3$; $r = 0.05$; $s = 0.15$; $y = 1$. The results are summarized in the following tables.

Table 1

Unemployment benefits effect for $\beta < 1/2$

	w_N	w_{KS}	w_e	w_{es}
b	+++	+	++	++

Table 2

Unemployment benefits effect for $\beta = 1/2$

	w_N	w_{KS}	w_e	w_{es}
b	++	+	++	++

Table 3

Unemployment benefits effect for $\beta \in]1/2;\bar{\beta}[$

	w_N	w_{KS}	w_e	w_{es}
b	++	+	+++	+++

Table 4

Unemployment benefits effect for $\beta \in]\bar{\beta};1[$

	w_N	w_{KS}	w_e	w_{es}
b	+	++	+++	+++

From these tables, we easily see that the Nash solution, which is most often used in matching models, overestimate the effect of unemployment benefits only when the bargaining power β of workers is less than *1/2*. We also note that the Equal sacrifice solution strengthens the increase in negotiated wage for all values of β. The use of this solution can change or reverse the results of several theoretical studies that often use Nash as a sharing rule. In addition, we establish the relation with Proposition 3 and we obtain the following remark:

Remark 1. *In the case where β =1/2 and c = 0, the effect of the unemployment benefits for all the solutions is the same.*

4.2. Minimum wage

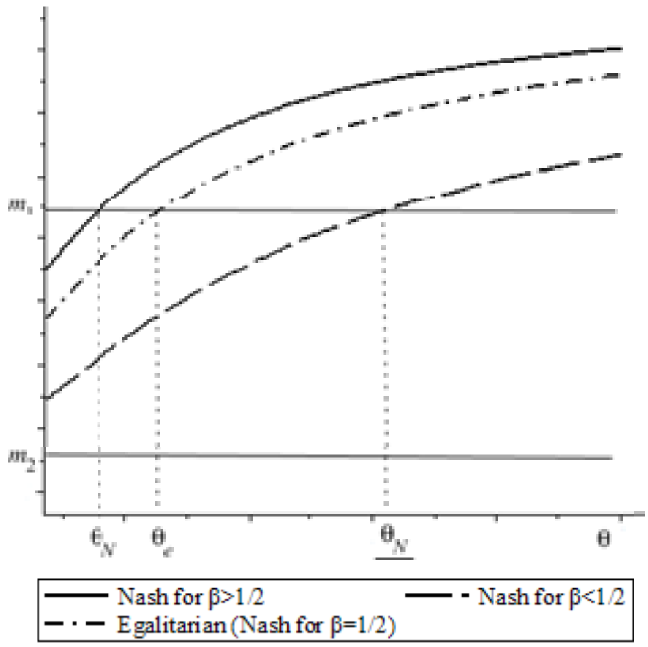
By introducing this device in our model, we have to study two situations: either the wage is constrained by the minimum wage (noted *m*) or the wage stays "free". Indeed, if the negotiated wage *w* is greater than *m*, then the wage setting is free. If the negotiated wage *w* is lower than *m*, then the wage at the equilibrium is equal to the minimal wage such as *w* = *m*.

4.2.1 Nash and Egalitarian solutions

The introduction of the minimum wage in the standard matching models does not have direct consequences on the negotiated wage for the Nash and the Egalitarian solutions. For these cases, the minimum wage only acts as a constraint. Consequently, we get the same wages as the previous section for the Nash and the Egalitarian solutions. We have to characterize two situations resulting from this constraint depending on the value of *m* (Figure 2).

Figure 2

Minimum Wage in the case of the Nash and Egalitarian Solutions



4.2.2 KS and Equal Sacrifice solutions

Concerning the KS and the Equal Sacrifice Solutions, the minimum wage acts both on the negotiated wage and as a constraint. Contrary to the previous solutions, the ideal utility is necessary to determine them. Consequently, the negotiated wage for KS and the Equal Sacrifice differ from those of the previous section. Indeed, it impacts the maximal utility of the firm. The best situation for the firm is to pay the lowest wage to the worker, *i.e.* the minimum wage *m* instead of the unemployment benefits *b*:

$$U_2^{\max} - d_2 = \frac{y - m + c}{r + s + 1} \tag{22}$$

Using this expression (19), we obtain a new negotiated wage for the KS and the Equal Sacrifice solutions:

$$w_{KS} = \frac{(y+c)(y-m) + b\Psi\left(y - m + c\right)}{y - b + \Psi\left(y - m + c\right)} \tag{23}$$

$$w_{ES} = \frac{y + c + \dfrac{r+s+q}{r+s+1}(m-b-c) + b\Psi}{1+\Psi} \tag{24}$$

Showing that the first derivatives of the equations (23) and (24) with respect to the minimum wage are positive ($\frac{\partial w_{KS}}{\partial m} > 0; \frac{\partial w_{ES}}{\partial m} > 0$), we can announce the following result:

Proposition 4 *In a matching model where wage bargaining is done according to the KS and Equal Sacrifice solutions, an increase in the minimum wage has the effect of strengthening the bargaining power of workers by increasing wages negotiated.*

By comparing the magnitude of the impacts of minimum wage in the case of the two solutions, we deduce that the effect of m is more important in the KS solution than in the ES solution for:

$$m > \frac{1}{\Psi}(y - b + \Psi(y+c) - (\frac{(y-b)(y-b+c)\left(r+s+1\right)(1+\Psi)\Psi}{r+s+q})^{1/2}) \tag{25}$$

The main purpose of the introduction of minimum wage in our model was to show that much of the literature related to matching models uses Nash solution. However, this solution does not capture the effect of minimum wage on the variables of the model since it acts indirectly on wage bargaining. While KS and Equal Sacrifice solutions seem more effective to study the effect of m which acts, in this case, directly on the mechanism of surplus sharing between the worker and the firm.

5. Final remarks

As we explained in the introduction of this paper, most of the literature about matching models generally retains the Nash solution, without justifying this choice, and discussing its relevance. However, other solutions can actually be applied, not without consequences. Indeed, by introducing other solutions (Kalai-Smorodinsky, Equal Sacrifice and Egalitarian) in a matching model, we show that the effects of public policies, particularly the minimum wage and unemployment benefits, are different. We conclude that the choice of the solution may actually be decisive and should therefore be subject to further and systematic analysis. Moreover, given the economic crisis in which the bargaining power of workers is weakened, it is important to be careful and thorough in choosing the bargaining solution. We also believe that a deeper analysis in terms of labor market participation and productivity could give us more details as to the suitability of the studied solutions in this article.

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