ИКОНОМИЧЕСКИ ИЗСЛЕДВАНИЯ
ECONOMIC STUDIES
Volume 30 (7), 2021

Dilian Vassilev[1]

A MODEL OF NATURAL INTEREST RATE: THE CASE OF BULGARIA[2]

The proposed model estimation of the natural interest rate for Bulgaria is based on the seminal model of Laubach and Williams (2003), as important modifications are implemented in order to capture the specifics of the Bulgarian economy. As a small and open economy, the real effective exchange rate is included in measurement equations as well as the Eurozone output gap. Second, we incorporate stylised facts and observations about the behaviour of the Bulgarian economy, such as the steady-state growth rates of potential output and initial guidance about the level of natural interest rates. We circumvent the "pile-up" issue by imposing certain assumptions about the level and growth rates of potential output and time preferences of economic agents. In order to validate the consistency and reliability of the assumptions, we counterfactually evaluate the past and present BG monetary conditions by estimating the real rate gap, i.e. compare the observed real interest rate (r) against the natural rate (r).*

We find that, contrary to many advanced economies, the natural real interest rate of the Bulgarian economy does not show a declining trend, i.e. the economy after 2008, i.e. it is not under the precondition of "secular stagnation". This means that BNB's monetary space is far from being exhausted so far. This is due to the fact, that Bulgarian productivity growth (as a catching-up economy) is predominantly exogenous (imported) and the growth rate of productivity proved sustainable even after 2008 and well compensates for the detrimental demographics. The results from the Taylor rule exercise confirm counterfactually, that the Bulgarian short term interest rates are justified, thus the transition to the inflation targeting regime of ECB is expected to be smooth.

Keywords: NIR; secular stagnation; inflation expectations; real interest rate
JEL: B15; N10; E50; O47

1. Introduction

The debate about natural interest rate (NIR[3]) is among the most heated in the recent years due to the concept of the "secular stagnation" hypothesis put forward by L. Summers (Summers, 2013, 2015). He advocated that there might be an exhaustion of monetary space

[1] *Dilian Vassilev, PhD student in Faculty of Economics, Sofia University "St. Kliment Ohridski", e-mail: dilian.vassilev@gmail.com.*

[2] This paper should be cited as: *Vassilev, D. (2021). A Model of Natural Interest Rate: The Case of Bulgaria. – Economic Studies (Ikonomicheski Izsledvania), 30 (7), pp. 46-72.*

[3] The natural rate of interest is a real short-term rate when the economy is at full employment and stable inflation.

in front of many advanced central banks. Summers argued that „*it may have become all but impossible to boost growth by using the old standard of lowering interest rates to encourage more investment and consumer spending*". A constantly lower natural interest rate could bring the economy in the unwanted and detrimental equilibrium of stagnating growth for a long time as the central bank could not open further the real rate gap (r*-r) due to ZLB (Zero lower bound of the interest rates). Theoretically, it is the real rate gap that affects in reality, the investment decisions of corporates and the consumption of durable goods of the households as it is the ultimate driving factor behind saving/investment decisions of the economic agents.

The recent contemporary understanding about natural rates is dominated by the seminal contribution of Michael Woodford (Woodford, Walsh 2005). He postulates that, the natural rate is a certain level of the real interest rate where inflation is stable and the output gap is zero in the environment of flexible prices. The natural interest rate is considered in the long-term horizon and is not affected by cyclical variations of prices and output gaps. The natural interest rate is a theoretical concept and as an unobservable variable has to be estimated by a model.

A central bank that targets inflation needs to form a general understanding of the level, dynamics and behaviour of the natural interest rate, in order to evaluate whether and how much accommodative or tightened is the level of the short term interest rate. In the Taylor-style rules[4] for monetary policy, this natural rate is often denoted by r-star (r*). The Taylor rule tells a monetary policymaker, that if inflation is at target and there is a zero output gap, the real policy rate should be set to equal the natural interest rate. If the Taylor rule is close to the actual nominal policy rate, the central bank is perceived to have a reasonable, rule-based approach to monetary policy. Initially, J. Tylor assumed that the natural interest rate equals the average of very long-run real interest rate. The later research proved that this approach has shortcomings, as it does not allow for structural variation in the key macroeconomic variables, for instance, the downward trend of potential growth rate due to demographic and/or productivity factors.

2. Literature Overview

The natural interest rate, as an unobservable variable and has to be estimated by a model. In addition, it is time-varying and strongly influenced by another unobservable variable, namely the potential output, respectively the output gap. This complicates enormously the task of pinpointing the natural interest rate at time t_0 with a comfortable degree of certainty. Nevertheless, the body of literature striving to capture the natural interest rate grew strongly in the period after 2003 and exploded after 2013 (Brand, Bielecki, Penalver, 2018).

[4] $i_t = \pi_t + r_t^* + a_\pi(\pi_t - \pi_t^*) + a_y(y_t - y_t^*)$ as i_t is the nominal policy rate, r_t^* is the natural interest rate, π_t is inflation, π_t^* is the inflation target, a_π and a_y are the response parameters (usually set to 0.5), and y_t and y_t^* are real GDP and potential GDP respectively.

*2.1. Laubach and Williams model for r**

The original monetary policy rule of Taylor holds the assumption that the natural rate is stable over a long period. In the long run, the output gaps average out to zero and so the average real interest rate will never be far away from the long-term natural interest rate. That assumption has been put into question by a key work of Laubach and Williams in 2003 (Laubach, Williams, 2003) and later enhanced by Holston in 2016 (Holston, Laubach, Williams, 2016). Laubach and Williams (hereafter LW), proposed a model for estimation of r* for the US economy that shows: 1) r* is not stable over long periods but time-varying due to structural factors; 2) is slowly declining towards 0, which raises the probability of moving the whole economy in a secular stagnation (due to exhaustion of monetary space).[5] In addition, they document noteworthy area of uncertainty around the point estimates.

LW have built a semi-structural model in a simple New Keynesian framework. Using a Kalman filter, they jointly estimate the three unobserved variables: natural interest rate (NIR), potential output and trend growth rate. LW adopt a semi-structural methodology, which makes use of the correlations among real output gap, core inflation and real interest rate gap, which is the difference between real interest and its equilibrium (natural) value. The observed variables include the real GDP and core inflation, while state variables include the trend growth rate, potential GDP, and a random-walk drift term mimicking households' time preferences.

LW model assumes an explicit relationship between the natural rate of interest r* and the estimated trend growth of GDP (g) and some time-preference parameters (z).

1) $r_t^* = c\, g_t + z_t$

Equation 1 says that the natural rate of interest r* is supposed to equal trend growth rate g in the absence of strong time preference of the economic agents z, namely households.

Technically, they use a state-space environment with two observation equations that represent a backwards-looking Philips curve and IS curve.

2) $\pi_t = B_\pi(L)\pi_{t-1} + b_y\overline{y}_{t-1} + b_i(\pi_t^I - \pi_t) + b_o(\pi_{t-1}^O - \pi_{t-1}) + \varepsilon_\pi$

Where the inflation π_t is determined by its own lag $B_\pi(L)\pi_{t-1}$, one lag in the output gap $b_y\overline{y}_{t-1}$ and two variables approximating shock in the relative prices – import inflation $b_i(\pi_t^I - \pi_t)$ and a lag in oil price $b_o(\pi_{t-1}^O - \pi_{t-1})$.

3) $\overline{y}_t = a_{t,1}\overline{y}_{t-1} + a_{t,2}\overline{y}_{t-2} + \frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r_{t-j}^*) + \varepsilon_y$

The output gap $\overline{y}_t$ is determined by its own two lags $a_{t,1}\overline{y}_{t-1} + a_{t,2}\overline{y}_{t-2}$, as well as a moving average of the real rate gap $\frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r_{t-j}^*)$ and an error term ε_y.

In the LW model, the state equations are three:

4) $y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_t$

[5] With low r*, when an economy enters recession, the policymakers would be unable to lower the policy interest rates to stimulate activity and inflation due to ZLB.

5) $g_t = g_{t-1} + \epsilon_t$

6) $z_t = D_z\,(L) z_{t-1} + \epsilon_t$

A complex issue in the estimation is finding the values for the standard deviation of the trend growth of potential output and for the households' time preferences, ($\boldsymbol{z_t}$) the so-called "pile-up" issue as discussed in (Stock and Watson 1998). This problem suggests that when the variation in the trend component is small, compared to the overall variation in the series, the maximum likelihood estimates of the signal-to-noise ratio are likely to be biased towards zero. In other words, if the variation of the trend growth rate is small – which seems plausible for most economic time-series – the maximum likelihood estimator of the variance of its changes is biased towards zero, because a large amount of probability *piles-up* at zero in the density function. LW overcame the *pile-up issue* by imposing two assumptions.

1) They assumed that the standard deviation of the trend growth of potential output, (σ_g), is the standard deviation of the i.i.d shocks in the growth rate of potential output, divided by the standard deviation in the potential GDP level and the value obtained is the standard deviations of the quarterly trend growth rate. $\boldsymbol{\lambda_g} \equiv \frac{\sigma_g}{\sigma_{y^*}}$ (For annualised trend growth rate, the value was multiplied by 4).
2) The standard deviation for the households' time preferences, ($\boldsymbol{z_t}$)denoted by $\boldsymbol{\lambda_z}$ and it is $\boldsymbol{\lambda_z} \equiv \frac{\sigma_z}{\sigma_{\bar{y}}} \frac{a_r}{\sqrt{2}}$

The approach taken by LW is a technical compromise that uses a Median Unbiased Estimation (MUE) to determine the size of ($\boldsymbol{\lambda_z}$). Some literature on the issues cast doubts of its soundness (see (Buncic 2020)), as LW approach is found that it cannot recover consistently the ratio of interest $\boldsymbol{\lambda_z} \equiv \frac{\sigma_z}{\sigma_{\bar{y}}} \frac{a_r}{\sqrt{2}}$ from MUE required for the estimation.

Finally, LW document a degree of uncertainty regarding estimates of the natural rate of interest and showed that any policy rules based on the assumption of a constant NIR or its mismeasurement lead to the imposition of wrong monetary policies. LW find estimates of the natural rate as useful, but insufficient in monetary policy decision making processes. Later, LW model has been applied to three other advanced economies – Canada, Euro Area, and UK. They find that large declines in trend GDP growth and natural rates of interest have occurred over the past 25 years in all four economies. These country-by-country estimates exhibit a substantial amount of comovement over time, suggesting an important role for global factors in shaping trend growth and natural rates of interest. In all four economies, the estimated trend growth rate has declined by 0.8 to 1 percentage points since 2007. This explains about half of the decline of the natural rates in the US and the euro area and 75 percent of the decline in Canada and the UK. The rest of the decline is attributed to unspecified factors.

2.2. Models modifying LW

The model of LW is exclusively designed for the body of US economy, as it captures the features of a big, relatively closed economy with stable structural issues regarding aggregate supply and long time series of reliable data. In order to apply their model for other types of economies, the LW model needs to be modified, as not many economies resemble the US one.

The second reason for the many attempts to modify the LW model is to narrow the uncertainty band around the estimates, or to fix some perceived flaws of the model design. Berger and Kempa (Berger, Kempa 2014) made an adaptation in the LW model for the small and open economy like Canada. They included as an important factor in observation equations the real exchange rate, as it is related to the output gap through the current account, influences inflation via its effect on import prices, and impacts the interest rate by inducing expectations of mean reversion of the real exchange rate towards its equilibrium level. They point out that, in a small open economy, both aggregate demand and the Phillips curve contain the real exchange rate as an argument. As the interest gap may also be associated with an exchange rate misalignment through a potential interest rate – exchange rate nexus, the model is even extended by an equation linking the real interest rate to the real exchange rate.

Brand and Mazelis (Brand, Bielecki, and Penalver 2018) opt for another modification. They close the original framework with a Taylor rule instead of using real interest rates as an endogenously determined process. This requires constructing an output gap that pins down inflation in line with the inflation objective, as incorporated in the Taylor rule. They switch to a non-accelerationist Phillips curve, with the output gap pinning down deviations of inflation from the inflation objective, rather than from a unit-root trend. They further deviate from the original approach by using model-consistent inflation expectations.

Georgy Krustev (Krustev, 2019) augments the LW model with a financial cycle and the labour market featuring a non-accelerationist Phillips curve. He finds that the financial cycle is a missing variable in observation equations and its inclusion improves the performance of the model estimates.

(Hledik, Vlcek 2018) opt for another modification as they link the natural rate of interest to equilibrium GDP growth, which is adjusted for real exchange rate appreciation. This adjustment is needed because GDP growth only measures yields from production and ignores the effect of currency appreciation. Second, they use a semi-structural model, which is closed by a monetary policy rule. This model allows to work with forward-looking model-consistent expectations and impose a comprehensive set of restrictions, i.e. model equations, determining the natural interest rate, to identify the natural rate of interest. Third, they use calibration instead of Bayesian estimation. More specifically, they calibrate the standard errors of the Kalman filter arbitrarily to obtain economically intuitive impulse responses. The final estimates of the natural rate depend mainly on the time-dependent estimates of the growth of potential GDP and real equilibrium appreciation of the exchange rate.

The semi-structural models provide reliable information for the recent historical behaviour of the natural interest rate and its evolution at lower frequencies as the transitory shocks die out. Unfortunately, they provide little help to a policymaker, who needs a model to pinpoint the r* in time t_0, i.e. now.

This problem is allegedly solved by a different approach, namely structural DSGE models. The structural DSGE models and error-correction models have sufficient ability to capture the changes of r* over the business cycle. This is important for a policymaker, but has its own flaws. For instance, DSGE suggest a very volatile natural interest as "advice" to a policymaker. A volatile natural interest rate is practically impossible to implement in the real world, as some smoothing and inertia are more preferred options for short term interest rate steering.

2.3. Models using an error-correction type of procedures

Lubik and Matthes (Lubik, Matthes, et al. 2015) compared the natural rate estimated using the Laubach and Williams approach with an estimate based on time-varying VAR models. They conclude that the two approaches provide similar results for the period since the 1980s. However, prior to this date, there was a significant difference in the results. Other researches find that the multivariate error-correction models can be used to estimate time-varying equilibrium using long-run relationships between macroeconomic variables. Estimates from this approach are done by (Fiorentini, et al., 2018). They use a long time series of a broad set of macroeconomic information, including total factor productivity and demographic developments. They set a local level model, which decomposes the observed real rate into a I(1) component, labelled r*, and an I(0) component, which resembles the real rate gap. Since the natural real rate in this model is a simple random walk, conditional forecasts are simply the most recently observed value. In the second part, Fiorentini et al. (2018) estimate a Panel error correction model (ECM) at an annual frequency over the period 1899-2016.

2.4. DSGE models for natural interest rate

In DSGE models, the natural rate of interest is an unobservable variable that can be extracted by estimating a fully structural model, in practice using a rich set of macroeconomic data. Most of the literature employs a definition of the natural rate of interest as the real interest rate that would prevail in a counterfactual economy under flexible prices and wages, and absent shocks to the mark-ups on goods and labour markets (Neri, Gerali 2019; Hristov, 2016). The whole class of DSGE models document a very volatile r* due to their ability to trace the impact of structural shocks on the natural rate of interest. They find that a risk premium shock is responsible for a significant part of the total variance of the natural rate of interest, as the risk premium shock modifies the households' effective discount rate for one-period risk-free (government) bonds.

3. Model Estimation of the Natural Rate: Bulgaria Case

To our knowledge, there is only one attempt so far for a r* estimation in order to derive a Taylor rule type for the Bulgarian economy. In 2004, "*The Currency Board: The only game in town*" (Hristov, 2004) shows that, in a hypothetical case, if BNB had followed (by then) inflation targeting regime with a monetary policy based on Taylor type rule, it would have steered more stringent short term interest rates compared the observed in the Bulgarian O/N market. In that paper, the author follows the method borrowed from the New Zealand central bank (Archibald, Hunter, et al., 2001) in order to calculate r* as the simple average of the real observable interest rate for 1994-2003 in Germany (Eurozone) plus BG country risk premium.[6]

There is a certain string of shortcomings in using the averaging method for r* + risk premium. First, if the time window for real interest rates includes periods with significantly high inflation, as it was the case with 1990-1994 Germany, then the interpretation of results could be misleading. For instance, in the period of 1990-1994, there was temporary high inflation in Europe due to the unification of Germany and not because of supply-demand real structural balance. In addition, mechanistic add-on of a country risk premium is difficult to interpret, as that premium (or spread) alone has its own factors which drive it. The second problem is the estimation of the real interest rates as " ex-post" instead of " ex-ante" that would require an estimation of inflation expectations of the economic agents in the absence (in the Bulgarian case) of market-based instruments as inflation-indexed bonds or inflation swaps. In order to overcome the above-mentioned shortcomings, we estimate the inflation expectations (See Appendix A) in order to calculate the observed "*ex-ante*" real interest rate and then we make a model estimation of the natural rate for the Bulgarian economy.

In order to make a model estimation of the natural rate for the Bulgarian economy, we follow the general principles of Laubach and Williams, 2003 with some modifications due to data specifics and the institutional framework of the Bulgarian economy – namely small open economy under CBA monetary framework.

As in LW, we use a model (semi-structural), assuming the following neoclassical growth model relationship between potential output growth g and natural rate r*. This model implies that the natural rate of interest varies over time in response to shifts in preferences and the growth rate of output. In a steady state of the economy, a representative household intertemporal utility maximisation yields the relationship between the steady-state real one-period interest rate r* and steady-state growth.

7) $r *= \frac{1}{\sigma} g_c + \theta$

Where r* is the natural interest rate, σ is the intertemporal elasticity of substitution in consumption[7], i.e. the slope of the curve, g potential growth rate of the real GDP, a θ is the rate of time preference. Since r* is an unobservable variable, we relate it with observable variables using Kalman filter as it is done in LW model, in order to simultaneously estimate r*, output gap yt and the level of potential growth rate g.

Given the theoretical link between the natural rate of interest and output growth noted above, we assume (as in LW[8]) that the law of motion for the natural rate of interest is as follows:

[6] Country risk premium = spread of the BG yield over identical German sovereign bond.
[7] The amplitude/elasticity of the response of g to change in r*.
[8] LW assume a one-for-one relationship between the trend growth rate of output and the natural rate of interest r*, which corresponds to assuming σ = 1.

8) $r^* = c\ g_t + z_t$

The natural rate of interest r* equals the estimated trend growth of GDP (g_t) and some time-preference parameters (z) of households.

The natural rate of interest is not necessarily constant, but can fluctuate under the influence of specific shocks, such as aggregated demand and productivity shocks, or changes in the preferences of households. If the economy is not in balance, for instance, because prices are not able to adjust freely, the real interest rate can deviate from the natural rate, which will lead to inflationary or deflationary pressure.

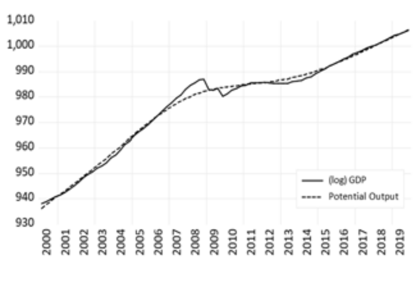
3.2. Modelling of $\boldsymbol{y_t}, \boldsymbol{r_t}, \boldsymbol{rer_t}$ *and data transformations*

a) <u>The output y_t</u>

LW model the potential growth rate as trend growth. We use different specifications of potential output using HP filter for the extraction of trend and cycle, which proved to be more feasible for the specifics of the Bulgarian economy. For instance, the Bulgarian economy, as a small open economy that is strongly dependent on exogenous factors for growth. Thus, HP filter is assumed to be sufficiently reliable especially having sufficient historical data. The shortcomings are well known – uncertainty about λ parameter (pre-defined preferences on the smoothness of the trend series) and end sample bias. Indeed, there are few publications that employ multivariate filtering approaches that deliver slightly better results as (Kasabov, et al., 2017), among many others (Ganev, 2004, 2015; Tsalinski, 2006) that deliver slightly more refined and reliable results. In that publication, they use a multivariate model with a production function, Phillips curve and Okun's Law. Figures to compares the estimated output gap with measures obtained from methods like a simple HP filter and a multivariate model in (Kasabov, et al., 2017), we find some slight differences. The reason for the differences is that the multivariate approach is more consistent with the undergoing structural change of the economy in 2000-2003 and 2011-2015.

Figure 1

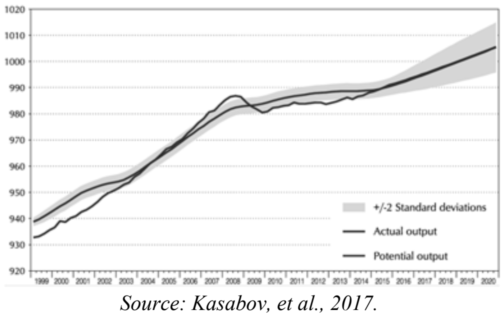
HP filtered potential output



Source: own calculations.

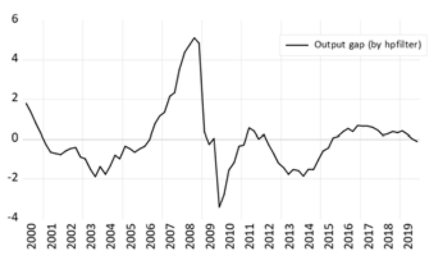
Figure 2

Multivariate model of potential output



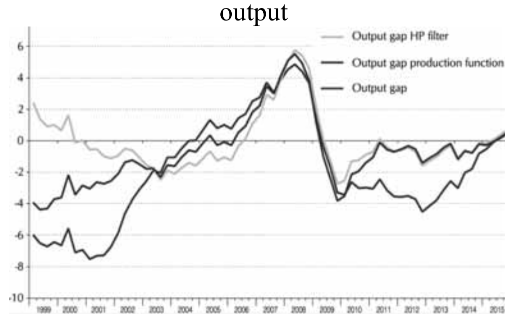
Source: Kasabov, et al., 2017.

Figure 3
Output gap (HPfilter) in % of potential output



Source: own calculations

Figure 4
Output gap (Multivariate) in % of potential output



Source: Kasabov, et al., 2017.

As a univariate filter, (HP) is not that strong in capturing structural changes in the economy, but the overall performance is reliable enough for the purposes of natural this (NIR) model as it is only one of the inputs, namely $\overline{y}_t$ in the observation equations (see eq. 12, 13). Most importantly, LW model jointly estimates the r_t^*, y_t^* and the g_t in state-space form using a Kalman filter which tends to smooth out the output variables in order to produce coherent results, thus negligible differences in potential output would not yield significant differences (Figure 10). In addition, a multivariate filter would complicate our already extensive LW-style model.

With that rationale, we chose to model the output (y_t) as decomposed into a stochastic trend component y_t^* (potential output) and a stochastic cyclical variation $\overline{y}_t$ (output gap) around the trend. To separate the trend from the cycle, we use HP filter as λ parameter = 1600.

9) $y_t = y_t^* + \overline{y}_t$

b) <u>The real interest rate</u>

The real interest rate r_t is decomposed to natural interest rate r_t^* and real rate gap $\overline{r}_t$.

10) $r_t = r_t^* + \overline{r}_t$

Though, the natural real rate r_t^* is unobservable variable, there are (in best case) quasi-observable real interest rates r_t that have key role in the assessment of the stance of the monetary policy. But the derivation of real interest rate has its own methodological issues that have to be addressed properly otherwise, the interpretation of results could be misleading.

The calculation of observed real interest rate is not a straightforward procedure, as it could be done by two differing approaches, 1) "ex-post" by subtracting observed inflation from nominal (i) rate or 2) "ex-ante" by subtracting of the model-based inflation expectation or by market-based financial instruments for inflation expectations (inflation linkers or inflation swaps). In Appendix A, we have elaborated on a very simple model framework for the derivation of "ex-ante" real interest rate and "ex-post" real interest rates.

Figure 5

Ex post and Ex-ante BG real interest rates

c) The real exchange rate

The same applies to the real exchange rate rer_t , as it is decomposed to equilibrium exchange rate and exchange rate gap $\overline{rer}_t$. The equilibrium exchange rate (rer_t^{eq}) concept is itself a dubious to estimate, so we took a shortcut to approximate it as a simple average of the existing data.

11) $rer_t = rer_t^{eq} + \overline{rer}_t$

3.1. Signal equations– Philips curve, IS curve

The likelihood-function and the unobserved states can be derived through the Kalman filter. This filter uses the economic restrictions in the empirical model, the relationships between the real interest gap, the output gap, inflation and the effective exchange rate gap, to filter the unobserved states in the model. The Kalman filter works on the principle that the estimate of the unobserved state is adjusted based on how far away the model's prediction of GDP is from actual GDP given the behaviour of other variables. If the prediction is true, the filter does not adjust the estimate of the natural interest rate. If, on the other hand, actual GDP is lower than predicted by the model, then the policy rate must have been less accommodative than the model had predicted and hence that the real rate gap was more positive than previously thought.

a) Philips curve

12) $\pi_t = B_\pi(L)\pi_{t-1} + b_y\overline{y}_{t-1} + b_{rer}\left(RER_t^{eq} - RER_t\right) + b_o(\pi_{t-1}^O - \pi_{t-1}) + \varepsilon_\pi$

In short, the aggregate supply side is represented by a backward Philips curve, where the inflation expectations are assumed to be driven by a backward process; hence, the inclusion of lagged inflation terms $B_\pi(L)\pi_{t-1}$. The impact of excess demand on inflation is captured by the first lag of the output gap $b_y\overline{y}_{t-1}$. We modify the LW equation and instead of import inflation, we use a real exchange rate gap ($b_{rer}\left(RER_t^{eq} - RER_t\right)$). LW designed their model for a big and relatively closed economy as US. The Bulgarian economy, on the contrary, is a small and very open economy, which is affected by the gaps of RER from equilibrium levels and is related to dynamics of inflation. For instance, a big negative RER gap would facilitate a higher inflation than (hypothetically) targeted. Thus, in the spirit of (Berger and Kempa 2014) we think that the real exchange rate is the most important relative price of a small open economy, and should be a key element in the identification of the transitory and permanent components of output, inflation and the interest rate. As with LW we add a lag in oil price $b_o(\pi_{t-1}^O - \pi_{t-1})$ as relevant regressor.

b) IS curve

In our specification, we add a new regressor (independent variable) in the IS-equation, namely a coefficient capturing the effect of the Eurozone output gap, as it has a very strong effect on the Bulgarian output gap and captures the external environment.

13) $\overline{y}_t = a_{t,1}\overline{y}_{t-1} + a_{t,2}\overline{y}_{t-2} + \sum_{j=1}^{2} a_r(r_{t-j} - r_{t-j}^*) + a_{ez}\left(y_{ez,gap}\right) + \varepsilon_{\overline{y}}$

The output gap $\overline{y}_t$ is determined by its own two lags $a_{t,1}\overline{y}_{t-1} + a_{t,2}\overline{y}_{t-2}$, as well as a moving average of the real rate gap $\frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r_{t-j}^*)$ and error term ε_y. In addition, we put another regressor namely, the output gap in the Eurozone $a_{ez}\left(y_{ez,gap}\right)$ as this captures the international environment, which strongly influences the very open Bulgarian economy.

3.2. State equations

We depart from LW approach about the "pile-up" issue and aim at a more simple technical solution that serves the purpose of managing the relationship between errors in the state equations. LW took the dubious procedure of MUE in order to estimate the initial values of that vector and make sure that there would be no clustering of errors to zero. We aim at avoiding the need of figuring out the "pile-up" problem and take a " shortcut" by calibrating values for the vector that contains the initial values using existing literature, which are also means of the variables. This is the way to implement previous knowledge and some stylised facts about historical developments in the economy.

14) $y_t^* = y_{t-1}^* + \frac{1}{4}g_{t-1} + (\lambda 1)\ \varepsilon_y$

The first state equation models the level of potential product y_t^* as an unobservable variable that depends on its own lag y_{t-1}^*, one lag of the trend growth rate $\frac{1}{4}g_{t-1}$ (multiplied by 0.25 because of quarterly data). The last element $(\lambda 1)\ \varepsilon_y$ is the variance of error from the IS equation (2). The coefficient (λ1) is the so-called "hyperparameter" that allows us to calibrate a number in order to control the variation of error term of y* and thus to simulate a shock on y*.

15) $r_t^* = g_{t-1} + z_{t-1}$

The natural interest rate r_t^* is modelled as one lag in the trend growth rate g_{t-1} and one lag in the time preference z_{t-1}.

16) $z_t = (c_1)z_{t-1} + (c_2)(\lambda 2)\epsilon_y$ $\qquad c_1=0.5,\ c_2 = 0.5$

The time preference z_t is the residual in $r^* = c\ g_t + z_t$ and follows an autoregressive process. The coefficient $\lambda 2$, is another "hyperparameter" that allows us to simulate shock on the level of the time preference of households. We calibrate equal weights of the coefficients c_1 and c_2.

17) $g_t = (c_1)g_{ss} + (c_2)g_{t-1} + (\lambda 3)\epsilon_y$ $\qquad c_1=0.5,\ c_2 = 0.5$

g_t is the trend growth rate of the potential product that is modelled as a random walk process anchored around long-term trend growth rate at a steady state g_{ss}[9]. We calibrate equal weights of the coefficients c_1 and c_2. We calibrate the value to $g_{ss} = 3.6$ because this is the average growth rate for the whole sample period.

Calibrating lambdas is practically the most sensitive moment and source of the vulnerability of the coefficient estimates. At the same time, the calibration provides us with powerful "hands-on" approach in defining the behaviour of z and g, as well as y*, r*. This is the moment when the existing literature varies most.

Table 1

Parameters in existing literature

Parameter	Holston, Laubach and Williams (2017)	Belke and Klose (2019)	Fiorentini et al. (2018)
λg	0.033	0. 1176	0. 043
λz	0.036	0. 0006	0. 013
r*	3.693	2.438	3.543
g	12.128	0.973	4.413
y*	0.520	0.7125	0.526

In addition, the crucial parameters g_{ss} and r_t^* , that instruct the model about where to start in searching for r*, need to be consistently justified and counterfactually validated by recursive simulations. Typically, central banks put a lot of efforts in the designation of the level of potential output (or output gap), as it key element of the inflation targeting

[9] Theoretically, this is the steady-state economic growth - defined as the rate of growth that the economy would converge to in the absence of new shocks, or the highest level of economic activity that can be sustained over the long term (Casadio, Paradiso, Rao, 2012).

operational framework. Thus, they already use various models that cross-check the numerical findings with the reality and historical evidence.

3.3. Model estimation

Before we run the model, we have to impose certain values for the key coefficients of λ1, λ 2 and λ 3 as the lambdas aim to affect the estimates of the standard deviations of the innovations to the state equations.

a) Hyperparameters assumptions

Table 2

Hyperparameters

Lambdas	Base line	Description
$\lambda 1$ =	0.010	Short-term (iid shocks in the level of y*)/sd(iid shocks in the IS curve)
$\lambda 2$ =	0.020	long-run sd (shocks in time preferences z)/sd(iid shocks in the IS curve)
$\lambda 3$ =	0.073	long-run sd (shocks in rate of growth g)/sd(iid shocks in the IS curve)

We calibrate λ1 as a ratio of the standard deviation of $\lambda_g \equiv \frac{\sigma_g}{\sigma_{y^*}}$ errors in the IS-curve equation and equation that models potential growth. The rest of the calibration of the lambdas is based on existing literature.

b) Steady-state growth rates and natural interest rates

In addition to the hyperparameters, the model needs two important parameters about:

1) What is the long-term, steady-state growth rate of potential output g_{ss}.

2) What is the long-term, steady-state natural interest rate r_{ss}.

Due to its structural format, the model allows us to incorporate a stylised facts and observations about the behaviour of the Bulgarian economy. That assumptions inform the model where to start in searching for suitable values for parameters as potential growth estimation and natural rates. Table 3 shows our calibrated parameters about steady-state trend growth rate and natural interest rates for Bulgaria. In the table, we’ve added for comparison only the same variables for Eurozone because they are an important starting point for justification of the rationality of our baseline scenario.

Table 3

Steady-state growth rate of potential output and natural interest rate

	Eurozone	Bulgaria
g_{ss} Steady state growth rate (in %)	1.5	3.6
r_{ss} initial guidance for r*(in %)	1.0	1.5

Figure 6
Volume index of Bulgarian real expenditure per capita in PPS (EU15=100)

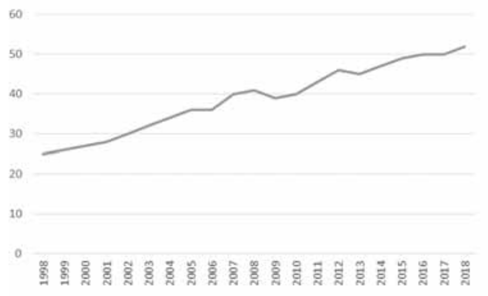
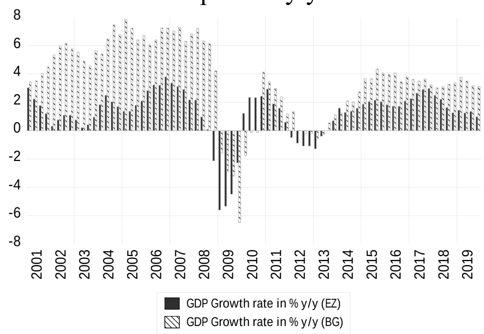


Figure 7
GDP growth rates of Bulgaria and Eurozone in percent y/y



Bulgaria is a catching-up economy, as its levels of incomes, GDP per capita and consumption are moving towards the Eurozone average, but currently they are only half of the identical variables for the Eurozone (Figure 2, Figure 3). On average, the rates of growth for GDP of the Eurozone for the period 1998-2019 are 1.5% and 3.1% for Bulgaria. In addition, ECB itself (Andersson et al., 2018) estimate the rate of potential output growth Eurozone and conclude that it has recovered most of its pre-crisis momentum and the current level is about 1.5%.

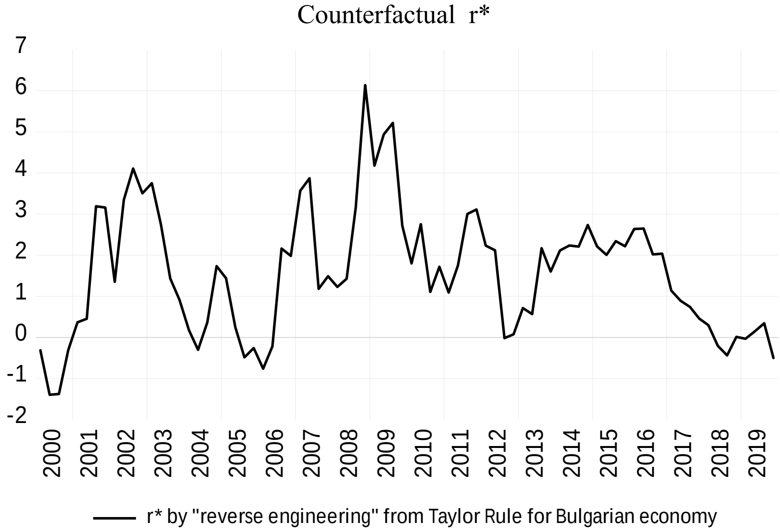
The explicit target for inflation in Eurozone is 2%[10] as the Bulgarian inflation expectations are anchored around 3.5% (Appendix A). The anchoring of Bulgarian inflation expectation around 3.5% gives us reason to assume that if hypothetically BNB followed inflation-targeting monetary regime, it would ”target” inflation near or range-bound 3.5% in medium to long run.

According to (Holston, Laubach, Williams, 2016), the r* for Eurozone is currently around 0%, but the average for the last 20 years is about 1.2%. In short, the Bulgarian economy, in the long run, grows twice faster and has twice higher inflation due to the catching-up effect. This means that the twice higher growth and twice higher inflation cancel out and the result is supposed to be identical level for the natural interest rate. We validate that proposition by ”reverse engineering” we rearranged a hypothetical Taylor rule for the Bulgarian economy assuming the market based nominal interest i_t is the intended policy rate of the central bank.

Taylor rule	“Reversed” Taylor rule
$i_t = \pi_t + r_t^* + a_\pi(\pi_t - \pi_t^*) + a_y(y_t - \bar{y}_t)$	$r_{t,rev}^* = i_t - \pi_t - a_\pi(\pi_t - \pi_t^*) - a_y(y_t - \bar{y}_t)$

[10] HICP annual growth rate in percent.

Figure 8



Where the output gap ($\overline{y}_t$) is a deviation from HP modelled potential y_t, π_t^* is our modelled Bulgarian inflation expectations (i.e. 3.5%), a_y and a_π are equal (0.5). We find it rational to assume that the natural interest rate is hovering around 1.5% in the long run.

c) Data and transformations

We use seasonally adjusted quarterly data for Bulgaria from 2000Q1 to 2019Q4 for inflation (annualised first difference of the log CPI excluding food and energy), GDP (logs multiplied by 100)[11] taken from quarterly national accounts and Eurostat database, the effective real exchange rate calculated by BIS (logs multiplied by 100), and the ex-ante real interest rate for alternative specifications. For the period 2000 Q1 to 2019 Q4, the yield to maturity of 1 year BG government treasuries is used. For Oil prices, we use Brent in Euros (logs multiplied by 100). Inflation expectations are calculated from the model described in Appendix A.

The choice of variables is of particular importance in order to achieve reliable results in model estimation as even small changes due to data availability could compromise the outcome. For instance, the choice of inflation index – *core* of *headline* inflation CPI index could affect the real observable interest rate and Philips curve estimation in the observation equation. Or, more importantly, the choice of nominal interest rate is arguably crucial in order to interpret results more consistently. For the nominal interest rate, there are a variety of existing rates that affect the economy.

[11] Logs are multiplies by 100 in order to use them as regressors in OLS equations and take advantage of interpretation as percent changes in estimated coefficients. This is the most typical data transformation for variables that grow exponentially.

Figure 9

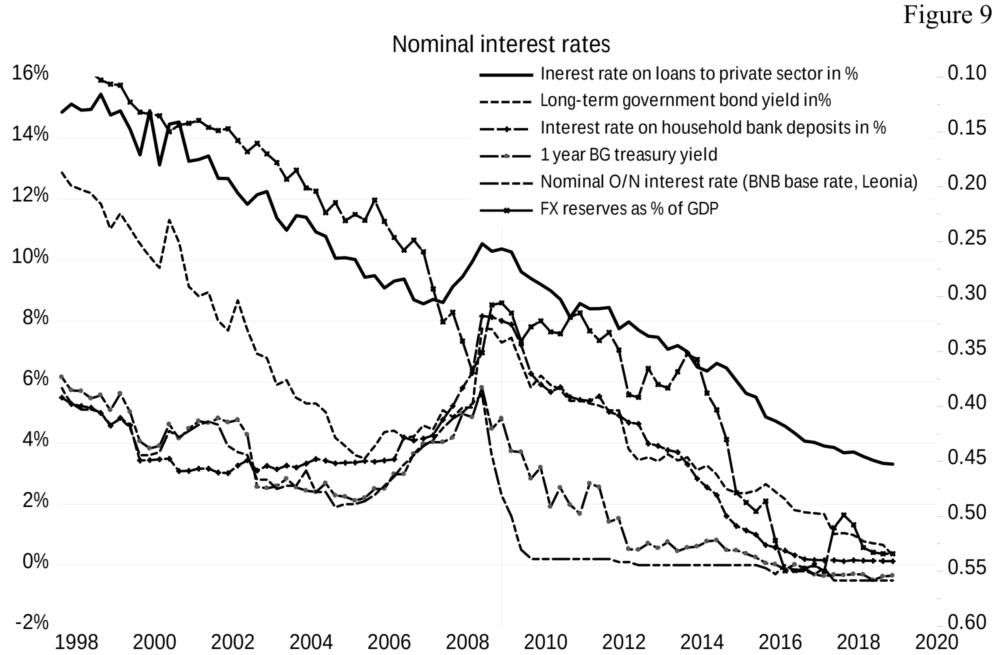


Figure 9 illustrates the existing data for the yields in percent of the relevant interest rates (on left-hand scale) in Bulgaria and (on right-hand scale inverted, bold black line) FX reserves to GDP ratio as a cross-check for financial conditions. We need the variable that contains maximum information about the financial conditions in the economy. Typical NIR models (as LW) use the O/N money market rates. Bulgarian monetary set-up is entirely defined by the Currency Board Agreement (CBA). This makes the O/N money market rates theoretically, obsolete as the central bank does not control the short term O/N rates in order to steer the monetary conditions in the economy. In addition, the money market daily turnover volumes are relatively small as the local banks do not use this vehicle to regulate their liquidity needs.

Table 4 presents the correlation of various rates to the ratio of FX reserves-to-GDP as a proxy for financial conditions.

The lending rate (Rate on loans to private sector) exhibits the strongest coefficient of correlation to FX reserves-to-GDP ratio. This finding confirms the theoretical expectation that this is the rate that most strongly affects the economic behaviour of economic agents. Unfortunately, the lending rate itself is quite problematic as it comprises significant liquidity and risk premiums that blur the overall calculation of the real interest rate. If we calculate the real interest rate as lending rate minus inflation rate, this could mechanically result in a higher real interest rate. Thus, we took the second-best, that it is the *1y BG treasury yield which* approximates reliably the financial conditions in the real economy – second highest correlation coefficient with FX reserves-to-GDP ratio, negligible liquidity and risk premiums and practically a part of the money market with a reliable time range of data.

Table 4

Correlation	Rate on loans to private sector	Long-term interest rate	Household deposit rate	1 year BG treasury	O/N interest rate	FX to GDP ratio
Rate on loans to private sector	1.00					
Long-term interest rate	0.94	1.00				
Household deposit rate	0.55	0.56	1.00			
1 year BG treasury	0.88	0.87	0.68	1.00		
O/N interest rate	0.83	0.76	0.43	0.91	1.00	
FX to GDP ratio	-0.96	-0.81	-0.48	-0.86	-0.79	1.00

4. Discussion of the Model Results

Table 5 present the results of the estimated model. The coefficients show the expected signs and the magnitude identical to results in other papers (Belke, Klose 2017). For instance, the two slope coefficients a_r and b_y have the expected signs and are statistically significant. We find this as an evidence that validates the choice of assumptions for hyperparameters and steady-state growth and r* are plausible and realistic.

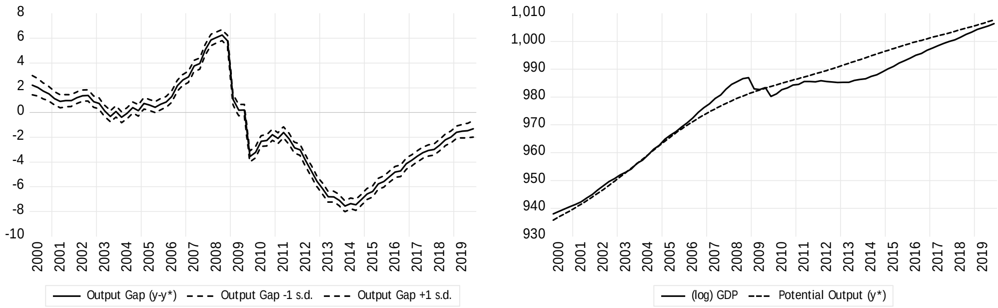
Table 5

Model estimation, Coefficients

Sspace: NEUTRAL_RATES				
Method: Maximum likelihood (BFGS / Marquardt steps)				
Sample: 2000Q1 2019Q4				
	Coefficient	**Std. Error**	**z-Statistic**	**Prob.**
IS Curve				
$a_{t,1}$	0.519	0.255	2.037	0.042
$a_{t,2}$	0.165	0.204	0.806	0.421
a_r	0.040	0.040	1.010	0.313
a_{ez}	0.956	0.115	8.279	0.000
Philips Curve				
B_π	1.009	0.003	362.7	0.000
b_y	0.085	0.026	3.228	0.001
b_{rer}	-0.039	0.015	-2.657	0.008
b_o	0.011	0.005	2.402	0.016
	Final State	**Root MSE**	**z-Statistic**	**Prob.**
r_t^*	2.612	1.275	2.049	0.041
z_t	0.139	1.101	0.126	0.900
g_t	2.491	0.689	3.616	0.000
Log likelihood	-188.909	Akaike info criterion		5.125
Parameters	11	Schwarz criterion		5.458
Diffuse priors	0	Hannan-Quinn criter		5.258

Figure 10

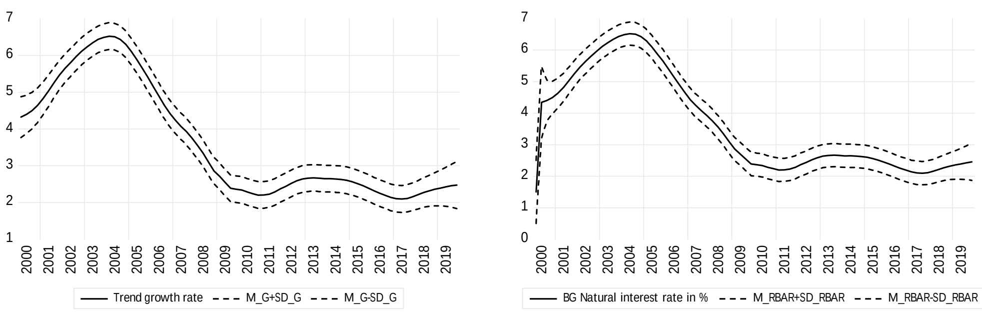
Potential growth, Output gap



The model estimate for trend growth (Figure 10, right chart) holds a slow-moving pattern as cyclical deviations from the trend take its toll on the long-term growth. This holds the assumption that in the short run, nevertheless peaks and troughs of cycles the economy keeps stable ability to grow. But in the medium to long run, a secular underperformance of growth pulls the potential output down in low-frequency mood. It is important to note that we aim at the estimation of real natural interest rate that is by definition a variable with a low frequency motion. Thus, the output gap goes with some margin around the trend (Figure 10, left) as the model itself approximates the cycle position of the economy. The Fig:10 shows that in the period right after 2000-2004, the economy has experienced mild negative output gap due to several factors as the recovery after the economic crisis 1996-1997 and the deep and profound structural change as a result from the reforms in a string of public and economic policies. In 2004-2008 the output gap has been closed as a period of strong output growth followed due to robust FDI inflow, housing prices surge and favourable external conditions. The global financial crisis in 2008 abruptly reversed the positive output gap and within three years, shifted the growth rate slightly below the trend. Since then, the real output slowly but consistently approaches the trend as up to 2019, the output gap is to be considered marginal.

Figure 11

Natural interest rate and trend growth rate of GDP



*4.1. Key observations from the results for Bulgarian r**

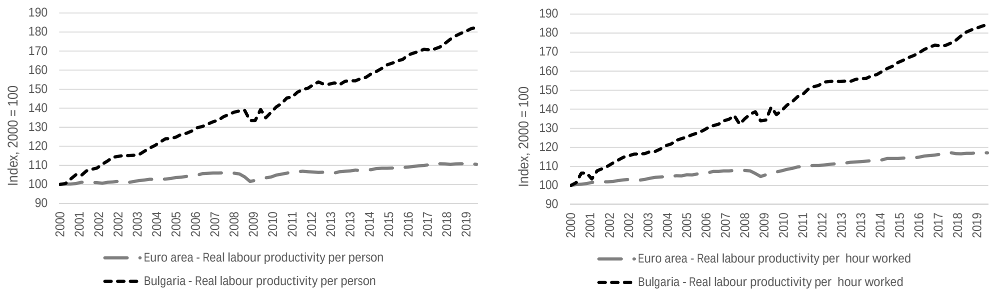
a) <u>Observation 1</u>

The most important observation about the natural interest rate (r*) is the absence of a sustainable declining trend over the whole sample period. Contrary to an overall decline in natural rates in many advanced economies in the last couple of decades, the Bulgarian economy does not show any measurable or pronounced sign of such a declining trend in r*. This is an essential difference, as it is the declining r* by and large the most important early warning indicator for the imminent era of ”secular stagnation”. The absence of a declining trend in Bulgarian r* after 2008 begs for an explanation as it seems counterintuitive at first glance.

The key explanation lies in the long-term driving factors for economic growth in Bulgaria. Bulgaria as catching-up economy “imports” productivity, thus Solow style[12] explanation for economic growth applies fully. The recent history suggests that the profound economic transformation after 1999, deep supply-side reforms that increased competition and mostly strong FDI flows from abroad is the profound driver of the productivity growth in the last 20 years (Figure 12). The reforms opened opportunities for *imported* technology, capital and knowledge.

Figure 12

Productivity growth, EA and BG, per person (left) and per hour worked (right)



b) <u>Observation 2</u>

Bulgarian r*, hoovers around 2.5%. Bulgaria is a catching-up economy, which grows twice faster than the Eurozone over the long term. This implies that the inflation is supposed to be higher during this catching-up process. Our simplified estimation, described in Appendix 1, confirms that the inflation expectations of the economic agents a centred around 3.5% on average in the long run as the inflation target in the Eurozone is (approximately) less than 2%. This fact suggests that it is reasonable to expect a higher natural interest rate.

---

[12] In Solow models (Solow, 1956, 1957), the productivity is exogenous variable and is the ultimate driver of growth in the long run.

Figure 13 illustrates a comparison between the r* of the Eurozone[13] and Bulgaria and shows that indeed Bulgarian r* is stably higher than the Eurozone r*. This is important because it allows a wider real rate gap (r-r*), thus a more accommodative monetary stance.

Figure 13
Natural Interest Rates (r*) BG and EZ

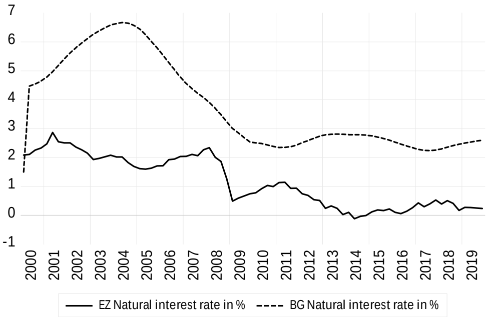


Figure 14
Output gap and real rate gap in %

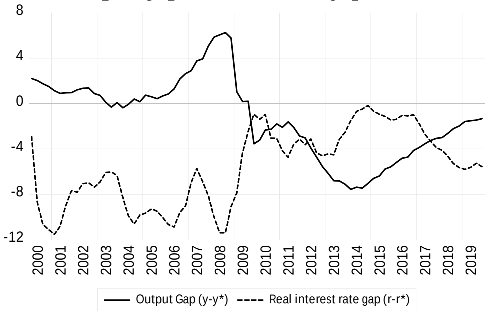


Figure 14 illustrates that in the case of Bulgaria, the negative real rate gap has supported a growth and a positive gap suppressed GDP growth in certain periods. Even visual inspections of Figure 14 gives a clearer evidence, that the real rate gap "works" and that all of our assumptions made in the model are justified. That proves that the results for the modelled Bulgarian natural interest rate are sufficiently reliable as general guidance for r*.

Figure 15
Scatterplot of Output gap to real rate gap

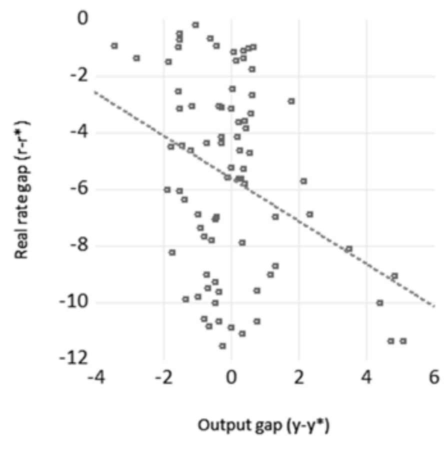
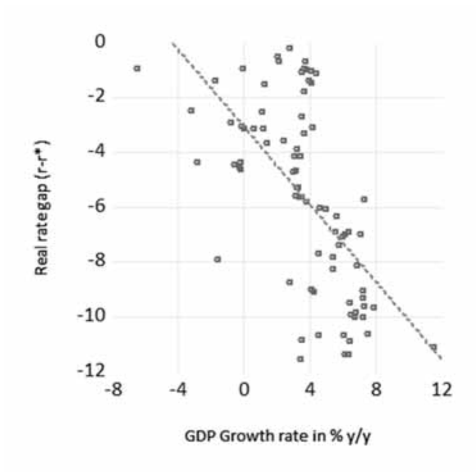


Figure 16
Scatterplot of real rate gap to GDP growth rate



[13] Eurozone r* source is Holston, Laubach, and Williams (2017) https://www.newyorkfed.org/research/policy/rstar.

For instance, in the period 2010-2016, the real rate gap happened to be sluggish, which led to anemic growth and very slowly closing of the output gap. The natural interest rate even moved to positive numbers in the period 2002-2008 due to a powerful rise of time preferences. This fact shifted the real rate gap negative for that period and precipitated significant overheating of the economy. The consequences were unsustainable asset prices overvaluations ("bubbles") in Bulgarian real estate, Stock indexes and an unsustainable current account deficit.

Figure 17
Scatterplot of real rate gap to investment growth

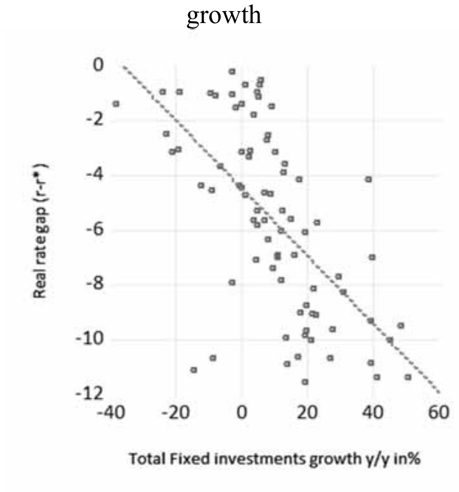


Figure 18
Investment by type as % of GDP

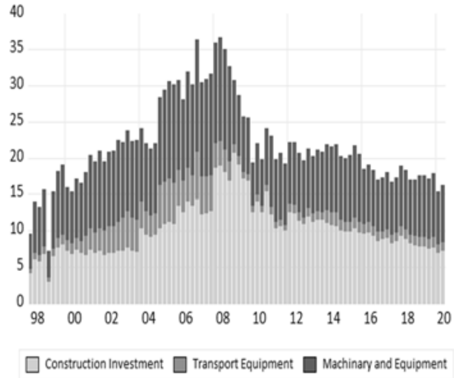


Figure 19
Scatterplot of real rate gap to credit growth

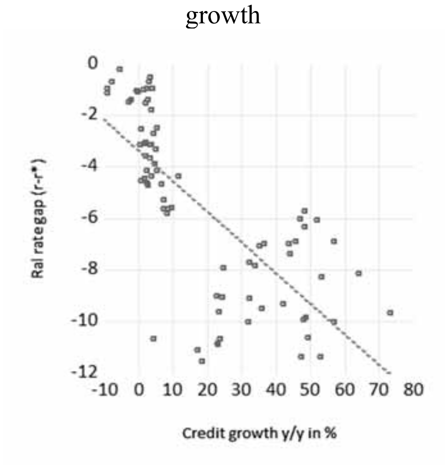
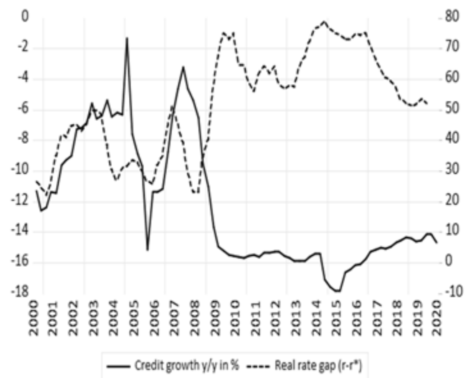


Figure 20
Dynamic of Real rate gap and credit growth



Figures 17 and 18 illustrate that the real rate gap affects moderately the decision process for the new investments. According to economic theory, the level of real interest rate is supposed to affect the saving/investment decisions of the economic agents. The theory of natural interest rate as defined in (Woodford, 2005) provides more robust results about the relationship between the real rate gap and saving/investment decision.

Figures 19 and 20 illustrate a similar picture that the real rate gap affects moderately the volume of credit in the economy.

4.2. Assumptions/result validation through a hypothetical Taylor rule

In order to check the reliability of our assumptions and the overall consistency of our model, we have calculated a simple Taylor model (Taylor 1993):

18) $i_t = \pi_t + r_t^* + a_\pi(\pi_t - \pi_t^*) + a_y(y_t - \overline{y}_t)$

As r* we plugged the outcome of the model, the output gap $(y_t - \overline{y}_t)$ is also another modelled input to the model, π_t^* is our outcome for the long-term Bulgarian inflation expectations (i.e. 3.5%), a_y and a_π are equal (0.5). Typically i_t is a short term interest rate steered by the central bank, but in our case, this is not relevant and we took a slight change in using 1y treasury yield. We intentionally let the Taylor rule without any restrictions or inertia component in order to see the unrestricted behaviour of the prescribed interest in comparison to the observed ST rate on the market. Both specifications show that in the period 2000-2008, the ST interest rate is way too accommodative, thus stimulating the growth beyond potential and inflation above the figurative target of 3.5%. In the period 2012-2016, both specifications diverged the outcome.

Figure 21

$\boldsymbol{a_\pi}$ = 1.5 and $\boldsymbol{a_y}$ =0.5

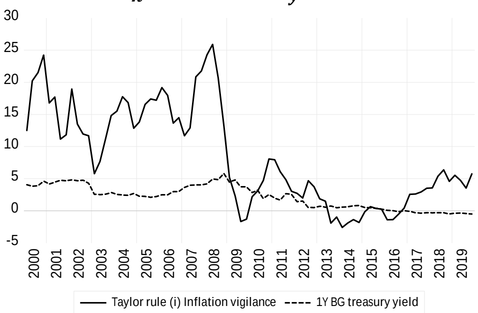


Figure 22

$\boldsymbol{a_\pi}$ = 0.5 and $\boldsymbol{a_y}$ =0.5

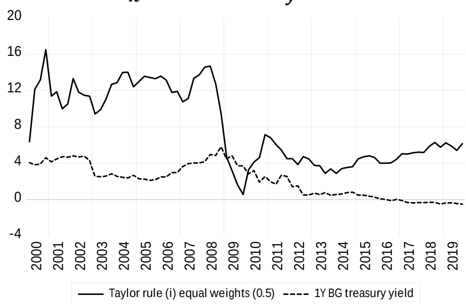


Figure12 says that a more inflation vigilant bank[14] should have lowered the short term rate further to achieve the inflation target. In that period Bulgarian economy experienced a period

[14] This implies that the central bank cares more about the inflation being on target than output gap being closed.

of very slow growth (even stagnation) and deflation. This fact adds evidence to the notion that if a central bank could not provide a negative real rate gap and/or negative nominal gap (against Taylor rule 'advice') than the aggregate demand and aggregate supply equilibrate through slower growth rates. Fig:13 represents a central bank that is equally concerned about inflation and welfare loss and defines short term rate nearly appropriate in the period 2012-2016 but gradually accommodative later on. It says contrary to the previous Figure, that in the period 2017-2019, the economy is running a little bit hot as it needed nominal short term rates near zero line.

A central bank could not allow a key interest rate, which it targets to go on wild swings or boats of volatility, thus some **inertia** is required in the Taylor rule procedures, which required a central bank to adjust interest rates only gradually (Clarida, Gali, Gertler, 2000; Rudebusch, 2005; Driffill, Rotondi, 2007). This implies the recognition that there are long and variable lags in the transmission of monetary policy, so there is a need to avoid tough "stop-and-go" policies and their consequences in terms of negative macroeconomic spillovers. In economic literature ρ usually goes around 0.8.

19) $i_t = \rho(i_{t-1}) + (1-\rho)[(\pi_t + r_t^* + a_\pi(\pi_t - \pi_t^*) + a_y(y_t - \overline{y}_t)]$

Figure 23

$\boldsymbol{\rho = 0.8}$ $\boldsymbol{a_\pi}$ = 1.5 and $\boldsymbol{a_y}$ =0.5

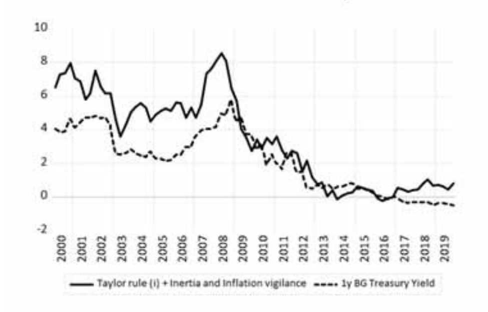


Figure 24

$\boldsymbol{\rho = 0.8}$ $\boldsymbol{a_\pi}$ = 0.5 and $\boldsymbol{a_y}$ =0.5

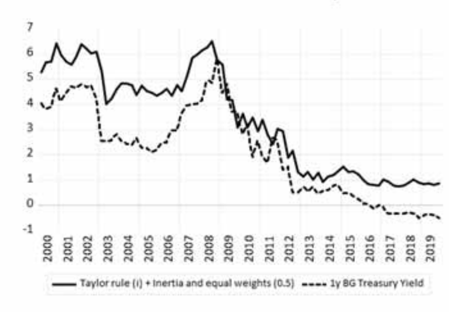


Figure 14 presents a Taylor rule with inertia as ρ = 0.8. That specification reflects the general understanding that a central bank is supposed to adjust slowly targeted short term rate to the ever-changing circumstances in the economy. The interpretation of the results is similar to Figure 13, as Figure 14 captures the most probable response function of BNB in the hypothetical case of steering short term rates.

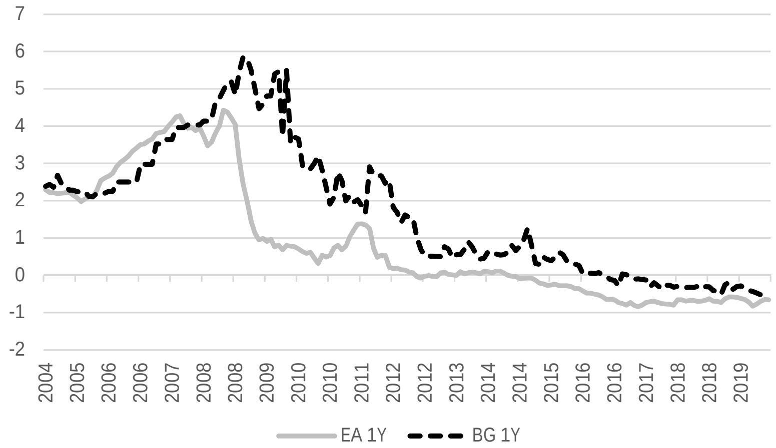
Though, we have elaborated a hypothetical case for (What if...) inflation targeting monetary set up instead of Currency Board Agreement (CBA[15]), the result from Taylor rule exercise proofs that even derived from purely market forces, the Bulgarian short term rates mimic what BNB would have done in case of inflation targeting regime.

[15] Bulgarian money market rates are driven purely on demand/supply balance without any BNB interference.

This is an important conclusion given that Bulgaria is on the doorstep of the Eurozone and the European Central Bank is going to steer directly the short term rates for the Bulgarian economy. Historical data show (Figure 16) that Bulgarian money market rates for a long period have been strongly influenced by the decision rate setting of ECB and due to the fixed exchange rate (EUR/BGN) and other transmission channels as trade, are transmitted easily to the local money market. For instance, a detailed and comprehensive research and analysis is done by Nenova, Ivanov, Ivanova (2019).

Figure 25

Yields on BG and EA treasuries 1Y maturity



The deviations happen mostly in times of market stress and reflect Bulgarian country-specific size of the risk-premium/liquidity premium over less risky Eurozone money market.

Very recent data show that the spread of BG rates to EZ rates practically disappeared. That means that Bulgarian short-term/money market rates would take direction continuously from ECB as it is now without any disruption to the present state of the monetary conditions in the economy. Trade and financial interlinkages between Bulgarian and Eurozone economies are more than intense that, as Taylor rules show, there is no even theoretical chance for a BNB's monetary policy substantially deviate from that of ECB.

5. Conclusion

The model estimation of natural interest rate (r*) is an important benchmark in assessing if a monetary policy of a central bank needs to be steered toward a positive or negative shift in order to achieve inflation on target and/or output on its potential level. The proposed model outcome is sensitive on input variables as the choice of ex-post vs. ex-ante real interest rates which needs to be addressed carefully. The model needs to be put in a context by building set of assumptions about crucial parameters as potential growth in steady state of the economy and prior knowledge or (intuition) about the most probable level of natural interest rate in steady state of the economy. In addition, the model needs some assumptions about (hyper) parameters that reflect a genuine understanding about the potential; level and growth rate of the economy in the long run. If these assumptions are realistic, then a behaviour of the real rate gap is the ultimate benchmark for our knowledge-to-reality check.

Even with a reliable set of assumptions, the model that provides an estimation of unobservable variable as natural interest rate is subject to uncertainty or error in the outcome. Such uncertainty complicates a rule-based monetary policy implementation but is inevitable for any type of model that mimics a complicated reality with only a handful of variables.

Nevertheless, the estimated Bulgarian natural interest rate plugged into the hypothetical Taylor rule provides evidence that the assumptions are plausibly calibrated and close to reality, thus sufficiently justified.

References

Andersson, M., et al. (2018). Potential output in the post-crisis period. – Economic Bulletin Articles, 7.

Archibald, J., Hunter, L., et al. (2001). What is the neutral real interest rate, and how can we use it?. – Reserve Bank of New Zealand Bulletin, 64.3, pp. 15-28.

Belke, A., Klose, J. (2017). Equilibrium real interest rates and secular stagnation: An empirical analysis for euro area member countries. – JCMS: Journal of Common Market Studies, 55.6, pp. 1221-1238.

Berger, T., Kempa, B. (2014). Time-varying equilibrium rates in small open economies: Evidence for Canada. – Journal of Macroeconomics, 39, pp. 203-214.

Branch, W. A. (2004). The theory of rationally heterogeneous expectations: evidence from survey data on inflation expectations. – The Economic Journal, 114.497, pp. 592-621.

Brand, C., Bielecki, M., Penalver, A. (2018). The natural rate of interest: estimates, drivers, and challenges to monetary policy. – ECB Occasional Paper, 217.

Buncic, D. (2020). Econometric Issues with Laubach and Williams' Estimates of the Natural Rate of Interest. Available at SSRN.

Casadio, P., Paradiso, A., Bhaskara Rao, B. (2012). Estimates of the steady state growth rates for some European countries. – Economic Modelling, 29.4, pp. 1119-1125.

Clarida, R., Gali, J., Gertler, M. (2000). Monetary policy rules and macroeconomic stability: evidence and some theory. – The Quarterly journal of economics, 115.1, pp. 147-180.

Driffill, J., Rotondi, Z. (2007). Inertia in Taylor rules. SSRN 1028711.

Fiorentini, G., et al. (2018). The rise and fall of the natural interest rate".

Ganev, K. (2004). Statistical Estimates of the Deviations from the Macroeconomic Potential: An Application to the Economy of Bulgaria. Agency for Economic Analysis and Forecasting.

Ganev, K. (2005). Measuring total factor productivity: Growth accounting for Bulgaria. – Bulgarian National Bank Discussion Paper 48.

Ganev, K. (2015). A Small Model for Output Gap and Potential Growth Estimation: An Application to Bulgaria. – Bulgarian Economic Papers, N BEP 04.

Hledik, T., Vlˇcek, J. (2018). Quantifying the Natural Rate of Interest in a Small Open Economy – The Czech Case. Czech National Bank.

Holston, K., Laubach, T., Williams, J. C. (2016). Measuring the Natural Rate of Interest: International Trends and Determinants. – Federal Reserve Bank of San Francisco Working Paper 2016-11.

Hristov, A. (2016). Measuring the natural rate of interest in the eurozone: a DSGE perspective. – CESifo Forum, Vol. 17. 1. Mu¨nchen: ifo Institut-Leibniz-Institut fu¨r Wirtschaftsforschung an der..., pp. 86-91.

Hristov, K. (2004). The currency board: The only game in town. – Discussion Paper 40.

Kasabov, D., Kotseva, P., Yanchev, M. (2017). Relationship between inflation, potential output and structural unemployment in Bulgaria. – BNB, DP, 104, p. 2017.

Krustev, G. (2019). The natural rate of interest and the financial cycle. – Journal of Economic Behavior & Organization, 162, pp. 193-210.

Laubach, T., Williams, J. C. (2003). Measuring the natural rate of interest. – Review of Economics and Statistics, 85.4, pp. 1063-1070.

Lubik, T. A., Matthes, C., et al. (2015). Calculating the natural rate of interest: A comparison of two alternative approaches. – Richmond Fed Economic Brief Oct, pp. 1-6.
Minea, A., Rault, C. (2008). Monetary policy transmission: old evidence and some new facts from Bulgaria".
Nenova, M., Ivanov, E., Ivanova, N. (2019). Monetary policy transmission: old evidence and some new facts from Bulgaria. – Bulgarian National Bank.115/2019.
Neri, S., Gerali, A. (2019). Natural rates across the Atlantic. – Journal of Macroeconomics, 62.
Rudebusch, G. D. (2005). Monetary policy inertia: fact or fiction?. – FRB of San Francisco Working Paper 2005-19.
Solow, R. M. (1956). A contribution to the theory of economic growth. – The quarterly journal of economics, 70.1, pp. 65-94.
Solow, R. M. (1957). Technical change and the aggregate production function. – The review of Economics and Statistics, pp. 312-320.
Stock, J. H., Watson, M. W. (1998). Median unbiased estimation of coefficient variance in a time-varying parameter model. – Journal of the American Statistical Association, 93.441, pp. 349-358.
Summers, L. (2013). 14th Annual IMF Research Conference: Crises Yesterday and Today. – November, 8, p. 2013.
Summers, L. H (2015). Demand side secular stagnation. – American Economic Review, 105.5, pp. 60-65.
Taylor, J. B. (1993). Discretion versus policy rules in practice. – Carnegie-Rochester conference series on public policy, Vol. 39, pp. 195-214.
Tsalinski, Ts. (2006). Two Approaches to Estimating the Potential Output of Bulgaria. – BNB, N 57.
Woodford, M., Walsh, C. E. (2005). Interest and prices: Foundations of a theory of monetary policy. – Macroeconomic Dynamics, 9.3, pp. 462-468.

APENDIX A

Ex post real interest rates $[\,r_t^p = i_t - \pi_{t-1}^p]$

"Ex-post" rates include two disturbing components which could render them a misleading proxy for non-observable ex-ante real interest rates: inflation risk-premium and agents' inflation expectation errors. Researchers are frequently are bound to use "ex post" real rates assuming that, there is low volatility in the time series or, the representative economic agent extrapolates the current value of inflation in future periods due to limited information or weak rationality. This means that inflation expectations in (t+1) are assumed to be the same as in t. That method holds that economic agents form their inflation expectations on the basis of past inflation experience and adjust the expectations accordingly to the realised inflation proportionally but less than 1-to-1. The empirical research show that many agents fit sufficiently in that description (Branch, 2004).

For instance, the available data for Bulgaria shows a strong relationship between observed inflation and survey based inflation expectations. European Commission publishes monthly data for Bulgaria[16], which compares to monthly CPI index data show robust dependence to yearly inflation data at moment t. EC uses survey with a balance[14] of answers to the question about expected inflation in the next 12 months.

A simple statistical regression shows (Table 6) that near 2/3 of the economic agents form their inflation expectations extrapolating current inflation (R^2 = 62%) for the period 2004-2019.

$$\pi_t = \alpha + \beta_1 Bal_{t+12|-12|} + s$$

[16] https://ec.europa.eu/economy_finance/db_indicators/surveys/documents/series/nace2_ecfin_1907/consumer_inflation_nace2.zip.

Figure 26
Scatterplot, Survey based inflation expectations vs. observed inflation rates for Bulgaria

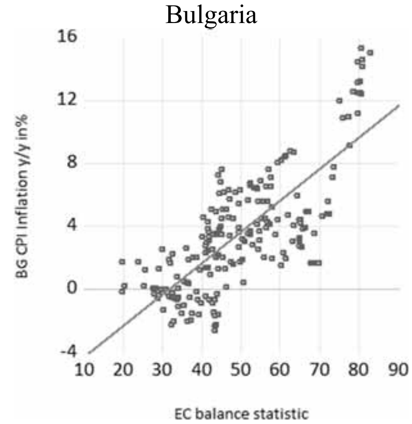


Figure 27
Survey based inflation expectations (rhs) and observed inflation rates y/y for Bulgaria

Table 6
Regression results: Equation 1

Coefficients	Period 2004-2009
intercept	-0.7291 (0.6615)
(EC) infl. expect	0.2143*** (0.0125)
R-squared	0.62
No. observations	182

Notes. $*p < 0.05$, $**p < 0.01$ $***p < 0.0001$

A formal regression analysis for the 2001-2019 show that there is a reliable significance of the estimated coefficients, but also some degree of autocorrelation in the error term and two periods of excess volatility due to global factors as economic recession in 2002-2003 and Eurozone crisis in 2010-2012. The visual perception of autocorrelation has been confirmed by formal ADF test. By adding additions variables as lags of observed inflation we can improve the stationarity or the residual.

Ex ante real interest rates [$r_t^e = i_t - \pi_t^e$]

Ex ante real interest rates are most easily derived from market instruments as inflation swaps or inflation linked government bonds. Unfortunately, the Bulgarian market has no such instruments. In addition, such instrument include not only expected inflation π^e, but also risk premium about forecast inflation and liquidity premium. The last two components could be significant factors in times of stress or volatility. That obstacle make us the move back to the previous example and overcome the problem with autocorelated residual. Thus, we add additional factors to survey based inflation expectation, namely inflation own lags

$$\pi_t = \alpha + \beta_1 Bal_t + \beta_1 \pi_{t-1} + s$$