Икономически изследвания
Economic Studies
Volume 32(4), 2023

Aysenur Tarakcioglu Altinay[1]
Mesut Dogan[2]
Bilge Leyli Demirel Ergun[3]
Sevdie Alshiqi[4]

THE FAMA-FRENCH FIVE-FACTOR ASSET PRICING MODEL: A RESEARCH ON BORSA ISTANBUL[5]

This study aims to test the validity of the Fama-French Five-Factor Model (FF5F) for Turkey. Within the scope of the study, throughout 468 weeks between September 2009 and August 2018, the returns over the risk-free interest rate of 18 different intersection portfolios are used based on value, profitability, and investment factors. A total of 8424 portfolios (18 portfolios x 468 weeks) are generated in the study. As a result of the analyses, it is determined that the Five-Factor Asset Pricing Model is valid for Borsa İstanbul. Subsequently, it is concluded that the Fama-French Five-Factor Model has a higher explanatory power in describing the stock returns of the portfolios formed with stocks of small-scale companies compared to the portfolios formed with stocks of large-scale companies. The findings are consistent with the literature.
Keywords: CAPM; Fama-French Five Factors Model (FF5F); Stock Returns; Borsa İstanbul
JEL: E44; G11; G12

1. Introduction

In the late 1950s, the soaring prestige of natural sciences, which attempted to explain natural events with the obtained data by applying to empirical instruments, encouraged the belief that it would be possible to mitigate problems pertinent to decision-making and equity allocation with the widespread use of optimization models and mathematical techniques (Dempsey, 2013).

---

[1] *Aysenur Tarakcioglu Altinay, Assoc. Prof., Usak University, +902762212121, e-mail: aysenur.altinay@usak.edu.tr.*
[2] *Mesut Dogan, Assoc. Prof., Bilecik Seyh Edebali University, +902282141311, e-mail: mesutdogan07@gmail.com.*
[3] *Bilge Leyli Demirel Ergun, Prof., Yalova University, +902268155766, e-mail: bilgeleyli@gmail.com.*
[4] *Sevdie Alshiqi, Dr., University of Prishtina, +38344124975, e-mail: sevdie.alshiqi@uni-pr.edu (Corresponding Authour).*
[5] This paper should be cited as: *Altinay, T. A., Dogan, M., Demirel, B. L. E., Alshiqi, S. (2023). The Fama-French Five-Factor Asset Pricing Model: A Research on Borsa Istanbul. – Economic Studies (Ikonomicheski Izsledvania), 32(4), pp. 3-21.*

In this context, Markowitz's (1952) "Portfolio Selection" and the modern corporate finance theory developed by Modigliani and Miller (1958; 1963) have contributed to the formation of normative literature on issues related to the functioning of financial institutions and the financial system. In the study of Markowitz (1952) on portfolio selection, the relationship between risk and return in securities investment is considered a turning point for modern finance and investment theories. "Modern Portfolio Theory", for which Markowitz was awarded a Nobel Prize in 1990, brought a new perspective to finance literature and rendered the traditional portfolio approach controversial. There are various models based on modern portfolio theory which develop by adding to this model. The best-known and most studied among these models is the Capital Asset Pricing Model (CAPM).

The Capital Assets Pricing Model (CAPM) argues that the risk premium expected from security should be proportional to the expected risk premium pertaining to the market. The model has been widely utilized in performance assessments, estimation of capital cost, portfolio selection, and measurement of abnormal returns. The CAPM has been considered an important turning point in modern finance theory since it was developed by Sharpe (1964), Lintner (1965), Mossin (1966), and Black (1972). The market model of William Sharpe (1964) was formulated as follows:

$$R_{it} = \propto_i + \beta_i * R_{Mt} + e_{it}$$

The variables in the formula are as follows:

$R_{it} = The\ return\ of\ stock\ i\ in\ period\ t$
$\alpha_i = The\ expected\ return\ of\ security\ i$
$\beta_i = The\ sensitivity\ of\ stock\ i\ to\ market\ movements$

$$R_{Mt} = The\ return\ on\ the\ market\ in\ period\ t$$

$e_{it} = The\ risky\ return\ of\ security\ i\ (defines\ the\ error\ term\ with\ statistically\ a\ zero\ mean\ and\ constant\ variance\ (\sigma_e^2)).$

The equation in this model explains the return on assets through the return on the stock market index. The β value in the equation denotes a risk indicator stemming from the relationship between market return and stock return. Following the model, the equilibrium model was developed. The difference between the market model and the equilibrium model emerges due to the relationship between excessive market returns and return on assets rather than market returns. The general formulation of the equilibrium model developed by Sharpe, Lintner, and Mossin is called the Capital Asset Pricing Model. The Asset Pricing Model is formulated as follows:

$$R_{it} = R_f + \beta_i * (R_{Mt} - R_f) + e_{it}$$

$R_{it} = Return\ of\ the\ stock\ i\ in\ period\ t$
$R_f = Risk - free\ interest\ rate$

$$R_{Mt} = Return\ on\ the\ market\ in\ period\ t$$

$e_{it} = The\ risky\ return\ of\ security\ i\ (defines\ the\ error\ term\ with\ statistically\ a\ zero\ mean\ and\ constant\ variance\ (\sigma_e^2)).$

In the new Capital Asset Pricing Model developed by Black (1972), unlike the previous model developed by Sharpe, Lintner, and Mossin; the assumption of risk-free borrowing and lending was included. Basu (1977) considered different time-series models and explained that the returns were positive and linear by associating them with the β coefficient. The equation of this model is as follows:

$$R_{it} - R_f = \alpha_i + \beta_i * (R_{Mt} - R_f) + e_{it}$$

Although the Capital Asset Pricing Model is still accepted as a widely used model in bond pricing, contradictory findings are being observed in empirical studies (Chiah et al. (2016). Therefore, the researchers studied more advanced models by including more descriptive variables in the model of stock return behaviour.

When investors select among the securities representing the ownership of their companies' activities, they assume that they pay reasonable prices considering what is known about the company (Fama, 1976). The foundation of modern finance theory is based upon such a generally accepted view in capital markets. The paradigm underlying this view argues that financial capital circulation is based on achieving the most attractive rates of return for its investors (Dempsey, 2013). This principle coincides with the assumption of the rational expectations hypothesis in economics that economic decision-making units would have all the information related to the variable. However, decision-making units do not have all the information related to the variable since another assumption of the hypothesis claims the existence of a certain level of cost to be incurred to gather information, it would not be possible to acquire a consistent return upon acting according to the assumption that merely partial information is accessible.

Insufficient disclosure of the expected returns of financial assets due to some deficiencies in the CAPM assumptions led to studies conducted on multi-factor models in determining stock returns. The first of these models was the "Three-Factor Model" developed by Eugene F. Fama and Kenneth R. French in 1993.

In this model, two new factors such as the company size (size) and the Book Equity / Market Equity ratio (value) were included in the CAPM which tests the relationship between the expected rate of return of any risky investment instrument or any portfolio and the rate of return of the market portfolio. Fama and French (1993) concluded that stocks of companies with market equities of less than $ 1 billion and stocks of companies with high book equities tend to yield higher returns than expected through the CAPM (Fama and French, 2015).

The equation for the Fama-French Three-Factor Model is as follows:

$$R_{it} - R_f = a_i + (R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + e_{it}$$

The variable previously denoted as α_i in the model is expressed as a_i in Chiah et al. (2016). There are no changes here except for the difference in notation.

The Size Factor (SMB-Small Size Minus Big Size) is the difference in return between portfolios with a low market value and portfolios with a large market value.

$$SMB = \frac{(\mathrm{SH} + \mathrm{SN} + \mathrm{SL}) - (\mathrm{BH} + \mathrm{BN} + \mathrm{BL})}{3}$$

The Value Factor (HML-High Value Minus Low Value) is obtained by subtracting low-value stocks from high-value stocks.

$$HML = \frac{(\mathrm{SH} + \mathrm{BH}) - (\mathrm{SL} + \mathrm{BL})}{2}$$

With the Fama-French Three-Factor Model, the β coefficient pertaining to the market sensitivity in the CAPM was excluded from the model and replaced by the size and the value factors based on the assumption that they would better handle the cross-sectional change. Fama and French developed the existing model in 2015 by including the investment and profitability factors in the Three-Factor Model since the Three-Factor model was not sufficient in explaining the cross-sectional changes having certain anomalies in expected returns related to investment and profitability. The new model has entered the literature as the Fama-French Five-Factor Model (Kubota and Takehara, 2018). The Fama-French Five-Factor Model, developed due to the failure of the Three-Factor Model in explaining the expected return, was formulated in the following form (Jan and Ayub, 2019);

$$R_{it} - R_f = a_i + (R_{Mt} - R_{ft}) + s_i\, SMB_t + h_i\, HML_t + r_i\, RMW_t + c_i\, CMA_t + e_{it}$$

The Investment Factor (CMA-Conservative Minus Aggressive) is obtained by subtracting high-risk (aggressive) stocks from low-risk (conservative) stocks.

$$CMA = \frac{(SC + BC) - (SC + BA)}{2}$$

The Profitability Factor (RMW-Robust Minus Weak) is the t-time difference in return between portfolios with high profitability ratios and portfolios with weak profitability ratios.

$$RMW = \frac{(SR + BR) - (SW + BR)}{2}$$

Table 1. Selected Portfolio Groups that Constitute the Fama-French Factors

SMALL	Book Equity/Market Equity (S-B/M)	High (SH)
		Neutral (SN)
		Low (SL)
	Profitability (SP)	Robust (SR)
		Medium (SM-)
		Weak (SW)
	Investment (S-INV)	Conservative (SC)
		Medium (SM)
		Aggressive (SA)
BIG	Book Equity/Market Equity (B-B/M)	High (BH)
		Neutral (BN)
		Low (BL)
	Profitability (BP)	Robust (BR)
		Medium (BM-)
		Weak (BW)
	Investment (B-INV)	Conservative (BC)
		Medium (BM)
		Aggressive (BA)

2. Literature Review

In the 1950s, the process of detecting stock price movements and determining the factors affecting this process became an interesting issue for researchers. In this process, many models have been developed pertaining to the issue. The first model of the field, as mentioned in the introduction part of the study, was Markowitz's (1952) modern portfolio theory. The process continued with the capital assets pricing model (CAPM) developed with the contributions of Sharpe (1964), Lintner (1965), and Black (1972). Following these studies, Fama and French (1993) developed a new three-factor model in which the criticism toward the CAPM was eliminated in the related field. In 2015, Fama and French designed a five-factor model by adding two more factors to the previous model developed in 1993. A large number of studies have been conducted in the domestic and foreign literature regarding the Fama-French Three-Factor Model. Nonetheless, upon examining the literature, it is apparent that there are merely a limited number of studies conducted on the validity of the Fama-French Five-Factor Model.

Chiah et al. (2016) made strong suggestions for the international capital markets which were excluded from the sample by testing the validity of the Fama-French Five-Factor Model in explaining the stock returns on the Australian Stock Exchange throughout 1982-2013. The study, in which the multi-factor model was found to be suitable for the Australian capital markets, emphasized the model's suitability also for the American capital market.

Fama ve French (2017), in which the Fama-French Five-Factor Asset Pricing Model was tested in the North American, European, Japanese, and Asian Pacific Stock Markets, detected that investments were negatively related to the returns whereas the increase in both the Book Equity / Market Equity ratio and profitability boosted the average returns in North America, Europe, and the Asia Pacific. In the study, it was determined that there was a positive and strong relationship between the average returns and the Book Equity / Market Equity ratio for Japan, however, it was concluded that the average returns were weakly related to profitability and investment factors.

Lin (2017), which the applicability of the Fama-French Five-Factor Asset Pricing Model was tested in the Shanghai and Shenzhen Stock Exchanges between 1997-2015, concluded that the Fama-French Five-Factor Model was far more successful than the Fama-French Three-Factor Asset Pricing model in explaining the returns in the Shanghai and Shenzhen Stock Exchange Markets. However, unlike the results obtained in Fama-French (2015), the study stated that the investment factor had no contribution in explaining the average returns.

Foye (2018) determined that the Five-Factor Model was more successful in explaining stock returns than the Three-Factor Model for Eastern Europe and Latin America by testing the applicability of the Fama-French Five-Factor Asset Pricing Model for the stock markets in Asia (China, India, Indonesia, Malaysia, Philippines, South Korea, Taiwan, and Thailand), Eastern Europe (Czech Republic, Hungary, Russia, Poland, and Turkey), and Latin America (Argentina, Brazil, Chile, Colombia, and Mexico) throughout 1996-2016.

Nevertheless, it was stated that the variables of profitability and investment are not explanatory for Asian countries, therefore, the Fama-French Five-Factor Asset Pricing Model failed to explain the returns for the Asian Region.

Huang (2018) tested the applicability of the Fama-French Five-Factor Asset Pricing Model in the Chinese Stock Exchange throughout 1994-2016. It was determined that the Fama-French Five-Factor Model for the Chinese Stock Exchange was more successful than other traditional asset pricing models in explaining stock returns.

Kubota ve Takehara (2018), who tested the stock returns of the Tokyo Stock Exchange throughout 1978-2014 with the Fama-French Five-Factor Asset Pricing Model, concluded that the profitability and investment factors were not statistically significant in explaining the stock returns.

Olive et al. (2018), this study analyses FAMA French Three Factor model and Capital asset pricing model in the Indian stock market. To maximize share price, the financial manager must learn to assess two key determinants viz., risk and return. Each financial decision presents certain risk and returns characteristics, and the unique combination of these characteristics has an impact on the share price. The risk and return of a single decision were discussed using CAPM and FAMA French Three-factor model. The objectives of the study included, measuring and analyzing the performance of the stock using FAMA French three-factor model and capital asset pricing model. This study was done for the Indian Stock market by choosing the first leading stock market Bombay Stock Exchange as the sample. The index selected for the study was S&P BSE 200, and only 120 companies were selected as the sample for conducting the study. CAPM being a single-factor model gave only 7.5 percent significant result to the single asset. This showed that market return alone cannot determine the risk and return of the company stock. The predictability of the variables of FAMA French Three Factor model namely market return, size (SMB), and value (HML) factors are also tested in this study. The result showed that about 58.3 percent of company stocks showed significant results towards SMB and 52.5 percent of company stocks are showing significant results towards HML. Hence, the findings were generally supportive of the FAMA French model applied to Indian equities.

Cox ve Britten (2019), by comparing the Fama-French Three-Factor Model and the Fama-French Five-Factor Model in terms of explaining the stock returns on the Johannesburg Stock Exchange between 1991 and 2017, concluded that the former model had higher explanatory power than the latter pertaining to the time-series of the size-value and size-profitability variables. The Fama-French Five-Factor Model was identified as the best model to explain revenue in general. It was concluded that the model is sufficient to reveal and explain the negative relationship of the size, beta and coefficient with the return.

Senarathne (2019) emphasized that the findings were insufficient in normal market conditions in his study where the applicability of the Fama-French Five-Factor Asset Pricing Model was tested for the stock market in Europe, Japan, Asia Pacific (Japan), and North America throughout 1990-2019. It is asserted that the reaction of investors to portfolios with common risks in the European and Japanese markets during the crisis depended on the opinion regarding the size of capital stocks, the size of the investment, the level of profitability, and Book Equity/Markey Equity.

Zhao et al. (2019) concluded that the Fama-French Five-Factor Model was more successful in mitigating asymmetric information in their study where the Fama-French Five-Factor Model was compared with the Bayesian Approach throughout 2000-2017. Li et al. (2019)

stated that the Fama-French Five-Factor Model was partially successful in explaining the stock returns of the Chinese Stock Market as a result of their study covering the period of 2005-2016, and the most important underlying reason was the fact that the Chinese stock market being rather fragile compared to the stock markets of developed economies.

Rugwiro and Choi (2019) tested the applicability of the Fama-French Three-Factor Asset Pricing Model for the Korean Stock Exchange throughout 1998-2016 and concluded that the Fama-French Three-Factor Model was insufficient to explain the liquidity factor compared to the Fama-French Five-Factor. Racicot et al. (2019) evaluated the sample consisting of 12 sector portfolio returns and market risk factors in terms of the applicability of the Fama-French Five-Factor Model for the S&P 500 throughout 1968-2016 and concluded that the model could be more sufficient to explain the returns upon inclusion of the estimators of the financial crisis and economic fluctuations into the model. Gonzalez and Jareno (2019), in their studies comparing the Fama-French Three-Factor and Five-Factor Models throughout 1989-2014, concluded that it could perform significantly better for low theta values during stagnation periods in explaining the sectoral returns on the US stock markets.

Ahmed et al. (2019) compare major factor models and find that the Stambaugh and Yuan (2016) 4-factor model is the overall winner in the time-series domain. The Hou, Xue, and Zhang (2015) q-factor model takes second place and the Fama and French (2015) 5-factor model and the Barillas and Shanken (2018) 6-factor model jointly take third place. The pairwise cross-sectional R-2 and the multiple model comparison tests show that the Hou et al. (2015) q-factor model, the Fama and French (2015) 5-factor and 4-factor models, and the Barillas and Shanken (2018) 6-factor model take equal first place in the horse race.

Pandey and Sehgal (2019), in this paper, authors experiment with the construction of alternative investor sentiment indices. Further, the authors evaluate the role of the sentiment-based factor in asset pricing to explain prominent equity market anomalies such as size, value, and price momentum for India. Based on the findings, the authors confirm that our Composite Sentiment index leads other sentiment indices currently in vogue in investment literature. The asset pricing models, including the more recent Fama French 5-factor model, are not fully able to explain the small firm effect which is captured by our sentiment-based factor which seems to proxy for the price over-reactions.

Pepenkov (2019), stock returns are generally difficult to explain, as they are comprised of many discrete channels of risk. Empirical asset pricing models (EAPM), such as the Fama-French five-factor model (FF5), have been used to partition these channels across a series of systematic risk factors, such as company size (total market equity), value (book-to-market ratio), investment, and operating profitability. Prior EAPMs only accounted for how such factors contributed to risk at the market level, ignoring any potential variation across the sectors. This study developed a sector-heterogenous model (SHM) which directly accounts for this variation by generalizing the Fama-French methodology to sector subsets of stocks. The results demonstrated that risk is meaning heterogeneous across sectors for each of the factors in the FF5, with different subgroups of factors being statistically significant within each sector. In a direct comparison of explanatory power, the SHM outperformed the FF5 and improved adjusted R-2 by an average of 5% for stocks across all sectors. Several applications of sector heterogeneity were then demonstrated for stock-picking purposes, including a high-beta portfolio strategy using the SHM-beta which outperformed the S&P

500 in backtesting. This study concludes that meaninsector heterogeneityneity exists in market risk. This information is materially useful to investors.

Chakraborty et al. (2019), This paper shows that asset prices are linear polynomials of various underlying explanatory factors, and asset returns being ratios of these polynomials, are rational functions that do not add linearly when averaging. Hence, average returns should be modelled based on stock prices. However, continuous returns may be treated as approximately linear across time and modelled directly. Our new Rational Function (RF) models, empirically outperform the traditional asset pricing models like the Capital Asset Pricing Model (CAPM) and the Fama-French three and five-factor models for both average and continuous returns. Moreover, the RF theory also provides a model to estimate asset volumes. The average change in asset volumes together with average returns provide the estimates for the average change in market values of assets. Thus, the RF model approach can be used to select assets that provide either the highest returns for profit maximization or the highest change in market values for wealth maximization for given levels of risk.

Ielasi, F., & Rossolini (2019), the aim of the paper is to compare the risk-adjusted performance of sustainability-themed funds with other categories of mutual funds: sustainable and responsible mutual funds that implement different approaches in portfolio selection and management, and thematic funds not committed to responsible investments. The study analyses a sample of about 1000 European mutual open-end funds where 302 are sustainability-themed funds, 358 are other responsible funds, and 341 are other thematic funds. Risk-adjusted performance is analyzed for the period 2007-2017 using different methodologies: a single-factor Capital Asset Pricing Model (CAPM), a Fama and French (1993) 3-factor model, and a Fama and French (2015) 5-factor model. Our main findings demonstrate that the risk-adjusted performance of ST funds is more closely related to their responsible nature than to their thematic approach. Sustainability-themed mutual funds are more similar to other socially responsible funds than to other thematic funds, as confirmed by performance analysis over time. They are also better than other thematic funds in overcoming financially turbulent periods and currently benefit from SRI regulation and disclosure.

Aït-Sahalia et al. (2020) authors use all traded stocks from NYSE, AMEX, and NASDAQ stock markets for 1996-2017 to construct the five Fama-French factors and the momentum factor at the 5-minute frequency. Second, the authors document the key empirical properties across all the stocks and the new factors and apply the nonparametric time series regression model with the new high-frequency Fama-French factors. Authors find that this factor model is effective in explaining the systematic component of the risk of individual stocks. In addition, the authors provide evidence that idiosyncratic jumps are related to idiosyncratic events such as earnings disappointments.

Liammukta et al. (2020), in this paper, authors have developed a Fama – French five-factor model (FF5 model) from Fama & French (2015) by using the concept the of time-varying coefficient. For a data set, the authors have used monthly data from Kenneth R. French's home page, it includes Japan portfolios (classified by using size and book-to-market) and 5 factors from July 1990 to April 2020. In the first analysis, the authors used the Augmented Dickey-Fuller test (ADF test) for the stationary test, from the result, all Japan portfolios and

5 factors are stationary. Next analysis, the authors estimated a coefficient the of the five-factor model by using a generalized additive model with a thin-plate spline to create the time-varying coefficient Fafive-factor five-factor model (TV-FF5 model). The benefit of the study is TV-FF5 model which can capture a different effect at different times of 5 factors but the traditional FF5 model can't do it. From the result, authors can show a time-varying coefficient in all factors and in all portfolios, for time-varying coefficients of Rm-Rf, SMB, and HML are significant for all Japan portfolios, time-varying coefficients of RMW are positively significant for SM, and SH portfolio and time-varying coefficients of CMA are significant for SM, SH, and BM portfolio.

Foye and Valentinčič (2020), and Fama and French (2015) recently proposed a five-factor model which adds investment and profitability terms to their seminal three-factor model. Motivated by the accounting-based nature of the new factors, the authors' test of variants of the models in Indonesia a country previous researchers have characterized by an idiosyncratic financial reporting environment and low earnings quality. Although multi-factor spanning tests imply these factors contribute to the explanation of average returns, tests using sets of LHS portfolios reveal all competing models produce large intercepts and the five-factor model offers at best only a trivial improvement to the description of average LHS returns.

Douagi et al. (2021), the novel contribution of this paper is to test if the Fama-French five- and six-factor models can explain the portfolio returns in the Regional Stock Exchange of Ivory Coast Securities (BRVM) between January 2007 and December 2018. For the Fama-French five-factor model, the results show that the only useful factors for describing the portfolio excess return are the market, value, and profitability when the OLS and the GARCH techniques are used. For the augmented Fama French six-factor model, the results report that only the market, value, profitability and illiquidity factors played an eminent role in explaining the portfolio's excess return. Moreover, using the OLS technique, it is found that the Fama-French five-factor model and the augmented Fama-French six-factor model can capture the portfolio returns. However, when the GARCH technique is used, the findings show that these models can fully explain the portfolio returns. The results found can help portfolio managers to identify extensive factors that have an impact on equity returns and to estimate the required return on the stock. Moreover, traders can employ these factor models to control investment risk.

Calice and Lin (2021), In this study, a comprehensive set of risk premia of country equity returns for 45 countries over the sample period 2002 – 2018 in both a single and a multiple-factor setting. Using a new three-pass estimation method for factor risk premia by Giglio and Xiu (2021), authors find that several factors, including default risk, are also priced in country equity excess returns, controlled by the Fama-French 5-factor and Carhart models. Moreover, the authors apply a novel approach to investigate the multi-factor impact on country equity returns. Authors find that the multi-factor information, constructed from the first principal component of the statistically significant single factors, provides a consistent and stronger prediction of anomalies in country equity returns.

Zhu et al. (2021), this paper tests a multi-factor asset pricing model that does not assume that the return's beta coefficients are constants. This is done by estimating the generalized arbitrage pricing theory (GAPT) using price differences. An implication of the GAPT is that when using price differences instead of returns, the beta coefficients are constant. Authors employ the adaptive multi-factor (AMF) model to test the GAPT utilizing a Groupwise Interpretable Basis Selection (GIBS) algorithm to identify the relevant factors from among all traded exchange-traded funds. The authors compare the performance of the AMF model with the Fama-French 5-factor (FF5) model. For nearly all time periods less than six years, the beta coefficients are time-invariant for the AMF model, but not for the FF5 model. This implies that the AMF model with a rolling window (such as five years) is more consistent with realized asset returns than is the FF5 model.

Cao et al. (2021), using a direct measure of investor attention generated from the Securities and Exchange Commission's EDGAR (Electronic Data Gathering, Analysis, and Retrieval) log files, the authors revisit the stock return predictability of the divergence of opinions in the presence of a varying degree of investor attention and information acquisition. They document a positive relationship between the divergence of opinions and future stock returns, consistent with the risk hypothesis, as opposed to the overvaluation hypothesis. More importantly, the authors find that the predictive power of divergence of opinions is more pronounced in stocks with lower investor attention. They further document the construction and profitability of divergence of opinions portfolios augmented with investor attention. A portfolio that goes long on stocks with low investor attention and the highest divergence of opinions and short on stocks with low attention and the lowest divergence of opinions generates a Fama-French 5-factor monthly alpha of 1.14%.

Platanakis (2021), for various organizational reasons, large investors typically split their portfolio decision into two stages – asset allocation and stock selection. In this study hypothesize that mean-variance models are superior to equal weighting for asset allocation, while the reverse applies for stock selection, as estimation errors are less of a problem for mean-variance models when used for asset allocation than for stock selection. This study confirms this hypothesis for US data using Bayes-Stein with no short sales and variance-based constraints. Robustness checks with four other types of mean-variance models, and a wide range of parameter settings support our conclusions. Authors also replicate our core results using Japanese data, with additional replications using the Fama-French 5, 10, 12 and 17 industry portfolios and equities from seven countries. In contrast to previous results, but consistent with our empirical results, authors show analytically that the superiority of mean-variance over 1/N is increased when the assets have a lower cross-sectional idiosyncratic volatility, which authors also confirm in a simulation analysis calibrated to US data.

Khoa and Huynh (2022), this study applied a machine-learning technique to compare the performance of the Fama-French 5-factor model (FF5). Two approaches are employed in the Fama-French model: Long Short Term Memory Recurrent Neural Network (LSTM-RNN) and Maximum Likelihood Estimation (MLE). From January 1, 2010, through March 3, 2022, the stock market in Ho Chi Minh City was experimentally researched. The rolling window approach is used in combination with the Root Mean Square Error (RMSE), and the results of the FF5 model with the LSTM-RNN algorithm are more efficient in prediction error than the MLE methodology. This contribution encourages investors and hedge fund managers to use the LSTM-RNN algorithm to boost forecasting efficiency.

Meng and Zhang (2022), this paper aims at analyzing the impact of corporate environmental information disclosure from the perspective of investors. To that end, the authors have collected environmental information disclosure data of all Chinese listed companies from 2004 to 2020 and controlled the impacts of annual reports on investor response. Authors apply the Fama-French five-factor model to calculate the accumulative abnormal returns of stocks during the event window periThe study'sdy's results suggest that environmental information disclosure can have a significant negative response among investors when authors take the impacts of annual reports into consideration. Moreover, the authors find that heavy-polluting companies and companies with high institutional shareholding are more likely to have negative reactions from investors. Notably, the negative response is found significant after the Ambient Air Quality Standard was revised in 2012. Furthermore, high environmental expenditure and strict environmental regulation will result in negative investor responses, while the political connection can alleviate the negative impacts of environmental information disclosure. The results remain robust in different ways. The findings suggest that listed companies may lack the incentive to engage in environmental management and are reluctant to disclose environmental information. Consequently, the government should formulate a mandatory disclosure policy and provide administrative support to environmental-friendly companies. Besides, companies should introduce innovative technologies to cut down environmental costs. Meanwhile, investors should be aware of the importance of corporate environmental behaviours and realize the long-term benefits of environmental management of listed companies.

3. Methodology

In this study, it is aimed to test the validity of the Fama-French Five-Factor Model for Borsa Istanbul. Within the scope of the study, the weekly data of the returns of 18 different intersection portfolios in excess of the risk-free interest rate are utilized throughout September 2009 – August 2018 (468 weeks) based on value, profitability, and investment factors. In the study, a total of 8424 portfolios (18 portfolios * 468 weeks) are formed. In the study, it is investigated which of the Fama-French Three, Four, and Five-Factor Models best explains the stock returns for Borsa Istanbul by developing estimators separately. In the Fama-French Five-Factor Model, besides the stock returns, the systematic risk premium βi (rm – rf) of stocks, market factor, size factor (SMB), value factor (HML), profitability factor (RMW) and investment factor (CMA) variables are used. In the study, companies in the financial sector, with equity capital and insufficient information are excluded from the analysis. The risk-free interest rate data used in the study are obtained from the official website of the Prime Ministry Undersecretariat of Treasury, and other data are obtained via the Finnet software. As the risk-free interest rate, the 1-month US Treasury bond interest rate is taken. In the study, a bias-adjusted robust estimator and the GRS-F test according to the Newey-West method are used.

The portfolios used in the study are presented in Table 2. After the companies are divided into 2 groups, big and small, while creating portfolios; portfolios are divided into 3 groups such as "Book Equity / Market Equity", "Investment" and "Profitability". Two portfolios, namely "Small-S" and "Big-B", are identified for the size effect. For the value effect, 3 portfolios such as "High (Big-B)," "Neutral (Neutral-N)" and "Low (Low-L)" are selected in accordance with the Book Equity/Market Equity. Later on, the 6 (2x3) value-weighted portfolio intersections according to Size and Book Equity / Market Equity are formed (Fama and French, 1995).

Table 2. Portfolios Used in the Study

Portfolio	Company Size	Value Effect	
SL[6]	Small	Low	Book Equity / Market Equity
SN	Small	Neutral	
SH	Small	High	
BL	Big	Low	
BN	Big	Neutral	
BH	Big	High	
SC	Small	Conservative	Investment
SM	Small	Medium	
SA	Small	Aggressive	
BC	Big	Conservative	
BM	Big	Medium	
BA	Big	Aggressive	
SW	Small	Weak	Profitability
SM-	Small	Medium	
SR	Small	Robust	
BW	Big	Weak	
BM-	Big	Medium	
BR	Big	Robust	

Stocks are divided into 3 groups (from smallest to largest). Then, regardless of this process, they are divided into 3 groups according to the DD/PD ratio. With the intersection of these two groups, the Firm Size DD/PD portfolio is created. In the model, the weighted average residual return ($Rit - Rft$) of the confluence of Firm Size-DD/PD, Firm Size Profitability and Firm Size-Investment portfolios is used as the dependent variable, as well as the residual returns of portfolios consisting of different combinations of these factors by dividing each factor into more percentages. Firms are ranked according to their profitability and divided into 3 percentiles. Bills in the 30th percentile and below are called "(Weak)", those in the 70th percentile and above (Strong), and the bills in the middle segment are called "(Medium)". Fama and French (2015, 2017) stated that in the profitability portfolios that they divided into three according to their profitability, the profitability portfolios with the highest third achieved higher returns. After the investment is calculated, the stocks are ranked according to the investment value and divided into 3 percentiles. Bills in the 30th percentile and below are called "(Conservative)", those in the 70th percentile and above are called "(Aggressive)" and the bills in the middle segment are called "(Medium)".

Within the scope of the study, the following models are developed to cover the aim of the study and the created portfolios:

$$Rit - Rft = \alpha i + \beta i(Rmt - Rft) + \varepsilon it$$

$$Rit - Rft = \alpha i + \beta i(Rmt - Rft) + si(SMBt) + hi(HMLt) + \varepsilon_{it}$$

$$Rit - Rft = \alpha i + \beta i(Rmt - Rft) + si(SMBt) + hi(HMLt) + ri(RMWt) + \varepsilon_{it}$$

$$Rit - Rft = \alpha i + \beta i(Rmt - Rft) + si(SMBt) + hi(HMLt) + ri(RMWt) + ci(CMAt) + \varepsilon_{it}$$

[6] *It refers to the return on a portfolio of stocks with small company size and low Book Equity / Market Equity ratio.*

In this context, the hypotheses of the GRS-F test are as follows (Gibbons, Ross & Shanken, 1989):

H_0: All alpha coefficients obtained from the CAPM, the Fama-French Three, Four, and Five-Factor models are equal to zero (αi = 0).

H_1: All alpha coefficients obtained from the CAPM, the Fama-French Three, Four, and Five-Factor models are not equal to zero (αi ≠ 0).

Table 3. Descriptive Statistics of the Intersection Portfolios in Excess of the Risk-Free Interest Rates

	N (Week)	Mean	Std. Deviation
SL	468	.0027	.03264
SN	468	.0037	.02982
SH	468	.0024	.02925
BL	468	.0031	.02504
BN	468	.0039	.02413
BH	468	.0023	.02623
SC	468	.0038	.03068
SM	468	.0032	.02964
SA	468	.0018	.03006
BC	468	.0033	.02698
BM	468	.0033	.02535
BA	468	.0035	.02786
SW	468	.0015	.02953
SM-	468	.0030	.02846
SR	468	.0051	.03206
BW	468	.0004	.02846
BM-	468	.0034	.02467
BR	468	.0042	.02346

Descriptive statistics regarding the created portfolios created in the study are given in Table 3. The highest average, the value-weighted weekly return is the SR portfolio, which consists of stocks with small and robust profitability ratios in terms of the company size The most profitable portfolios are SR, BR, and BN, respectively.

Table 4. Correlation Analysis Results regarding the Factor Premiums

	RM-RF	SMB	HML	CMA	RMW
RM-RF	1				
SMB	0.096	1			
HML	0.063	-0.223	1		
CMA	0.116	.018	0.166	1	
RMW	-.005	0.133	-0.064	-.010	1

Correlation analysis results regarding the factor premiums are given in Table 4. There is a negative relationship between SMB and HML factors, whereas a positive and weak relationship between CMA and HML factors. It is seen that there is a very low correlation among the independent variables used in the study. It can be claimed that this situation can prevent multicollinearity problems and spurious regression results that may occur in the model.

Table 5. Unit Root Test Results

Variables	LLC Test		PP Fisher Test	
	T-test	Probability(p)	Statistic	Probability (p)
SL	-5.76	0.000	37.21	0.000
SN	-17.27	0.000	87.86	0.000
SH	-28.50	0.000	56.33	0.000
BL	-4.99	0.000	42.58	0.000
BN	-17.31	0.000	72.84	0.000
BH	-22.91	0.000	69.21	0.000
SC	-25.40	0.000	72.54	0.000
SM	-29.61	0.000	117.70	0.000
SA	-28.04	0.000	77.87	0.000
BC	-24.22	0.000	70.68	0.000
BM	-9.69	0.000	45.39	0.000
BA	-14.73	0.000	36.73	0.000
SW	-7.76	0.000	36.79	0.000
SM-	-7.17	0.000	40.73	0.000
SR	-9.46	0.000	43.99	0.000
BW	-20.06	0.000	61.21	0.000
BM-	-8.22	0.000	43.97	0.000
BR	-21.21	0.000	76.54	0.000
Rm-Rf	-26.40	0.000	76.45	0.000
SMB	-8.77	0.000	45.22	0.000
HML	-7.54	0.000	76.55	0.000
CMA	-12.45	0.000	44.80	0.000
RMW	-15.60	0.000	66.40	0.000

The hypotheses for unit root tests of the variables are as follows:

H_0: The series contains a common unit root (H_0: $p_i = p = 1$).

H_1: The series contains no common unit root (H_1: $p_i = p < 1$).

Series must be stationary in order to obtain econometrically significant relationships among the variables. If the time series of the variables contain a trend, the relationship reflects the spurious (bogus) regression (Sevinç, 2013, pp. 235-236). Table 5 presents the unit root test results indicating the stationarity of the variables. In this study, the Fisher ADF Root Test is used. Also, the LLC and the PP Fisher tests are performed. Both root test results indicate that the series is stationary. In other words, the null hypothesis (*H_0*), which claims that the variables contain unit roots due to the stationarity of the series, is statistically rejected.

4. Findings

In this part of the study, the validity of the Fama-French Five-Factors Asset Pricing Model (FF5F) is tested for Turkey.

Table 6. Regression Results

Ri – Rf	a	β	s	h	r	c	Grs-f	dw	f-statistic	Adjusted R^2
CAPM	0.003 (0.285)	0.254 (2.564)**	-	-	-	-	1.55 (0.15)	1.986	35.78 (0.000)	0.384
Fama-French Three Factor (Model 1)	0.003 (0.285)	0.254 (2.564)**	1.023 (5.754)*	.121 (1.032)	-	-	1.32 (0.23)	1.986	35.78 (0.000)	0.447
Fama-French Four Factor (Model 2)	0.004 (0.292)	0.261 (2.631)**	1.135 (5.954)*	.143 (1.195)	.656 (4.034)*	-	1.13 (0.38)	1.867	38.25 (0.000)	0.448
Fama-French Five Factor (Model 3)	0.011 (0.322)	0.282 (2.945)**	1.265 (6.551)*	.160 (1.280)	.756 (4.344)*	-.234 (1.001)	1.02 (0.41)	1.982	42.91 (0.000)	0.455

The CAPM, Fama-French Three-Four, and Five-Factor regression results are presented in Table 6. Upon examining the analysis results, it is understood that 4 models constituted with 18 portfolios are significant and there is no autocorrelation. The R^2 values of the CAPM, the Fama-French Three-Factor, the Fama-French Three-Factor, the Fama-French Four Factor, and the Fama-French Five-Factor Models are 38.4%; 44.7%; 44.8%, and 45.5%, respectively. This obtained result indicates the Fama-French Five-Factor Model as the model with the highest explanatory power in explaining stock returns. However, it is seen that alpha coefficients are equal to zero and there is no pricing error in the developed models. Moreover, the market factor β coefficients in models are positive and significant. The value factor "h" coefficient is not statistically significant in the Fama-French Three-Four and Five-Factor regression models. On the other hand, in the Fama-French Four and Five-Factor regression models, the profitability factor "r" coefficient is positive and significant. Furthermore, the investment factor "c" coefficient is not determined to be statistically significant in the Fama-French Four and Five-Factor regression models.

Consequently, the H_1 hypothesis is accepted for the CAPM, the Fama-French Three, Four, and Five-Factor Models after performing the GRS-F test. In other words, the CAPM, the Fama-French Three, Four, and Five-Factor Models are determined to be valid for Borsa İstanbul, since there is no pricing error in the models.

Table 7 indicates the regression results of the CAPM, the Fama-French Three, Four, and Five-Factor Models in terms of portfolios formed with stocks of large-scale companies. Upon examining the analysis results, it is understood that all 4 models generated with 9 portfolios are significant and there is no autocorrelation. R^2 values of the CAPM, the Fama-French Three-Factor, Four Factor, and Five-Factor Models are 34.2%; 42%; 42.1%, and 43%, respectively. This result obtained reveals that the model with the highest explanatory power in explaining the stock returns of large-scale companies is the Fama-French Five-Factor Model.

Table 7. Regression Results regarding the Portfolios Formed with Stocks of Large-Scale Companies

Ri – Rf	α	β	s	h	r	c	Grs-f	dw	f-statistic	Adjusted R^2
CAPM	0.003 (0.202)	0.182 (1.973)**	-	-	-	-	1.43 (0.13)	1.871	28.12 (0.000)	0.342
Fama-French Three Factor	0.004 (0.221)	0.202 (2.021)**	1.129 (5.239)*	.132 (1.190)	-	-	1.29 (0.26)	1.922	31.20 (0.000)	0.420
Fama-French Four Factor	0.002 (0.234)	0.245 (2.230)**	1.344 (5.722)*	.143 (1.216)	.643 (4.228)*	-	1.16 (0.36)	1.791	32.11 (0.000)	0.421
Fama-French Five Factor	0.002 (0.304)	0.265 (2.642)**	1.342 (5.986)*	.132 (1.322)	.698 (4.328)*	-.264 (1.121)	1.04 (0.39)	1.885	34.23 (0.000)	0.430

Table 8. Regression Results regarding the Portfolios Formed with Stocks of Small-Scale Companies

Ri – Rf	α	β	s	h	r	c	Grs-f	dw	f-statistic	Adjusted R^2
CAPM	0.002 (0.242)	0.198 (2.091)**	-	-	-	-	1.39 (0.15)	1.745	43.33 (0.000)	0.453
Fama-French Three Factor	0.002 (0.286)	0.289 (2.783)**	1.253 (5.854)*	.146 (1.243)	-	-	1.33 (0.23)	2.025	58.15 (0.000)	0.547
Fama-French Four Factor	0.002 (0.296)	0.291 (2.801)**	1.543 (5.987)*	.165 (1.440)	.598 (4.108)*	-	1.10 (0.38)	1.991	59.01 (0.000)	0.548
Fama-French Five Factor	0.001 (0.312)	0.310 (3.019)**	1.353 (6.294)*	.147 (1.270)	.676 (4.245)*	-.264 (1.087)	1.01 (0.46)	1.980	62.20 (0.000)	0.554

Table 8 indicates the results of the CAPM, the Fama-French Three, Four, and Five-Factor regression results in terms of portfolios formed with stocks of small-scale companies. Upon examining the analysis results, it is understood that all 4 models generated with 9 portfolios are significant and there is no autocorrelation. R^2 values of the CAPM, the Fama-French Three-Factor, Four Factor, and Five-Factor Models are 45,3%; 54.7%; 54.8%, and 55.4%, respectively. This obtained result reveals that the model with the highest explanatory power in explaining the stock returns of small-scale companies is the Fama-French Five-Factor Model.

5. Conclusion

Accurate calculation of the expected returns of the stocks is one of the issues that has been in dispute since the 1950s and there is still no consensus in the literature on the issue. Many models have been developed concerning the concept. Distinctive models are still being created. The common point in the generated models is that they are all based on the CAPM. In models developed with respect to the CAPM and the most recent studies, the aim is to calculate the stock returns more accurately. For this purpose, many models have been developed in which new explanatory factors, which are thought to influence calculations, are included. The Fama-French Five-Factor (2015) model is one of the models developed for this purpose.

In this study, the validity of the Fama-French Five-Factor Model for Borsa İstanbul is tested. Within the scope of the study, based on value, profitability, and investment factors, the weekly data of the returns of 18 different intersection portfolios in excess of the risk-free interest rate are utilized between September 2009 and August 2018 (468 weeks). In the study, a bias-adjusted robust estimator and the GRS-F test according to the Newey-West method is used. As a result of the analysis, R^2 values of the CAPM, and the Fama-French Three, Four, and Five-Factor Models are determined as 38.4%; 44.7%; 44.8%, and 45.5%, respectively. This result singles out the Fama-French Five-Factor Model as the model with the highest explanatory power in explaining stock returns. Also, as a result of the GRS-F test, the H_0 hypothesis is accepted for the CAPM, the Fama-French Three, Four, and Five-Factor Models. In other words, the CAPM, the Fama-French Three, Four, and Five-Factor Models are determined to be valid for Borsa İstanbul, since there is no pricing error in the models.

In the study, the regression results of the CAPM, the Fama-French Three, Four, and Five-Factor Models are analyzed in terms of portfolios formed with stocks of large- and small-scale companies. According to the analysis results, the R^2 values of the CAPM, the Fama-French Three-Factor, Four, and Five-Factor Models are 34.2%; 42%; 42.1% and 43%, respectively; in terms of portfolios formed with stocks of large-scale companies.

On the other hand, R^2 values are 45.3, 54.7%, 54.8%, and 55.4%, respectively; in terms of the portfolios formed with the stocks of small-scale companies. In other words, the Fama-French Five-Factor Model has higher explanatory power to explain the portfolios formed with the stocks of small-scale companies compared to the portfolios formed with the stocks of large-scale companies. This situation can be justified by the fact that small-scale companies have higher risks and higher returns. The findings obtained within the scope of the study are compatible with certain studies in the literature such as Fama and French (1993) which concluded that companies with a Market Equity of less than $ 1 billion and stocks of companies with high book equities tended to yield higher returns than estimated through the CAPM, and Kubota and Takehara (2018) which explained the reason of that situation as a common risk factor.

References

Ahmed, S., Bu, Z., Tsvetanov, D. (2019). Best of the best: a comparison of factor models. – Journal of Financial and Quantitative Analysis, 54(4), pp. 1713-1758.

Aït-Sahalia, Y., Kalnina, I., Xiu, D. (2020). High-frequency factor models and regressions. – Journal of Econometrics, 216(1), pp. 86-105.

Basu, S. (1977). Investment Performance of Common Stocks in Relation to Their Price-Earnings Ratios: A Test of The Efficient Market Hypothesis. – The Journal Of Finance, Vol. 32, N 3, pp. 663-680.

Black, F. (1972). Capital Market Equilibrium with Restricted Borrowing. – The Journal of Business, Vol. 45, N 3, pp. 444-455.

Calice, G., Lin, M. T. (2021). Exploring risk premium factors for country equity returns. – Journal of Empirical Finance, 63, pp. 294-322.

Cao, Z., Kilic, O., Wang, X. (2021). Investor attention, divergence of opinions, and stock returns. – Journal of Behavioral Finance, 22(3), pp. 265-279.

Chakraborty, N., Elgammal, M. M., McMillan, D. (2019). Rational functions: an alternative approach to asset pricing. – Applied Economics, 51(20), pp. 2091-2119.

Chiah, M., Chai, D., Zhong, A., Li, S. (2016). A Better Model? An empirical investigation of the Fama-French five-factor model in Australia. – International Review of Finance, 16(4), pp. 595-638.

Cox, S., Britten, J. (2019). The Fama-French five-factor model: evidence from the Johannesburg Stock Exchange. – Investment Analysts Journal, 48(3), pp. 240-261.

Dempsey, M. (2013). The Capital Asset Pricing Model (CAPM): The History of Failed Revolutionary Idea in Finance?. – Abacus, Vol. 49, Supplement, pp. 7-23.

Douagi, B. M. F. W., Chaouachi, O., Sow, M. (2021). The portfolio management: investigation of the Fama-French five-and six-factor asset pricing models. – Polish Journal of Management Studies, 23.

Fama, E. F., French, K. R. (2017). International tests of a five-factor asset pricing model. – Journal of financial Economics, 123(3), pp. 441-463.

Fama, E. F., French, K. R. (2015). A Five-Factor Asset Pricing Model. – Journal of Financial Economics, 116, pp. 1-22.

Fama, E. F. (1976). Foundation of Finance. Basic Books.

Fama, E. F. (1993). Fama, E. F. ve French, K. R. (1993). Common Risk Factors in The Returns On The Stocks and Bonds. – Journal of Finance Economics, Vol. 47, pp. 3-56.

Fama, E. F. ve French, Kenneth, R. (1995). Size and Book-to-Market Factors in Earnings and Returns. – Journal of Finance, 14 (1), pp. 131-155.

Foye, J., Valentinčič, A. (2020). Testing factor models in Indonesia. – Emerging Markets Review, 42, 100628.

Foye, J. (2018). A Comprehensive Test of the Fama-French Five-Factor Model in Emerging Markets. – Emerging Markets Review, 37, pp. 199-222.

Gibbons, M., Ross, R., Stephan, A., ve Shanken, J. (1989). A Test of the Efficiency of a Given Portfolio. – Econometrica, 57 (5), pp. 1121-1152.

González, M., Jareño, F. (2019). Testing extensions of Fama & French models: a quantile regression approach. – The Quarterly Review of Economics and Finance, 71, pp. 188-204.

Huang, T. L. (2018). Is The Fama and French Five-Factor Model Robust in the Chinese Stock Market. – Asia Pacific Management Review.

Ielasi, F., Rossolini, M. (2019). Responsible or thematic? The true nature of sustainability-themed mutual funds. – Sustainability, 11(12), 3304.

Jan, M. N., Ayub, U. (2019). Do The Fama And French Fıve-Factor Model Forecast Well Usıng Ann?. – Journal of Business Economics and Management, Vol. 20, 1, pp. 168-191.

Khoa, B. T., Huynh, T. T. (2022). Long Short-Term Memory Recurrent Neural Network for Predicting the Return of Rate Underframe the Fama-French 5 Factor. Discrete Dynamics in Nature and Society.

Kubota, K., Takehara, H. (2018). Does The Fama and French Five-Factor Model Work Well in Japan?. – International Review of Finance, 18:1, pp. 137-146.

Li, C., Hu, Z., Tang, L. (2019). Re-examining the Chinese A-Share Herding Behaviour With A Fama-French Augmented Seven-Factor Model. – Applied Economics, 51(5), pp. 488-508.

Liammukda, A., Khamkong, M., Saenchan, L., Hongsakulvasu, N. (2020). The Time-Varying Coefficient Fama-French Five Factor Model: A Case Study in the Return of Japan Portfolios. – The Journal of Asian Finance, Economics and Business, 7(10), pp. 513-521.

Lin, Q. (2017). Noisy Prices and the Fama-French Five-Factor Asset Pricing Model in China. – Emerging Markets Review, 31, pp. 141-163.

Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. – The Review of Economics and Statistics, Vol. 47, N 1, pp. 13-37.

Markowitz, H. (1952). Portfolio Selection. – The Journal of Finance, Vol. 7, N 1. (Mar., 1952), pp. 77-91.

Meng, J., Zhang, Z. (2022). Corporate environmental information disclosure and investor response: Evidence from China's capital market. – Energy Economics, 108, 105886.
Modigliani, F., Miller, H. M. (1958). The Cost of Capital, Corporation Finance and the Theory of Investment. – The American Economic Review, Vol. 48, N 3 (Jun., 1958), pp. 261-297.
Modigliani, F., Miller, H. M. (1963). Corporate Income Taxes and the Cost of Capital: A Correction. – The American Economic Review, Vol. 53, N 3 (Jun., 1963), pp. 433-443.
Mossin, J. (1966). Equlibrium in a Capital asset Market. – Econometrica, Vol. 34, N 4, pp. 768-783.
Olive, P. F., Rohini, A., Deepa, N., Selvanayaki, S. (2018). An econometric investigation of FAMA french three-factor model and capital asset pricing model in Indian stock market. – Indian Journal of Economics and Development, 14(3), pp. 430-438.
Pandey, P., Sehgal, S. (2019). Investor sentiment and its role in asset pricing: An empirical study for India. – IIMB Management Review, 31(2), pp. 127-144.
Papenkov, M. (2019). An Empirical Asset Pricing Model Accommodating the Sector-Heterogeneity of Risk. – Atlantic Economic Journal, 47(4), pp. 499-520.
Platanakis, E., Sutcliffe, C., Ye, X. (2021). Horses for courses: Mean-variance for asset allocation and 1/N for stock selection. – European Journal of Operational Research, 288(1), pp. 302-317.
Racicot, F. E., Rentz, W. F., Tessier, D., Theoret, R. (2019). The Conditional Fama-French Model And Endogenous İlliquidity: A Robust İnstrumental Variables Test. – PloS one, 14(9).
Rugwiro, S., Choi, S. B. (2019), Re-examination of Fama-French Models in The Korean Stock Market. – Asia-PasificFinancial Markets, 26(1), pp. 23-45.
Senarathne, C. W. (2019), Do Fama-French Common Risk-Factor Portfolio Investors Herd on A Daily Basis? Implications for Common Risk-Factor Regressions. – Journal of Capital Markets Studies, Nov. 19, pp. 137-157.
Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk. – The Journal of Finance, Vol. 19, N 3, pp. 425-441.
Zhao, Y., Stasinakis, C., Sermpinis, G., Fernandes, F. D. S. (2019). Revisiting Fama–French Factors' Predictability With Bayesian Modelling And Copula-Based Portfolio Optimization. – International Journal of Finance & Economics.
Zhu, L., Jarrow, R. A., Wells, M. T. (2021). Time-invariance coefficients tests with the adaptive multi-factor model. – The Quarterly Journal of Finance, 11(04), 2150019.