ИНСТИТУТ ЗА ИКОНОМИЧЕСКИ ИЗСЛЕДВАНИЯ НА БЪЛГАРСКАТА АКАДЕМИЯ НА НАУКИТЕ
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DYNAMIC MEASURES OF SOVEREIGN SYSTEMIC RISK[2]

This paper introduces a dynamic dependence framework to calculate various indicators of systemic sovereign default risk. Our analysis reveals a notable increase in systemic fragility among euro-area sovereigns since the onset of the Subprime Crisis, particularly during the First Greek Bailout in May 2010. Furthermore, our measures successfully capture key events within the euro area, including Mario Draghi's impactful "whatever-it-takes" speech in mid-2012 and the Cypriot Banking Crisis of 2012-2013. The incorporation of dynamic dependence into our measures provides a more comprehensive depiction of systemic risk within the euro area sovereign system, often demonstrating distinct dynamics when compared to their static counterparts. These findings carry significant policy implications and contribute to enhancing our understanding of systemic risk among euro-area sovereigns.
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1. Introduction

In the past decade, the issue of sovereign default has gained significant prominence, particularly within the context of the euro area (EA). The potential adverse consequences of a default by an EA government have prompted numerous sovereign and bank bailouts, while also impacting interest rates, capital flows, trade dynamics, and overall economic growth in Europe and beyond. Consequently, there is a pressing need for an extensive examination of sovereign risk levels and their implications for the broader financial system within the EA. This necessity has given rise to fundamental questions that demand thorough investigation: How can the systemic risk of sovereigns be quantified? What are the mechanisms for

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[2] This paper is an outcome of a project undertaken at the European Central Bank (ECB) using proprietary Intercontinental Exchange Competition and Markets Authority (ICE CMA) data, downloaded via Datastream. While Datastream has its own in-house CDS provider (Thomson Reuters), the quality of the data, as well as the duration of the time series for many sovereigns and banks is not satisfactory. However, Datastream provides CDS data from other sources (in this case – ICE CMA), at the cost of purchasing an additional license. The ECB agreed to provide data for this project for the duration of the research stay but declined any subsequent updates.
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measuring feedback and contagion effects stemming from a sovereign default? Is it likely that defaults of smaller peripheral governments would trigger defaults of larger EA sovereigns? Despite their crucial relevance to the formulation of consistent and timely macro- and micro-prudential policies, these questions remain largely unanswered.

This paper makes a valuable contribution to this discourse by proposing an innovative approach to analyze systemic risk and contagion among sovereigns, drawing upon market expectations regarding default risk. By employing this approach, we enhance several significant measures of sovereign systemic risk within the regulators' toolkit. To this end, our study adopts a comprehensive methodology for assessing joint default risk among sovereigns, aiming to augment existing measures that capture conditional systemic default risk within the euro area sovereign system, developed in Radev (2022c).

It is imperative for regulators and policymakers to examine and monitor euro area sovereign default in conjunction with banking default due to the significant exposures of EA banks to EA government debt observed during the sovereign debt crisis (see, for instance, ECB 2010b; EBA 2011; IMF 2011; Acharya, Thakor 2016). The potential transmission of negative shocks from sovereigns to the banking sector can lead to the collapse of the entire EA financial system. Although sovereign defaults are relatively rare, they carry substantial welfare costs not only for the parties involved in the debt contract but also for third parties. On one hand, defaulting sovereigns experience reputational losses and limited future access to international debt markets (see, for example, Panizza et al. 2009). On the other hand, defaults have direct adverse effects on domestic and foreign private and public investors who, in the worst-case scenario, may also face distress (see, for instance, Arteta and Hale 2008). Moreover, contagion can propagate not only through financial channels but also through real economy channels due to the strong economic interconnections within the common currency area, the European Union (EU), and globally (Gorea, Radev 2014).

To expand the regulatory toolkit for measuring systemic risk, we refine the currently employed measures utilized by the European Central Bank (ECB). Specifically, we introduce dynamic dependence into a set of minimum cross-entropy systemic risk measures developed by Radev (2022b), and Radev (2022c). These measures include the conditional probability of a sovereign defaulting given the default of another sovereign, the conditional probability of a sovereign defaulting given the simultaneous defaults of two other sovereigns, and the multivariate conditional probability of at least n sovereigns defaulting given the default of a specific sovereign. The first two conditional measures are vital for investigating and monitoring specific channels through which default risk is transmitted within the sovereign system. In contrast, the multivariate conditional measure captures the overall risk in the system by considering the complete dynamic dependence structure inherent among euro area sovereigns.

Our approach consists of three steps. Firstly, we employ credit default swap (CDS) spreads to infer the perceived individual default risk of ten euro area sovereigns. This is accomplished using a CDS bootstrapping technique (Hull, White, 2000; Gorea, Radev, 2014; Radev, 2022a). By utilizing derivatives such as CDS that are more sensitive to default risk, we address the issue of the infrequent occurrence of sovereign defaults in Europe. Additionally, our bootstrapping procedure enables us to estimate the expected probability of default based on a single CDS spread observation, thereby significantly reducing data requirements.

To model the multivariate probability density of the euro area sovereign system consistently with individual probabilities of default (PoDs), we employ the minimum cross-entropy procedure proposed by Kullback (1959) and extended by Segoviano (2006) and Segoviano and Goodhart (2009). Segoviano (2006) refers to this method as Consistent Information Multivariate Density Optimization (CIMDO). The cross-entropy approach draws on the Merton Model's insight that an institution defaults on its debt when its assets can no longer cover its liabilities. However, by utilizing traded default-sensitive credit default swap (CDS) data, it avoids the need to rearrange assets to fit within the Merton Model framework, as required by Gray, Bodie, and Merton's Sovereign Contingent Claims Analysis (2007) and Gray's work (2011). Our paper presents a significant modelling innovation in comparison to the measures proposed by Segoviano and Goodhart (2009), Radev (2022b), and Radev (2022c) by introducing dynamic correlation into the measurement of conditional systemic risk of sovereigns using rolling windows of changes in 5-year CDS spreads. This approach enables us to derive measures that more accurately reflect the level of systemic risk at any given time. Subsequently, after obtaining the dynamic multivariate probability density of the sovereign system, we progress to the final stage of deriving a series of systemic risk indicators that analyze the sovereign system's vulnerability to default events.

Our findings indicate a substantial increase in sovereign systemic fragility since mid-2007. Various events appear to influence this dynamic, including the Subprime Crisis, Greek fiscal issues, and subsequent efforts by European authorities to mitigate the Sovereign Debt Crisis in the euro area. The dynamic dependence versions of our measures offer a more comprehensive depiction of conditional default risk in the European sovereign system and, in many instances, exhibit distinct dynamics compared to their static counterparts. This underscores the significance of acknowledging changes in dependence that may occur during crisis periods when assessing systemic default risk in the sovereign system.

We conduct essential extensions and robustness checks to enhance the applicability of our paper to policymakers and regulators. Due to data limitations, we are unable to update the full sample of sovereigns in our analysis to the present day. However, we managed to update the measures until mid-2017 for several prominent euro area sovereigns operating across the continent, namely Spain, the Netherlands, and Italy. The updated results demonstrate the impact of the later stages of the Sovereign Debt Crisis, such as Mario Draghi's "whatever-it-takes" speech and the Cypriot Banking Crisis of 2012 and 2013, on euro area sovereigns.

Our paper contributes to the literature on systemic risk measures and cross-entropy-based measures of sovereign risk. Measures such as CoVaR (Adrian, Brunnermeier, 2016), Marginal and Expected Systemic Shortfalls (Acharya, Richardson, 2009; Acharya et al., 2017), and SRISK (Brownlees, Engle, 2017) have been widely used but failed to capture the complete dependence structure among banks in the financial system. Segoviano and Goodhart (2009) introduced multivariate CDS-based measures using the CIMDO approach (Segoviano, 2006). However, these measures assume Gaussian distributions and independence among banks, limiting their informational content. Gorea and Radev (2014) addressed this limitation by discussing correlated joint probabilities of default during the Sovereign Debt Crisis. Radev (2022b) and Radev (2022c) extended the set of systemic risk measures with the Conditional Joint Probability of Default and the unconditional Systemic Risk Measure (SFM) using the cross-entropy approach. Radev (2022d) expanded the Systemic Fragility Measure by incorporating the LOO approach (Hue et al., 2019). These measures offer valuable insights into systemic risk. In our paper, we introduce dynamic dependence into conditional measures of systemic default risk of sovereigns. By considering the evolving dependence structure, we aim to provide a richer depiction of conditional default risk in the sovereign system.

We make several significant contributions to literature. Firstly, we contribute to the broader research agenda focused on understanding the vulnerability of the European financial system during recent crises, as explored in studies by Gorea and Radev (2014), Acharya and Steffen (2015), Radev (2022b), Radev (2022c), and Radev (2022d). Secondly, our work enhances the multivariate probability measures of sovereign systemic risk developed by Segoviano and Goodhart (2009), Radev (2022b), Radev (2022c), and Radev (2022d) by incorporating dynamic dependence into the modelling of conditional systemic risk.

The paper is organized as follows: Section 2 outlines our methodology for deriving marginal and joint probabilities of default. In Section 3, we introduce our probability measures and provide guidelines for their calculation. Section 4 provides a brief overview of our dataset, while Section 5 presents our empirical findings. Finally, Section 6 concludes the paper.

2. Methodology

Our multivariate conditional probability estimation approach involves three steps. Firstly, we obtain probabilities of default (PoDs) from CDS spreads using the CDS bootstrapping procedure outlined in Hull and White (2000). Secondly, we employ a minimum cross-entropy method, following the work of Kullback (1959) and Segoviano (2006), to construct a multivariate probability distribution that aligns with the individual PoDs. Additionally, we account for the dynamic dependence among sovereign entities within the system, as discussed in Radev (2022d). Lastly, we calculate various systemic distress measures to assess the risk within the sovereign system of the euro area.

2.1. Deriving Marginal Probabilities of Default

This paper employs a CDS bootstrapping procedure to estimate probabilities of default (PoD) based on the approach outlined in Hull and White (2000) and used in Radev (2022a). The method utilizes a cumulative probability model that incorporates recovery rates, refinancing rates, and cumulative compounding. By leveraging CDS contracts with varying maturities, hazard rates are calibrated for specific time horizons, enabling the estimation of cumulative default probabilities.

The rationale behind the CDS bootstrapping procedure lies in utilizing default-sensitive contracts traded in the insurance market, such as credit default swaps, which aim to protect the buyer against underlying asset defaults. This enables the reverse-engineering of PoDs that satisfy the no-arbitrage condition, ensuring fair market value without any arbitrage advantage. While there are several approaches and proxies available for deriving PoDs from CDS contracts, the modelling procedure described by Hull and White (2000) stands out as popular, robust, and consistent, particularly when utilizing the complete term structure of CDS spreads for individual entities. Additional details on the CDS bootstrapping procedure can be found in the online appendix.

The aforementioned method is applicable for estimating probabilities of default for both sovereign and corporate entities. The resulting risk measures represent risk-neutral probabilities of default, meeting the no-arbitrage condition in financial markets (Hull, 2006). Risk-neutral probabilities refer to the probabilities that render market participants indifferent to buying or selling an asset given the prevailing market conditions. It is worth noting that risk-neutral probabilities differ from actual (or physical) probabilities, which consider the risk aversion of market participants. Empirical approximations of risk aversion, such as Sharpe's Ratio (Sharpe, 1966), can be used to derive physical probabilities from risk-neutral probabilities. In this analysis, we present risk-neutral probabilities as they tend to be larger than their physical counterparts, providing more conservative estimates of default risk. Given the rarity of the events examined in this paper, it is argued that policymakers should employ more conservative estimates to effectively monitor the default risk of the sovereigns falling under their purview.

We utilize the complete range of maturities spanning from 1 to 5 years of CDS spreads to estimate sovereign probabilities of default (PoDs). To account for quarterly premium payments and accrual interest, as suggested by Adelson, Bemmelen, and Whetten (2004), appropriate adjustments are made. For risk-free rates, we consider all available maturities of AAA Euro Area bond yields within the 1 to 5-year range. The recovery rate is consistently set at 40%, aligning with the prevailing assumption in both literature and practical applications.[3] The resulting series represents cumulative probabilities of default. To obtain probabilities with a one-year horizon, which is of particular interest to policymakers, an annualization process is performed using the following formula:

$$PoD_t^{\text{annual}} = 1 - (1 - PoD_t^{\text{cum}})^{\frac{1}{T}}, \tag{1}$$

where T is the respective time horizon (T=5 for 5-year $\boldsymbol{PoD}$) and $\boldsymbol{PoD_t^{\text{annual}}}$ is the annualized version of the cumulative $\boldsymbol{PoD_t^{\text{cum}}}$.

2.2. Recovery of the Multivariate Probability Density

Given the scarcity of traded joint credit events in the insurance market, it becomes necessary to impose a specific structure on the multivariate probability density of the system in order to facilitate the transition from individual probabilities of default (PoDs) to joint probabilities. Our methodology is rooted in the concept of cross-entropy, originally introduced by Kullback (1959). By minimizing an objective function based on cross-entropy, we iteratively update a

[3] For a discussion on how different recovery rates affect the PoD estimates, please refer to Gorea and Radev (2014).

prior distribution to derive a posterior distribution that satisfies a set of constraints designed to ensure the consistency of the joint probabilities with the individual PoDs.

Cross-entropy exhibits close ties and shares much of its terminology with Bayesian statistics. The main objective of this method is to obtain a reasonable approximation of the unknown joint asset distribution, capturing the inherent characteristics of the available data, including dependence, fat tails, skewness, and more. This is achieved by adjusting a prior multivariate distribution, which serves as an initial estimate, to align with the data. In practical terms, the outlined procedure redistributes the probability mass from the central region of the joint distribution towards its tails beyond a predetermined threshold, mimicking the Merton-like properties. Importantly, this adjustment is performed in a manner that ensures the tail mass remains consistent with the individual PoDs derived from individual credit default swap (CDS) spreads. Since financial markets are incomplete and traded baskets of CDS for all possible scenarios are not readily available, these approximations are considered reasonable and widely employed.

Within banking literature, it is common to adopt a prior distribution of normal shape without considering cross-entity correlation (Segoviano, 2006; Segoviano, Goodhart, 2009). However, empirical evidence presented by Gorea and Radev (2014) and further analytical proofs by Radev (2022a) highlight the significance of selecting an appropriate correlation structure for the prior distribution, especially considering the correlation between bank and sovereign assets. Consequently, the authors advocate for using a static correlation matrix as a prior distribution, representing an improvement over the zero-correlation model proposed by Segoviano (2006). Building upon this, Jin and De Simone (2014) extend the research by incorporating time-varying covariance using the BEKK model developed by Engle and Kroner (1995) for a portfolio of five banks operating in Luxembourg. Additionally, Gorea and Radev (2014) conduct sensitivity checks, demonstrating that employing a prior distribution with fatter tails, such as a t-distribution with a low number of degrees of freedom, only leads to marginal differences in joint probabilities of default. This observation arises from the fact that the focus of multivariate probability measures in the literature lies primarily in summarizing the tail mass of the joint distribution rather than its precise shape. Considering the limited benefits and the substantial computational burden associated with more intricate distributions, particularly in higher dimensions, Radev (2022a) argues in favour of utilizing a joint normally distributed prior as a pragmatic choice.

To proceed, let the financial system be represented by a portfolio of n sovereigns: $\boldsymbol{X_1}$, $\boldsymbol{X_2}$, to $\boldsymbol{X_n}$., with their log assets being $\boldsymbol{x_1}$, $\boldsymbol{x_2}$, to $\boldsymbol{x_n}$. The cross-entropy approach then minimizes the following Lagrangian:

$$
\begin{aligned}
L(p,q) = & \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,\dots,x_n)\ln\left[\frac{p(x_1,x_2,\dots,x_n)}{q(x_1,x_2,\dots,x_n)}\right] dx_1\cdots dx_{n-1}dx_n \\
& +\lambda_1\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,\dots,x_n)\mathbf{I}_{[\bar{\mathbf{x}}_1,\infty)} dx_1\cdots dx_{n-1}dx_n - PoD_t^1\right] \\
& +\lambda_2\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,\dots,x_n)\mathbf{I}_{[\bar{\mathbf{x}}_2,\infty)} dx_1\cdots dx_{n-1}dx_n - PoD_t^2\right] \\
& +\cdots \\
& +\lambda_n\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,\dots,x_n)\mathbf{I}_{[\bar{\mathbf{x}}_n,\infty)} dx_1\cdots dx_{n-1}dx_n - PoD_t^n\right] \\
& +\mu\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1,x_2,\dots,x_n) dx_1\cdots dx_{n-1}dx_n - 1\right]
\end{aligned} \tag{2}
$$

The first integral in Equation (2) represents the cross-entropy probability difference (see Kullback (1959)) that minimizes the distance between a prior distribution guess $q(x_1,x_2,\dots,x_n) \in R_n$ and a posterior distribution $p(x_1,x_2,\dots,x_n) \in R_n$ that reflects empirical market data on individual probabilities of default. PoD_t^1, $PoD_t^2$to PoD_t^n stand for the expected probabilities of default of the respective entities, derived from CDS prices. With $\mathbf{I}_{[\bar{\mathbf{x}}_1,\infty)}$, $\mathbf{I}_{[\bar{\mathbf{x}}_2,\infty)}$ to $\mathbf{I}_{[\bar{\mathbf{x}}_n,\infty)}$ we denote a set of indicator variables that take the value of one if the respective entities' default thresholds x_1, x_2, to x_n are crossed and zero otherwise. The default thresholds are the same as in the classic structural model (Merton, 1974). μ, λ_1, λ_2 to λ_n are the Lagrange multipliers of the constraints. The optimal posterior distribution is then:[4]

$$
p^*(x_1,x_2,\dots,x_n) = q(x_1,x_2,\dots,x_n)\exp\left\{-\left[1+\mu+\sum_{i=1}^{n}\lambda_i\mathbf{I}_{\mathbf{x}_i,\infty)}\right]\right\} \tag{3}
$$

Hence, to obtain the most favourable posterior distribution, three key components are required: a prior distribution with a suitable dependence structure (such as a multivariate Gaussian density with an empirical correlation matrix), optimal Lagrange multipliers, and individual default thresholds.[5] The resultant posterior joint distribution exhibits two significant characteristics. Firstly, it captures the market's collective perspective regarding the default region of the unobservable asset distribution within the system. Secondly, it possesses fat tails, even when the initial assumption is based on a multivariate Gaussian distribution.

[4] *See Radev (2022a) for a complete solution of the multivariate minimum cross-entropy problem.*
[5] *Data and codes to replicate the bivariate case of the cross-entropy method are available at: https://sites.google.com/site/deyanradev/data-and-online-appendices.*

2.3. Dynamic Dependence

Following Radev (2022d), we employ dynamic correlation matrices to compute our multivariate conditional measures by calculating pairwise correlations based on the 5-year CDS spreads observed three months (60 days) prior to period t. By introducing a dynamic dependence structure to our approach, in conjunction with the dynamics of individual probabilities, we can derive measures that more closely reflect the level of systemic risk at any given time.

3. Dynamic Measures of Sovereign Systemic Risk

This section describes the conditional bivariate and multivariate measures, introduced in Radev (2022c). Section 5 will compare visually the dynamic correlation measures to their static versions in Radev (2022c).

3.1. Dynamic Probability of A Defaulting Given B Defaults

We start with the simplest extension beyond the unconditional joint probability framework: the probability of default of sovereign A (say Italy) given sovereign B (say Spain) defaults (P(A|B)), introduced in Radev (2022c). Deriving P(A|B) is a direct application of the Bayes rule:

$$P(\, A \mid B \,) = \frac{P(A, B)}{P(B)}, \tag{4}$$

where $\boldsymbol{P(A, B)}$ is the joint probability of default of sovereigns A and B, while $\boldsymbol{P(B)}$ is the marginal probability of default of sovereign B.

This measure is useful in analyzing particular channels of contagion from one sovereign to another or vice versa. Since $P(\, A \mid B \,)$ is rarely equal to $P(\, B \mid A \,)$,[6] we can discern which of both sovereigns in the couple is more vulnerable to a default of its counterpart. For policymaker purposes, it can be incorporated in tables or heat maps with average conditional PoD containing all possible bivariate couples, akin to correlation tables. In contrast to correlation tables, however, the corresponding values across the main diagonal of the PoD table will not be equal.

3.2. Dynamic Probability of A Defaulting Given B and C Default

The next indicator introduced in Radev (2022c) measures the conditional probability of default of a sovereign, given two other sovereigns default simultaneously. In the Bayes' framework, mentioned above, this probability of default is defined as

[6] *Actually, both measures are equal if and only if the individual unconditional probabilities are equal.*

$$\boldsymbol{P}(\boldsymbol{A} \mid \boldsymbol{B}, \boldsymbol{C}) = \frac{\boldsymbol{P}(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})}{\boldsymbol{P}(\boldsymbol{B}, \boldsymbol{C})}, \tag{5}$$

with $\boldsymbol{P}(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})$ and $\boldsymbol{P}(\boldsymbol{B}, \boldsymbol{C})$ being, respectively, the joint probabilities of sovereigns A, B and C, and of sovereigns B and C defaulting. For instance, this will measure the probability of default of Italy, given Spain and France *jointly* default.

The procedure for the calculation of the measure is similar to the method in the previous subsection, but this time it involves 3- and 2-dimensional joint probabilities of default. The measure is particularly useful when measuring the risk of a sovereign run on several sovereigns to spread further throughout the system.

3.3. Dynamic Conditional Probability of at Least N Sovereigns Defaulting

Our final (and most complex) probability measure is the probability of at least *n* sovereigns defaulting, given a particular sovereign default (PAN). This measure is a generalization of the probability of at least one (PAO) bank defaulting, introduced in Segoviano and Goodhart (2009) and in our case aims at gauging the expected severity of a crisis stemming from a particular sovereign, and hence, the rate of *contagion penetration* in the financial system. In contrast to the Systemic Fragility Measure introduced in Radev (2022c), which reflects the overall *unconditional* fragility of the system, the PAN is a *conditional* measure that gauges the level of systemic fragility in case of a default of one of its sovereign participants.

To define the measure, let us consider again a system of three sovereigns, A, B and C.[7] The probability of at least one additional sovereign defaulting given a particular sovereign (say C) defaults is then

$$PAN(\text{ at least } 1 \mid C) = P(A \mid C) + P(B \mid C) - P(A, B \mid C), \tag{6}$$

where $\boldsymbol{P}(\boldsymbol{A} \mid \boldsymbol{C})$, $\boldsymbol{P}(\boldsymbol{B} \mid \boldsymbol{C})$ and $\boldsymbol{P}(\boldsymbol{A}, \boldsymbol{B} \mid \boldsymbol{C})$ are the respective conditional probabilities for all possible default contingencies. Using this intuition, it is easy to proceed one step further and derive the probability of at least two sovereigns (in this case A and B) defaulting given sovereign C defaults:

$$PAN(\text{ at least } 2 \mid C) = P(A, B \mid C). \tag{7}$$

In the limit (i.e. for N-1 additional entities defaulting), the PAN converges to the Conditional Joint Probability of Default (CoJPoD), introduced in Radev (2022b):

$$\text{CoJPoD}\left(\text{ System }_{-\boldsymbol{C}} \mid \boldsymbol{C}\right) = \boldsymbol{P}\left(\text{ System }_{-\boldsymbol{C}} \mid \boldsymbol{C}\right), \tag{8}$$

[7] *The extension to higher dimensions, although more involving, is straightforward, as long as we keep account of the default contingencies to be added or subtracted.*

where CoJPoD $\left(\text{System }_{-C} \mid C \right)$ is the probability of the remaining sovereigns in the system to default, given sovereign C defaults.

3.4. Practical Considerations for Policymakers

This paper improves upon the family of sovereign risk measures introduced by Radev (2022c) by applying a dynamic dependence structure to the prior distribution in the CIMDO procedure. For the purposes of practical implementation of the discussed measures, it is important to note that since their calculation involves a different number of sovereigns, they are distinct measures and not variations of a single measure. Therefore, these measures should be interpreted with care. Although we use a 10-dimensional distribution of sovereigns in this paper, to arrive at the simpler conditional measures in Equation (4) and (5), we reduce the dimensions to the needed joint probabilities and individual probabilities by integrating over the values of the sovereigns that we do not need. For instance, $P(A \mid B)$ involves a portfolio of two sovereigns, A and B, and assumes independence with the rest of the sovereigns in the system, and hence is based on a bivariate distribution, achieved by integrating over the remaining 8 sovereigns. $P(A \mid B, C)$ involves three sovereigns and therefore is based on a trivariate distribution, achieved by integrating over the remaining 7 sovereigns. $PAN(\text{ at least } 1 \mid C)$ is based on all 10 sovereigns and therefore stems from a 10-dimensional distribution.

Since the number of dimensions of the multivariate distribution matters in probability theory, the values of these three measures are not directly comparable. To see this, consider the numerators $P(A, B)$ and $\mathrm{P}(A, B, C)$ in equations (4) and (5), respectively. Intuitively, increasing the number of defaulting sovereigns means that it is less likely that *all* sovereigns will default, and therefore $\mathrm{P}(A, B, C)$ is smaller than $P(A, B)$. However, since we use a bivariate distribution in the latter case, we assume the independence of sovereigns A and B with sovereign C. Therefore, in the bivariate setting of equation (4), adding sovereign C in the joint PoD will be represented as $P(A, B) \cdot P(C)$, which is different to the $\mathrm{P}(A, B, C)$ in Equation (5) which is derived from a trivariate distribution with a non-zero correlation structure. However, we can compare the values of PAN for a different number of defaulting sovereigns, because they will all stem from the same 10-dimensional distribution, where we sum up the regions where at least n sovereigns default given a particular sovereign, say BNP Paribas, defaults.

To sum up, we can compare the values of the pairwise probabilities $P(A \mid B)$ across different pairs, because they stem from bivariate (albeit different) distributions. We can also compare the different combinations of $P(A \mid B, C)$ probabilities since they come from trivariate (albeit different) distributions. But we cannot compare the levels across $P(A \mid B)$ and $P(A \mid B, C)$ probabilities. These measures serve different purposes for policymakers and, e.g., may measure the vulnerability of a sovereign to the default of a particular sovereign or the joint default of a couple of sovereigns. Since there may be unlimited contingencies policymakers may be interested in, there is an unlimited probability measures that may be developed.

4. Data

We recover marginal probabilities of default using CDS premia for contracts with maturities from 1 to 5 years for the period 01.01.2007 and 31.12.2011. The bootstrapping procedure requires as additional inputs refinancing interest rates, which we choose to be the AAA euro area government bond yields for maturities from 1 to 5 years. The CDS spreads and the government bond yields are at daily frequency, which is also the frequency of the resulting probabilities of default. Our analysis covers 10 EA sovereigns used by the European Central Bank to calculate various probability-based systemic risk measures, such as the Systemic Fragility Measure (Radev and Alves, 2012). The sample is presented in Table 1.

Table 1. List of euro area sovereigns used in our analysis

Euro Area Sovereigns		
	Country code	Name
1	AT	Austria
2	BE	Belgium
3	FR	France
4	DE	Germany
5	GR	Greece
6	IE	Ireland
7	IT	Italy
8	NL	Netherlands
9	PT	Portugal
10	SP	Spain

Table 2 presents the descriptive statistics of the 5-year CDS spreads of the 10 sovereigns in our sample. The average 5-year CDS spread in the cross-section ranges from 32,26 basis points for Germany to 843,07 basis points for Greece. We also notice a substantial increase of CDS premia even for the safest sovereign at the beginning of the sample, France, from 0.5 to 247,08 basis points. However, this does not compare to the dynamics of the price for protection against the default of Greece, which starts at 4,40 basis points at the beginning of the period and reaches a maximum of 14395,72 basis points. We also notice that, on average, French, German and Dutch sovereigns exhibit the lowest volatility in the price for protection against default.

Table 2. Descriptive statistics of the 5-year CDS spread series of 10 sovereigns

	AT	BE	FR	GE	GR	IE	IT	NL	PT	ES
Minimum	0.50	1.40	0.50	0.60	4.40	1.75	5.30	1.00	3.40	2.47
Mean	66.45	84.73	52.28	32.26	843.07	259.38	128.37	38.29	254.63	133.88
Maximum	273.00	403.01	247.08	120.59	14395.72	1286.91	595.68	136.21	1308.51	490.86
Std. dev.	56.98	84.09	52.14	26.74	1780.04	277.35	124.46	32.50	342.73	123.42
Nr. of obs.	1305	1305	1305	1305	1305	1305	1305	1305	1305	1305

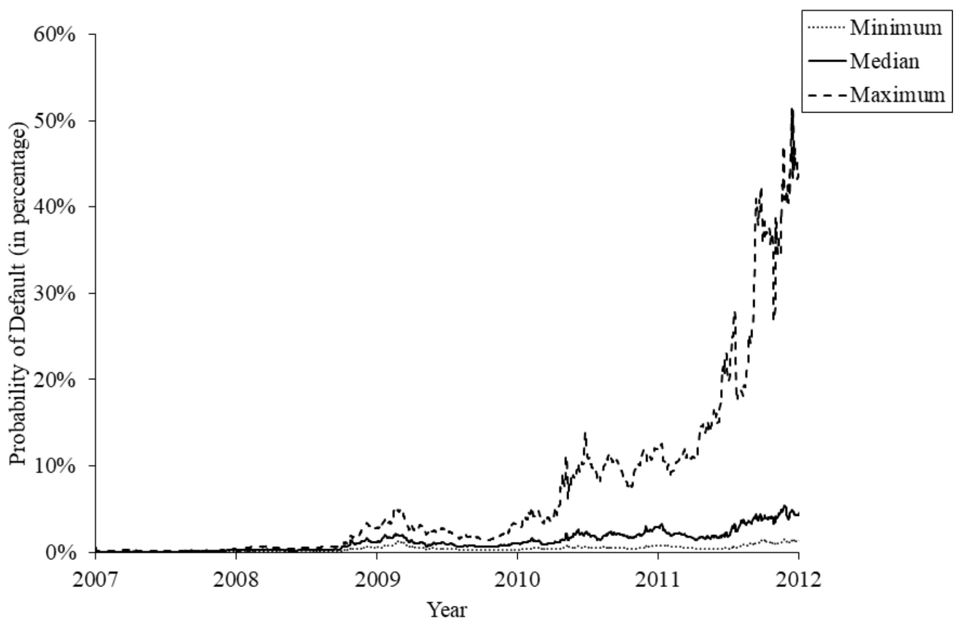
Descriptive statistics of the 5-year CDS spread series of 10 sovereigns: Austria (AT), Belgium (BE), France (FR), Germany (GE), Greece (GR), Ireland (IE), Italy (IT), Netherlands (NL), Portugal (PT), Spain (ES). The data are in basis points. Period: 01.01.2007 – 31.12.2011.

5. Empirical Results

5.1. Marginal Probability of Default Results

This section presents the findings regarding the probabilities of default for each individual sovereign in our sample. Figure 1 displays the annualized probabilities of default derived from 5-year CDS spreads for the 10 sovereigns in our study. The series illustrates that prior to the onset of the Subprime Crisis in August 2007, the market perceived euro-area sovereigns as relatively low risk in terms of default. However, the first significant increase in marginal default risk occurred around the time of the Bear Stearns bailout in the spring of 2008. Throughout the financial crisis and subsequent global recession, we observe that individual default risk remained relatively stable at around 2%. Notably, the dynamics of the probabilities of default during the latter half of the sample period can be divided into two subperiods: the first being between the escalation of the sovereign debt crisis in the first quarter of 2010 and the second quarter of 2011, and the subsequent period onwards.

Figure 1. Minimum, Median and Maximum of 5-Year Annualized CDS-Implied Bootstrapped Probabilities of Default for 10 sovereigns

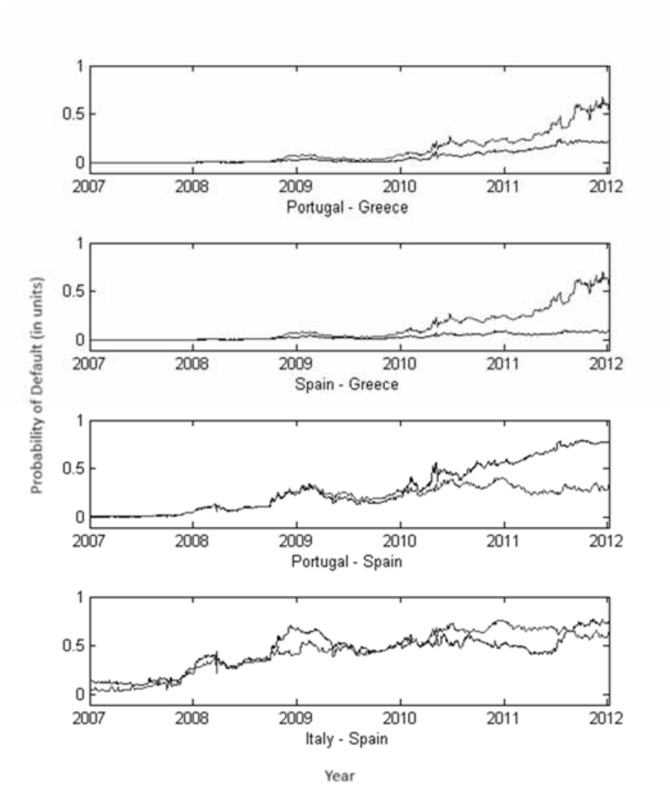


Minimum (dotted line), Median (solid line) and Maximum (dashed line) of 5-Year Annualized CDS-Implied Bootstrapped Probabilities of Default for Austria, Belgium, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain. Euro-denominated CDS spreads are used. Period: 01.01.2007 – 31.12.2011. Source: own calculations.

5.2. Conditional Probabilities Results

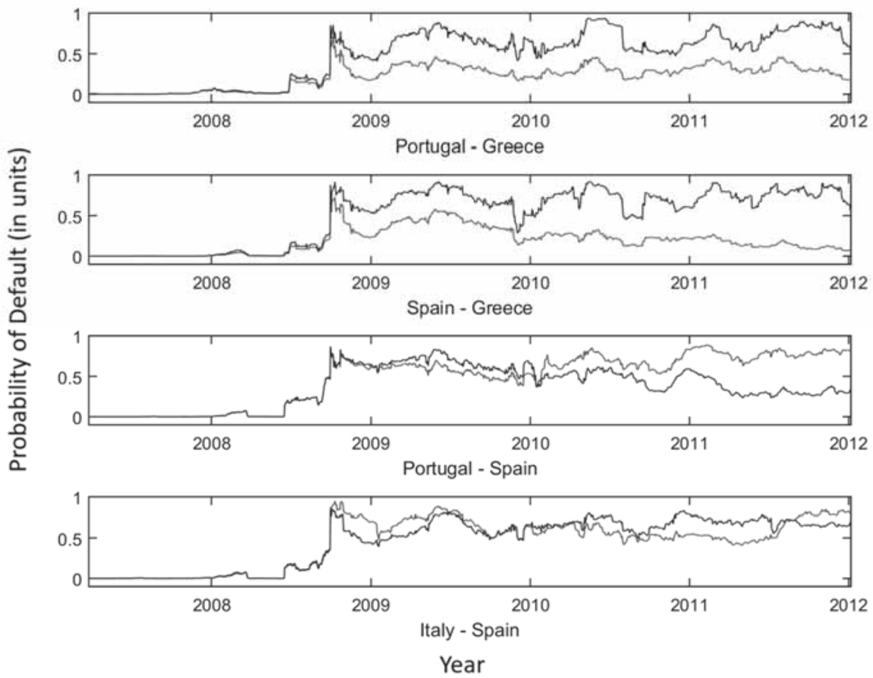
Figure 2 and Figure 3 illustrate the probabilities of default for individual sovereigns in two scenarios: static (Figure 2) and dynamic (Figure 3). In the static case depicted in Figure 2, intriguing patterns emerge in the sovereign measures. In most cases, the conditional probabilities of default within each pair closely mirror each other, indicating a similarity in their individual unconditional probabilities of default (as denoted by the denominator in Equation 4). For instance, the conditional probabilities of default for Portugal and Spain exhibit a close alignment until the beginning of 2011. However, during 2011, the probability of Portugal defaulting given Spain's default becomes higher than its counterpart. This suggests that international investors perceived Spain as a safer sovereign, and in the event of Spain's default, they would anticipate a higher likelihood of Portugal following suit.

Figure 2. Sovereign conditional probability of default given a particular sovereign default: Static Case



5-year annualized conditional probabilities of default of selected sovereign couples in the period 01.01.2007 – 31.12.2011. The black (grey) line corresponds to the probability of default of the first (second) sovereign listed in the couple, given the second (first) sovereign defaults. E.g., the black line in the top plot represents the probability of a default of Portugal given Greece defaults, while the grey line corresponds to the probability of a default of Greece given Portugal defaults. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective sovereign' 5-year CDS spreads. Source: own calculations.

Figure 3. Sovereign conditional probability of default given a particular sovereign default: Dynamic Case



5-year annualized conditional probabilities of default of selected sovereign couples in the period 01.01.2007 – 31.12.2011. The black (grey) line corresponds to the probability of default of the first (second) sovereign listed in the couple, given the second (first) sovereign defaults. E.g., the black line in the top plot represents the probability of a default of Portugal given Greece defaults, while the grey line corresponds to the probability of a default of Greece given Portugal defaults. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective sovereigns' 5-year CDS spreads over a 3-month (60 business days) rolling window. Source: own calculations.

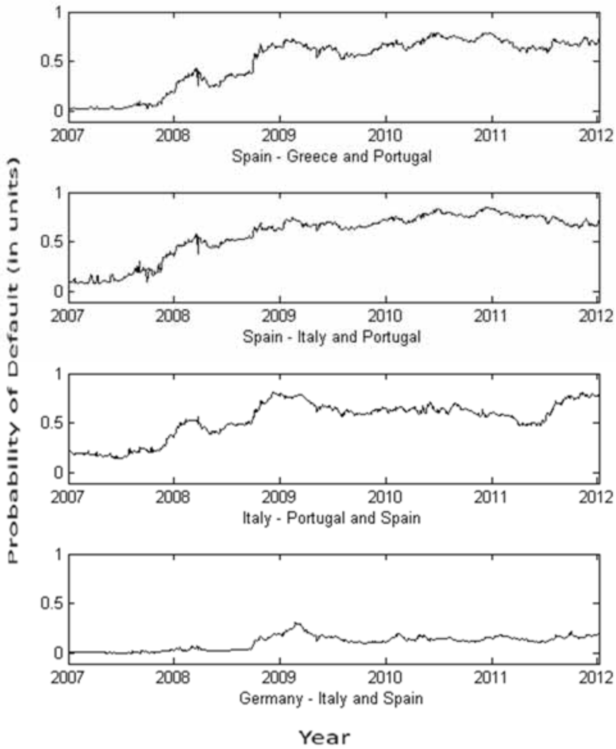
Furthermore, we observe significant disparities in the levels and dynamics of probabilities of default across the subplots, even when involving the same sovereign (e.g., comparing the subplots for Portugal-Spain and Portugal-Greece). These variations can be attributed to distinct levels of dependence among the respective pairs.

The dynamic scenario depicted in Figure 3 exhibits significant deviations from the static scenario presented in Figure 2. Notably, we observe an escalation in the level of default risk and more prominent spikes compared to the static plots. As both figures employ the same input data, we can attribute the contrasting dynamics solely to the incorporation of dynamic correlation, which better captures the evolving patterns of dependence during both crisis and tranquil periods.

Furthermore, we observe that the probabilities of default derived from the averaged static correlations are seldom higher than those obtained using a rolling window. This finding indicates that the former approach fails to capture the nonlinear dynamics present in the joint distribution of sovereign assets. By relying on static correlations, we overlook crucial changes in the interrelationships among sovereigns over time.

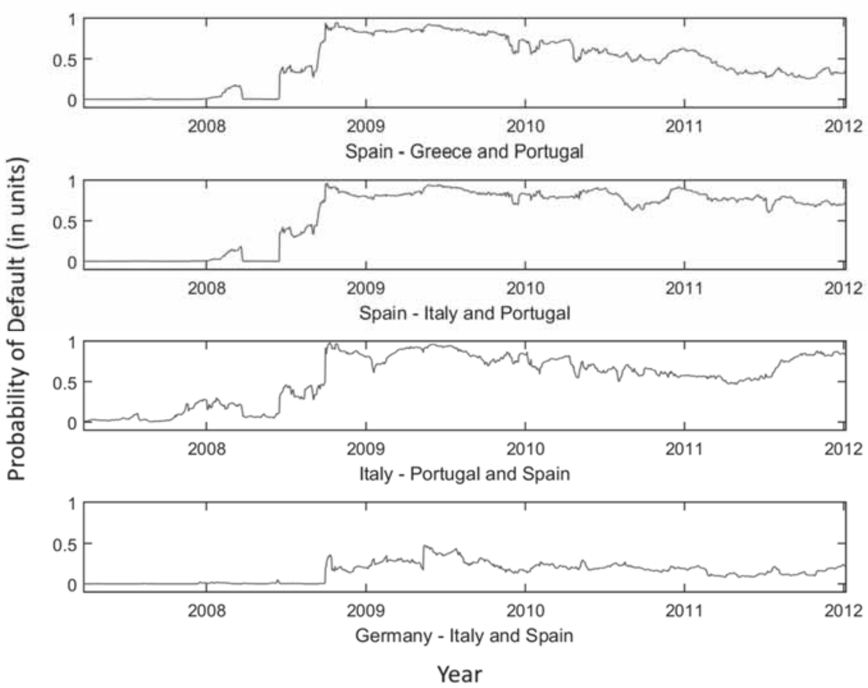
In Figure 4 and Figure 5, we present univariate sovereign probability results, conditional on two sovereigns defaulting, for the static and dynamic cases, respectively. The upper subplot in Figure 4 demonstrates a substantial effect on the default perceptions regarding Spain when there is a joint default of Greece and Portugal. This pattern is observed consistently across all triplets, indicating that the simultaneous default of any two sovereigns would have a significant and detrimental impact on the default risk assessment of a third sovereign.

Figure 4. Sovereign conditional probability of default given two sovereigns default: Static Case



5-year annualized conditional probabilities of default of selected sovereigns in the period 01.01.2007 – 31.12.2011. The black line corresponds to the probability of default of the first sovereign listed in the couple, given the remaining two listed sovereigns default simultaneously. E.g., the line in the top plot represents the probability of default of Spain given Greece and Portugal's default. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective sovereigns' 5-year CDS spreads. Source: own calculations.

Figure 5. Sovereign conditional probability of default given two sovereigns default: Dynamic Case



5-year annualized conditional probabilities of default of selected sovereigns in the period 01.01.2007 – 31.12.2011. The black line corresponds to the probability of default of the first sovereign listed in the couple, given the remaining two listed sovereigns default simultaneously. E.g., the line in the top plot represents the probability of default of Spain given Greece and Portugal's default. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective sovereigns' 5-year CDS spreads over a 3-month (60 business days) rolling window. Source: own calculations.

The inclusion of dynamic correlation in Figure 5 provides a deeper comprehension of joint default risk dynamics. The spikes in default risk exhibit heightened prominence, notably around pivotal events such as the Greek bailout in May 2010. The observed dynamics in Figure 5 significantly diverge from those in the static correlation scenario, particularly within the first and last subplots encompassing Italy-Portugal and Spain, as well as Germany-Italy and Spain, respectively. In the former case, a discernible decline in the conditional default risk of Italy following the First Greek Bailout is evident, a phenomenon that lacks similar emphasis in the static correlation depiction. These fluctuations can be ascribed to the dynamic interdependencies among these sovereigns, which our methodology effectively captures and represents.

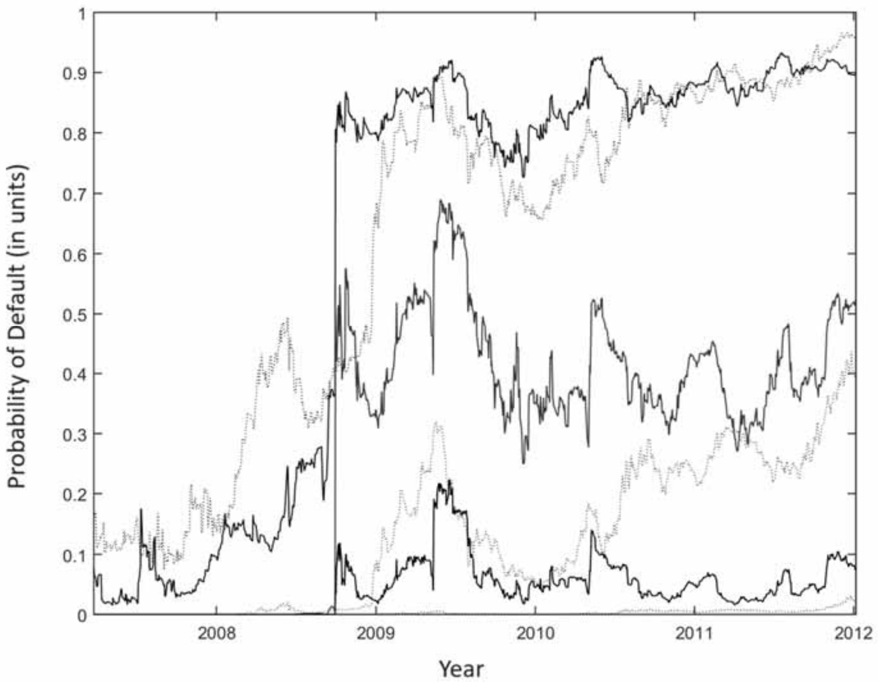
Figure 6 presents our final metric, referred to as the Probability of at least n additional sovereigns defaulting (PAN), given a specific sovereign default. To enhance clarity, we utilize solid lines to denote the dynamic case and dotted lines to represent the static case. Each curve in Figure 6 represents the cross-sectional median values of the respective probability for the sake of presentation.

Given our fixed 10-dimensional sovereign system, it is expected that the likelihood of an additional sovereign default decreases as we require more sovereigns from the system to default. Consequently, within our 10-dimensional sovereign system, the conditional probability of 8 additional sovereigns defaulting is inherently lower than the conditional probability of 7 additional sovereigns defaulting. Notably, the conditional probability of 1 additional sovereign defaulting, depicted by the top two lines, exhibits the highest values in both the static and dynamic cases, surpassing the probabilities associated with at least 5 additional sovereigns defaulting (middle two lines) and at least 9 additional sovereigns defaulting (bottom two lines).

The outcomes confirm the previous findings derived from our various metrics, signifying that distress within the sovereign system commenced as early as mid-2007. The dynamic measures demonstrate greater volatility, characterized by significant spikes in August 2007 (coinciding with the outbreak of the Subprime crisis) and May 2010 (corresponding to the First Greek Bailout). It is worth noting that the conditional probability of at least one sovereign defaulting rapidly approaches the upper limit of the probability domain (with unreported maximum values even closer to 1 than depicted), rendering the dynamics of this measure, often referred to as the probability of spill-over effects (introduced in Segoviano and Goodhart, 2009), relatively uninformative. Consequently, we contend that our generalized approach, which examines different numbers of defaulting sovereigns, offers a more comprehensive depiction of the extent to which default spill-over effects permeate the financial system.

Interestingly, the static case of PAN exhibits higher values than the dynamic case for extended periods. This disparity arises from employing fixed correlation matrices calculated over the entire sample in the static case, while in the dynamic case, we calculate the matrices using a rolling window of 60 days. The dynamic approach enables us to account for fluctuations in dependence, which typically occur during times of crisis and stability (see, e.g., Forbes, Rigobon, 2002; Radev, 2022e). Thus, we posit that dynamic conditional measures present a more precise portrayal of the level and direction of changes in systemic risk.

Figure 6. Probability of at least n additional sovereigns defaulting given particular sovereign defaults: Dynamic and Static Cases



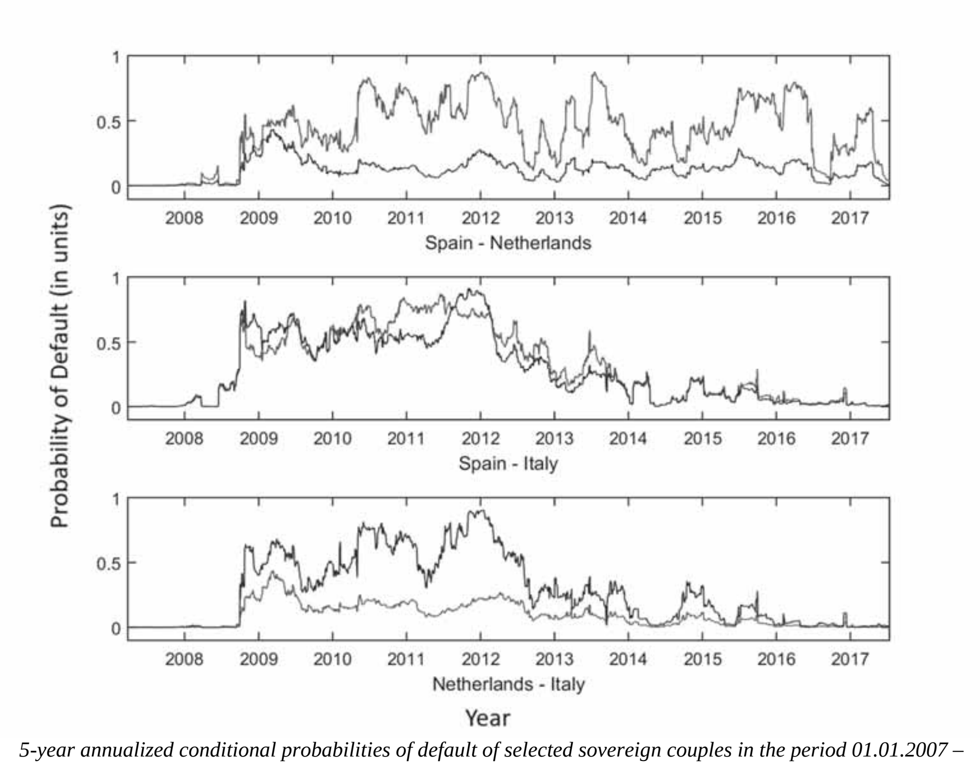
Dynamic Case (solid line) and Static Case (dotted line), involving 10 euro area sovereigns in the period 01.01.2007 – 31.12.2011. The median values across the cross-section of the respective probabilities are reported. The top two lines correspond to the probability of at least 1 additional sovereign defaulting for the dynamic case (solid line) and static case (dotted line). The middle two lines correspond to the probability of at least 5 additional sovereigns defaulting for the dynamic case (solid line) and static case (dotted line). The bottom two lines correspond to the probability of at least 9 additional sovereigns defaulting for the dynamic case (solid line) and static case (dotted line). The probabilities derivation incorporates empirical correlation, calculated between changes of the respective sovereigns' 5-year CDS spreads over a 3-month (60 business days) rolling window (solid line) and between changes of the respective sovereigns' 5-year CDS spreads (dotted line). Source: own calculations.

5.3. Extension of Time Period

In this section, we incorporate additional data for some of the sovereigns to extend the estimation period until 30.06.2017. Thomson Reuters has provided us with in-house credit default swap (CDS) data for certain sovereigns in our sample, allowing us to update several of our probability measures. Specifically, we have obtained CDS data for Spain, the Netherlands, and Italy until 30.06.2017.

While our data is limited in sample size and time period, it provides insights into the dynamics of our lower-dimensional measures beyond December 2011. For instance, we examine the probability of a sovereign defaulting given the default of another sovereign (Figure 7), as well as the probability of a sovereign defaulting given the joint default of two other sovereigns (Figure 8). In the upper subfigure of Figure 7, we observe that Spain is more vulnerable to the default of the Netherlands than the reverse case. Interestingly, the conditional probabilities of default for Spain and Italy (second subplot) closely track each other, indicating that both sovereigns have similar individual unconditional probabilities. In the third subplot, the conditional probabilities of the Netherlands and Italy exhibit a narrower tracing pattern. Overall, Spain and Italy appear to exhibit higher riskiness compared to the Netherlands and are more sensitive to the hypothetical default of the latter.

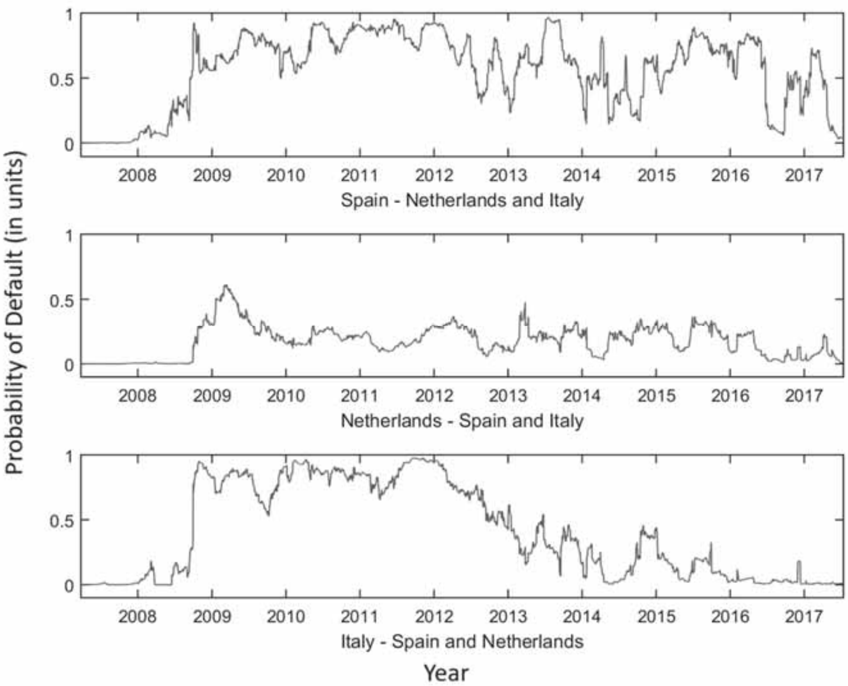
Figure 7. Sovereign conditional probability of default given particular sovereign defaults: Dynamic Case



5-year annualized conditional probabilities of default of selected sovereign couples in the period 01.01.2007 – 30.06.2017. The black (grey) line corresponds to the probability of default of the first (second) sovereign listed in the couple, given the second (first) sovereign defaults. E.g., the black line in the top plot represents the probability of default of Spain given the Netherlands defaults, while the grey line corresponds to the probability of default of the Netherlands given Spain defaults. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective sovereigns' 5-year CDS spreads over a 3-month (60 business days) rolling window. Source: own calculations.

Moving to Figure 8, we note that Spain demonstrates greater sensitivity to the joint default of the Netherlands and Italy compared to the other two constellations. Furthermore, Italy exhibits the least sensitivity to the joint default of the remaining two sovereigns.

Figure 8. Sovereign conditional probability of default given two sovereigns default: Dynamic Case



5-year annualized conditional probabilities of default of selected sovereigns in the period 01.01.2007 – 30.06.2017. The black line corresponds to the probability of default of the first sovereign listed in the couple, given the remaining two listed sovereigns default simultaneously. E.g., the line in the top plot represents the probability of default of Spain given the Netherlands and Italy jointly default. The probabilities derivation incorporates empirical correlation, calculated between changes of the respective sovereigns' 5-year CDS spreads over a 3-month (60 business days) rolling window. Source: own calculations.

Observing the overall dynamics, we find that the riskiness of conditional probabilities is influenced by significant events within the euro area throughout the extended period. Notable spikes occurred around the time of the Private Sector Involvement agreement in late 2011 and early 2012, which signalled the de facto default of Greece on its government debt. Additionally, the "whatever-it-takes" speech delivered by Mario Draghi in mid-2012, which reassured the markets and essentially pledged ECB support to safeguard the euro, also impacted the conditional probabilities. Moreover, the Cypriot Banking Crisis in late 2012 and early 2013 contributed to fluctuations in these probabilities. In all cases, the conditional probabilities exhibit a decline by the end of the time period, particularly after mid-2016.

6. Conclusion

This research enhances existing default risk measures for euro area sovereigns through a consistent approach for assessing individual and joint default risk. We introduce dynamic dependence into the cross-entropy method underlying our framework, allowing us to better capture changing dependence patterns across different market conditions. Our analysis reveals an escalation in sovereign default risk since the Subprime Crisis, particularly around the First Greek Bailout in May 2010. We also effectively capture significant events like Mario Draghi's "whatever-it-takes" speech in mid-2012 and the 2012-2013 Cypriot Banking Crisis. Our dynamic dependence measures provide a more comprehensive view of conditional default risk within the euro area sovereign system, often showing different dynamics compared to static measures.

This study contributes to the ongoing discourse on joint default risk measures, enhancing our understanding of market perceptions of default risk and the implications of regulatory interventions and economic reforms. The incorporation of dynamic dependence and our proposed measures expands policymakers' tools for assessing systemic sovereign risk. Furthermore, our approach holds promise for evaluating the impact of financial system reforms, such as bank resolution regimes and Basel III, as well as major crises and global events like Brexit, the COVID-19 pandemic, and the War in Ukraine, offering fertile ground for future research.

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