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MEAN-VARIANCE ENVIRONMENTAL INVESTMENT OPTIMIZATION OF BULGARIAN PRIVATE PENSION FUNDS[3]

Pension funds' investments are increasingly linked to the changes in climate and environmental protection. The integration of environmental, social and governance (ESG) factors into their investment process is still varying in different countries and regions. The limited number of studies on the application of the ESG investment approach by private pension funds in Bulgaria shows that the country lags behind the trends in Europe.

Although pension funds do not perceive environmental investments as riskier or less profitable than conventional ones, many of them remain cautious due to the shorter-term financial performance data of green assets. To achieve adequate retirement savings and a high replacement rate saving "more and for longer" is not enough. As far as the topic of portfolio investment performance is on the agenda, one would reasonably ask what the reflection of environmental investments would be on the widely diversified portfolios of pension funds in the country.

The present research is dedicated to a mean-variance (MV) portfolio optimization involving a selection of conventional and green assets under different constraints and "shades of green" by using historical data. The empirical results from the portfolio optimizations performed shed light on the questions raised and complement the motivational spectrum "in favour of" or "against" environmental investment.

Keywords: private pension funds; environmental investments; ESG factors; portfolio optimization; mean-variance model; efficient frontier
JEL: J32; G11; Q56

1. Introduction

The critical analysis of the investment policy of voluntary pension funds (VPFs) in Bulgaria shows that the information disclosed about it is of extremely similar content and, in some cases, the formulation of objectives with respect to risk and profitability is entirely consistent.

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In general, there are no distinctive features and specific spirit of the different policies.[4] The ESG investment approach, which has become increasingly applied among pension funds in Europe, *is still poorly represented in Bulgaria*. The share of green investment in the portfolios of VPFs in the country during the period 2012-2021 fluctuates between 0.55% and 1.92% of total assets, reaching 3-4% in certain funds (Beneva, 2023a, 311). The values mentioned are totally acceptable with a view to the lack of a formal investment policy on matters related to sustainability.

During the studied 10-year period (2012-2021), only VPF “Doverie” declared the conduct of socially responsible investment policy based on its Code of Conduct. At present, only three out of nine funds have identified environmental, social and governance (ESG) risks as potential investment risks (“UBB”, “Doverie” and “Saglasie”) and their integration into the investment policies is comparatively recent (in November 2021, October 2022 and March 2023, respectively). *The information disclosed is limited and not sufficiently specific*. Pension funds declare the use of negative screening as a strategy to manage ESG risks, however, without specifying the criteria and/or without mentioning the materiality thresholds when doing the screening or the lists of “forbidden” companies and countries, which is the practice with environmental leaders all over the world.

VPFs licensed in the country not only refrain from considering the way sustainability matters affect their results, status and development (the “outside-in” perspective); they further fail to take into account the impact of their investment decisions on people and the environment (the “inside-out” perspective). The investment policy so applied by VPFs in Bulgaria puts the country in the group of the ones lagging behind the process of ESG investment.

The research thesis of the article is that the environmental investments of the VPFs in Bulgaria do not harm portfolio return and vice versa help for better performance of their portfolios. The objective of this paper is to empirically study the effects of including green assets in the investment portfolio of VPFs in Bulgaria. Optimization has to reveal whether green financial instruments could contribute to a better compromise between the risk and return of the investment portfolio, i.e. whether they outperform their traditional equivalents.

The research focuses on one of the ESG factors, particularly – the environmental one. The arguments are related to the policy of the European Union in the field of environmental protection, the commitments undertaken (Green Deal, Paris Agreement, etc.) and the objectives declared with respect to climate finances. The selection of green indices and funds for the portfolio optimization covers the topics, related to climate change, fossil fuels, carbon emissions, clean energy, waste management, water treatment, etc. Benchmarks of assets associated with the wider principles of ESG and sustainable investment by incorporating social and governance criteria are outside the scope of analysis, as there is no way to isolate their environmental component as a factor for the assets' financial performance.

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[4] The analysis is based on the investment policies of VPFs disclosed by pension insurance companies on their official websites (last accessed on 1 November 2023). Survey includes all VPFs licensed in Bulgaria into 31 December 2021 except VPF "DallBogg: Life and Health", which starts activity a month before.

The green indices used in the optimization procedure were selected based on the environmental criteria that were in effect as of June 2022. Given that the process of formulating and revising definitions, standards, labels, road maps, benchmarks, etc., in the field of green finance is extremely dynamic, the authors are not responsible for possible deviations from the stated environmental criteria that may generate a greenwashing risk.

The optimization problem is considered from the perspective of voluntary funds for several reasons. The choice of employers and employees for voluntary pension insurance is based on the perceived need for secure, stable and decent income in old age. About 1/5 of the economically active population of Bulgaria owns an individual party in a voluntary pension fund, but the amount of accumulated funds is still small. If the investment policy of the pension funds corresponds to the consumer's expectations of risk, return and investment beliefs, the potential of the market can be developed. As investments in VPFs are less regulated (Bulgarian legislation envisages lower investment restrictions as compared to the ones for mandatory pension funds), they have a real opportunity to offer better product design, helping to boost confidence in the sector.

The study is fully in line with stipulated regulatory requirements for the investment activity of VPFs, but the questions related to legislative restrictions expediency and possibilities to quantitative limits liberalization remain outside the scope.

2. MV optimization as one of the possible portfolio approaches

The significance of the problem of ensuring adequate income at retirement age explains the large number of studies on the strategic capital allocation and optimization of the investment portfolios of pension funds. The studies are distinguished by their varieties of models and assumptions, however, they are all based on the traditional investment approach, e.g.:

- maximising the expected utility in the case of different preferences by the investor (utility functions) – risk-adjusted function (Blake, 1999), power function (Deelstra, Grasselli, Koehl, 2004), functions of the Constant Absolute Risk Aversion, Constant Relative Risk Aversion or Hyperbolic Absolute Risk Aversion type (Menoncin, Scaillet, 2006; Vigna, 2009; Vigna, 2011);
- optimal allocation of the portfolio with a guaranteed minimum of pension income (Boulier, Huang and Taillard, 2001; Deelstra, Grasselli, Koehl, 2003);
- accounting for the individual characteristics of the insured persons, including contribution rate, income replacement rate and change in risk preferences (Haberman, Vigna, 2002; Liu, et al., 2023; Vigna, Haberman, 2001; Walker, 2005);
- optimization for pension plans with defined contributions and return clause of premium with an interest rate (Akpanibah, Osu, 2018);
- application of the "mean-variance" model (Vigna, 2009; Vigna, 2011), "mean-VaR" (Zhu, Dong, Wu, 2022), "mean-CvaR" (Hollenwaeger, 2017);

- guidelines to savers on how to manage their pension plans before and after retirement according to economic and personal characteristics (Konicz, Mulvey, 2015).

The interest in socially responsible investing over the past few years has provoked a number of studies on the possibilities for portfolio optimization by using green assets or assets with ESG characteristics. A significant part of the studies apply the "mean-variance" approach for portfolio optimization. Shen and LaPlante (2019) compare portfolio performance in the context of different climate scenarios, He and Cai (2012) make a parallel between the conventional and green portfolio and Herzel, Nicolosi and Stariča (2012) study the efficient frontiers of a conventional and ESG portfolio with respect to each of the three dimensions. There are several publications that demonstrate the performance of sustainable portfolios as compared to portfolios comprising of conventional and sustainable assets (Pedersen, Fitzgibbons and Pomorski, 2021; Porage, 2020; Wong, 2020). And while most studies are based on portfolio optimization by using stock, Mascelluti (2018) studies the impact of incorporating green bonds in the portfolio. A recent study (Beneva, 2023b) tests pension portfolio optimization with green assets, based on the UPM/LPM (Upper Partial Moment/ Lower Partial Moment) model. Despite the advantages, the UPM/LPM algorithm is not often used in practice, due to its computational complexity.

Some of the considered models are not applicable for Bulgarian pension funds' optimal portfolio selection, because of the specifics of the system. The legislation does not provide a guaranteed income, return clause of premium, possibility to take into account insured persons' different degrees of risk aversion or preferences on portfolio composition, etc. That's the reason the choice of a framework for the present optimization of a VPF's portfolio in Bulgaria by incorporating green assets focuses on the MV approach. Furthermore, the model has been arguably the most widely used portfolio optimization method in scientific literature. Of course, it cannot be claimed that MV is the most reliable optimization method, but it is distinguished by easy recognition, computational convenience and adaptability. It is also a good basis for comparing the results, generated by the other optimization studies.

The MV approach, which is based on Markowitz's (1952, 1959) traditional portfolio theory, is applied to identify the optimal ratio between the different asset classes in the diversified portfolio and to determine the efficient portfolios out of a set of possible portfolios. Variance as a measure of risk has certain advantages with respect to other risk indicators related to costs, convenience and recognition. The application of the "mean-variance" approach is reasonable where the distribution of return is close to a multivariate normal distribution or where the investor has a quadratic utility function. Practically, the optimal portfolio in the "mean-variance" model can be calculated by minimising the following expression (Markowitz, 1952):

$$Var\left(R_p\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \quad (1)$$

subject to the constraints:

$$\sum_{i=1}^{n} w_i . \bar{R}_i = \bar{R}_p \qquad (2)$$

$$\sum_{i=1}^{n} w_i = 1 \qquad (3)$$

$$w_i \geq 0, i = 1, \dots \dots n \qquad (4)$$

where: $Var(R_p)$ – variance of portfolio return; w_i – asset weight i in the portfolio; $\overline{R_i}$ – expected return on asset i; $\overline{R_p}$ – expected return on the portfolio; n – number of assets in the portfolio; σ_{ij} – the covariance between asset i and asset j.

The covariance matrix (symmetric and positive semi-definite) of n risky assets is:

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nm} \end{pmatrix} \qquad (5)$$

Alternatively, the model could be stated as maximizing portfolio's expected return, for a given level of portfolio's acceptable risk, Var (R_p):

$$\overline{R_p} = \sum_{i=1}^{n} w_i . \bar{R}_i \qquad (6)$$

subject to the constraints:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} w_i . w_j . \sigma_{ij} = Var(R_p) \qquad (7)$$

and analogous constraints on the asset's weight (formulas (3) and (4)).

MV optimization of portfolios of private pension funds in this study is performed through the functions and instruments of Financial Toolbox from the software system of MATLAB®. The integrated functions allow effective calculation of the asset's moments, setting numerous types of constraints, optimization of the portfolio and visualisation of the efficient frontier. The selected solver is for quadratic programming – *quadprog* by application of the interior point algorithm for convex optimization problems.

First, a „standard" portfolio object has to be created – incorporate the list of assets, the risk-free rate, and the moments of asset returns into the object. The next step is setting up the MV portfolio optimization problem with default constraints (non-negative weights that must sum to 1) and adding other constraints.

The optimization problem is solved by finding the minimum of the target function that is subject to the constraints. The algorithm has two possible pathways depending on the type of the Hessian matrix, which influences the necessary time for solving the optimization problem. The first option is to have a full matrix and the second one is to have a sparse matrix. The algorithm performs the following steps:

- presolve/postsolve – an attempt is made to mitigate the problem by removing the excess information (checks are made whether any linear equality/inequality constraint includes only one variable, whether any linear matrix with constraints has null rows, whether the limits of the linear constraints are consistent, etc.) and by simplification of constraints;
- generation of an initial point;
- predictor-corrector – the initial objective is to find the point where the Karush-Kuhn-Tucker (KKT) conditions (necessary and sufficient conditions to find the global optimum) are maintained. Then, a step based on Newton-Raphson's formula is foreseen, followed by the calculation of the corrector step;
- stopping conditions – the predictor-corrector algorithm is repeated until a point is reached, where the constraints within the tolerance are satisfied and where the relative sizes of the step are small;
- infeasibility detection – the quadprog solver calculates a function measuring the similarity between the data and the problem-solving model (merit function) of each iteration. In the case of high values of the function, the solver stops and declares that the problem cannot be solved (MathWorks, 2021).

Solving the "mean-variance" optimization problem ends with the development of an efficient frontier. This is a curve formed by multiple portfolios that meet the conditions for minimum variance at a certain level of return or maximum return at a given risk level, by observing the predefined constraints. It is possible to locate specific efficient portfolios for given target values of return or risk, especially useful when comparing portfolios with different restrictions, as well as generate a portfolio that maximizes the Sharpe ratio (a measure of return-to-risk that plays an important role in portfolio analysis, and also represents the tangency portfolio on the efficient frontier from the mutual fund theorem).

To assess the impact of the green investment approach, multiple conventional and green indices have been selected, which are a kind of benchmark for the performance of the different groups of investment instruments within the diversified portfolio. Global and regional indices and funds from both developed and emerging markets are included.

The selection of indices and funds adheres to the requirements for the individual financial instruments envisaged in Bulgarian legislation (constraints regarding issuers, investment grade, trading markets, registration, etc.).

The benchmarks have been selected based on the following criteria:

1) they should reflect the performance of investment instruments Bulgarian VPFs can actually invest in;

2) they should be either launched on the market or there should be back-tested data available, if the investment pool is substantially restricted (which is characteristic mainly for green indices) for a period of at least the last 10 years (i.e. as of 31.12.2011);
3) the indices are calculated by leading global providers of benchmarks, analyses and data about investors and stock exchanges (e.g. S&P Dow Jones, MSCI Inc., etc., which often use the large pension funds around the world as a benchmark for the performance of their investments);[5]
4) they are calculated as total return indices. This is the better indicator of actual income from financial assets and allows comparison between assets of both the same class and different classes;
5) the selection of collective investment schemes (CIS) (including funds traded on the stock exchange) focuses on: top managers of assets offering a wide product range (Invesco, iShares, VanEck, First Trust, etc.); large funds (with more than 500 million EUR of managed assets, as far as the size of the funds is an indicator of their profitability and efficiency); lack of requirements for minimum amount of investment.

SSC envisages a restriction of up to 30% for VPF investments in assets denominated in a currency other than BGN or EUR, except in cases of concluded risk hedging transactions. From this perspective, for the optimization performed indices and funds calculated in BGN or EUR have been preferred, where possible. The yield on AAA-rated 10-year government bonds issued in the Eurozone and published by the European Central Bank is taken as the risk-free rate of return.

This study adopts three target levels of environmental performance in VPF investment portfolios tied to “floors” with respect to the weight of the green instruments in them. The portfolios are defined as “light green”, “medium green” or “dark green”, with a minimum share of green assets in them of 5%, 10% or 20%, respectively. The idea is analogical to the classification of green investors applied by Mascelluti (2018) and the “shades” of green bonds distinguished by the Center for International Climate Research.

“Mean-variance” optimizations take place at two levels. First, based on the data about the performance of investment assets during the period December 2011 – December 2021 (120 months) due to the long-term orientation of environmental factors. Second, based on 72-

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[5] Official data sources are as follows: MSCI’s 6 indices – M SI (www.msci.com); LEONIA + – Bulgarian National Bank (www.bnb.bg/Statistics/StBIRAndIndices/StBILeoniaPlus); S&P DJ’s 16 indices – S&P Dow Jones Indices (www.spglobal.com/spdji/en); EONIA – European Central Bank (www.ecb.europa.eu/stats); Schroder ISF Global Climate Change – Schroders (www.schroders.com); Invesco Morningstar US Energy Infrastructure MLP – Invesco (www.invesco.com/corporate/en); Eurekahedge Hedge Fund Index – Eurekahedge (www.eurekahedge.com); Triodos Groenfonds NV – Triodos Investment Management (www.triodos-im.com); DV Tresury Bonds Index – DV Asset Management (dvam.bg/en); Nikkei 225 – Investing.com (www.investing.com). Data of ChinaBond’s 2 indicies, Solactive Green Bond Index, SOFIX and BGREIT are provided through personal correspondence from China Central Depositiry&Clearing Co., Ltd. (yield.chinabond.com.cn/cbweb-mn), Solactive AG (www.solactive.com) and Investor.BG AD (www.investor.bg). The data of the others 10 indices and funds – Yahoo Finance (finance.yahoo.com).

month data by annual rebalancing of the portfolios in the interval between 2017-2021. In this way, 5 "rolling" periods are observed based on the example shown by Dechant and Finkenzeller (2011), He and Cai (2012), Wojt (2010) and others.

In order to form efficient frontiers based on the 10-year data, the following problems are solved:

1) optimization without investment constraints and a wider investment pool (38 assets – 22 conventional and 16 green);
2) optimization with a set of conventional and green assets (35 assets)[6] with a different set of constraints:
   - test 1 – basic quantitative limits (the quantitative limits by investment instruments as envisaged in the SSC – up to 5% in one CIS, up to 10% in REITs, up to 10% in investment properties);
   - test 2 – additional constraints (a requirement for a minimum share of liquid assets – at least 1.5% in the LEONIA+ index is added to the basic quantitative limits) and a currency constraint (the share of currencies other than BGN and EUR is limited to 30%);
3) optimization with a set of conventional and green assets with basic quantitative limits and different shades of the "green" portfolios: "light green" portfolio (min. 5% in green assets), "medium green" portfolio (min. 10% in green assets) and "dark green" portfolio (min. 20% in green assets).

The use of a "rolling" optimization procedure allows us to account for the changing characteristics of the assets and to generate a time series of portfolio allocations. The rebalancing of portfolios provides more realistic allocations as compared to the static optimization procedure which ends with one efficient frontier. In the case of yearly rebalancing, optimizations form 5 time intervals: A (2012-2017), B (2013-2018), C (2014-2019), D (2015-2020) and E (2016-2021). The following problems are solved for each period:

1) optimizations with conventional and green assets with basic quantitative limits;
2) optimizations with conventional and green assets with additional constraints (basic constraints, minimum of liquid assets and currency constraint);
3) optimizations by using additional assets ("new" instruments are added, where no time series are available for the entire period studied (2012-2021), but just for part of this period) with basic quantitative limits.

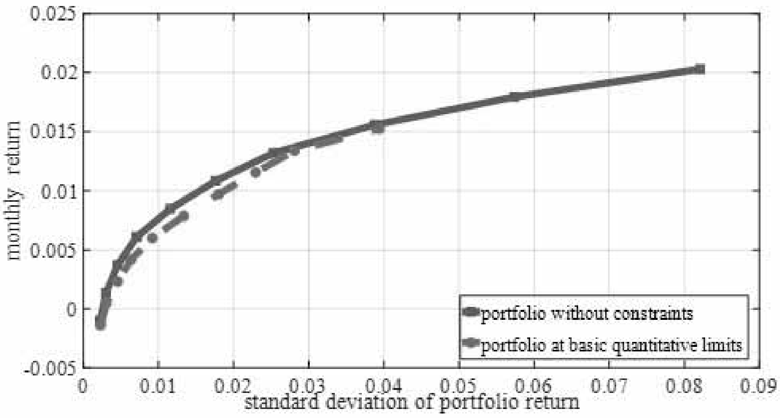
[6] The investment pool consists of 21 conventional and 14 green indices – 3 assets less than optimization without constraints, because some of assets do not meet the legislative requirements (eligible markets and investment rating for green bond indices, and availability of a license for alternative investment funds).

3. A palette of efficient frontiers in the MV approach

3.1. Efficient investment decisions based on 10-year data

The portfolios from the efficient frontier generated without investment constraints are actual investment opportunities if the pension funds apply the principle of prudence. VPFs in Bulgaria, however, are faced with a limited investment choice, which graphically translates into a lower frontier of efficient portfolios as compared to the efficient frontier in the absence of regulatory constraints (Figure 1).

***Figure 1. Efficient frontiers under the presence and absence of regulatory restrictions*[7]**



Source: Authors' calculations.

Practically, the efficient portfolios for Bulgarian VPFs carry an identical risk with lower return and, moreover, with limited potential. The difference in the monthly return from efficient portfolios with the same risk varies between 0.04% and 0.14%, which forms of interval from 0.52% to 1.76% on an annual basis. Actually, the most significant deviation is observed in medium-risk portfolios, which provide annual returns between 3% and 10%. The end portfolios report lower contrast. If we take the investment horizon into account, the inequalities would be significant.

The selection of the investment pool of assets and the setting of quantitative limits in accordance with the requirements of the SSC *has a significant impact on the share of green assets in efficient portfolios*. If there are no regulatory constraints, the weight of green assets will exceed 10% in any case or may even be more than 1/4 or even more than 1/2 of the

[7] The efficient frontier of a portfolio without constraints is generated by an investment pool of 38 assets (22 conventional and 16 green) under absence of quantitative limits. The efficient frontier of a portfolio at basic quantitative limits is constructed by an investment pool of 35 assets (21 conventional and 14 green) under the presence of following restrictions: up to 5% in one CIS, up to 10% in REITs, up to 10% in investment properties. The additional optimizations with such constraints show that the efficient frontier of combined assets is better than a set only with one of the asset groups.

investment instruments in some portfolios.[8] Even though green investments find their place at the efficient frontier and with basic quantitative limits (table 1), their weight varies from 3.58% to 20.25%.

Table 1. Investment instruments weights in the efficient frontier portfolios under different constraints[9] (%)

Investment instrument	Portfolio's number									
	1	2	3	4	5	6	7	8	9	10
	test 1 (basic quantitative limits)									
Conventional bonds	7.94	26.34	42.20	58.40	49.18	34.76	19.31	4.09	0.00	0.00
Conventional stocks and shares	0.38	5.53	9.74	13.84	24.65	33.90	43.60	57.50	69.70	80.00
Bank deposits	83.01	55.00	27.89	0.76	-	-	-	-	-	-
Alternatives	1.83	7.31	14.17	20.88	22.59	23.53	23.82	21.92	10.06	5.00
Green bonds	1.22	-	-	-	-	-	-	-	-	
Green stocks and CISs	5.62	5.83	5.99	6.11	3.58	7.81	13.27	16.49	20.25	15.00
	test 2 (additional constraints)									
Conventional bonds	7.94	22.40	31.62	45.67	52.86	37.09	20.62	5.80	0.00	0.00
Conventional stocks and shares	0.38	5.88	9.67	15.37	24.07	35.91	46.73	62.35	71.33	78.50
Bank deposits	83.01	58.51	39.33	16.89	1.50	1.50	1.50	1.50	1.50	1.50
Alternatives	1.83	7.53	13.77	16.69	19.03	21.13	21.38	19.30	10.00	5.00
Green bonds	1.22	-	-	-	-	-	-	-	-	
Green stocks and CISs	5.62	5.68	5.62	5.38	2.54	4.37	9.77	11.05	17.17	15.00

Source: Authors' calculations.

The application of additional constraints for liquid assets and currency also reduces the weight of green instruments in optimised solutions. The maintenance of funds in current accounts is generally compensated by the investment resources in conventional assets. The introduction of the currency constraint, however, contributed to a decline in the green instruments in efficient portfolios by between 0.15 and 5.44 percentage points, where the smaller differences were observed at the beginning of the efficient frontiers. It is expected that this problem will fade away over time considering the constantly growing green investment opportunities in EUR, particularly after the regulatory changes started by the

[8] Bulgarian REITs, Chinese green bonds and some green CISs stand out with a high weight in the efficient portfolios. Descriptive statistics for ChinaBond China Green Bond Index reveal low volatility and relatively good average return, so the index has the lowest coefficient of variation among the investment set. It also has a negative correlation with Asian stock indices and a very weak positive correlation with bond and bank deposit indices (between 0.10 – 0.23). The risk-return ratio of the BGREIT index also places it in the leading positions. In addition, it is observed to be negligigly correlated with almost all indices in the investment set. The exception is 3 of them (SOFIX, S&P Eurozone Investment Grade Corporate Bond Index, TGF), where the correlation coefficient is in the range 0.3 to 0.4, revealing a weak dependence. On the other hand, CIS QCLN is characterized by high risk but also high return. For this reason, it finds a place among the second half of efficient portfolios.

[9] Investments in property (including through REITs), infrastructure and private equity are marked as "alternatives".

European Union. This will inevitably contribute to a wider investment choice for VPFs with the possibility of limiting currency risk.

Green bonds have minimum participation in the formation of the efficient frontier and this is so only in the lowest-risk portfolio. When interpreting the results, one should not underestimate the lack of variety among the indices for this instrument and the fact that they have existed only for a short period. Investments in green stocks are dominated by the low-carbon S&P500 Fossil Fuel Free Index, which has an excellent risk-return profile. The index stands out with a moderate to very high correlation with some equity indices and green CISs, however, the correlation with the Bulgarian equity and property market, Indian equity market and global infrastructure is very weak, so it generates diversification benefits. By contrast, the risk profiles of green CISs are quite heterogeneous, which is also highlighted by the weights of the schemes in the efficient portfolios. Some CISs only participate in the first, lowest-risk efficient portfolio, others – in portfolios from 1 to 5, while others are included in the second half of the efficient frontier and two of the CISs only participate in the 10th portfolio, which has the highest-risk and, respectively, a high return. In practice, investments in funds reflecting global markets for low-carbon energy, water management and green project financing *have the potential to add value to VPFs portfolios*.[10]

The introduction of the currency constraint leads to a reduction in the weights of the indices that follow the US treasury and municipal bonds, which is compensated by including the sovereign debt of Eurozone countries in investment portfolios. The impact of constraints on the structure of investment portfolios also causes a change in their risk-return profile.

Table 2. Risk and return of the efficient portfolios under different investment restrictions *(annualized data, %)*

Portfolio's number	1	2	3	4	5	6	7	8	9	10
Basic quantitative limits (test 1)										
Return	-1.63	0.57	2.82	5.12	7.46	9.85	12.29	14.78	17.32	19.91
Standard deviation	0.79	1.06	1.59	2.19	3.18	4.62	6.23	7.94	9.74	13.65
Additional constraints (test 2)										
Return	-1.63	0.54	2.75	5.01	7.31	9.66	12.06	14.50	17.00	19.54
Standard deviation	0.79	1.06	1.72	2.50	3.40	4.67	6.19	7.85	9.60	13.46

Source: Authors' calculations.

The optimization solutions show that limiting investments in currencies other than BGN and EUR leads to an increase in the standard deviation in 7 out of the 10 efficient portfolios. In

[10] The involvement of TGF and FIW in the low segment of the efficient frontier deserves attention. Although the risk-return ratio is not particularly favorable, TGF is characterized by very low risk, which gives it the ceiling for this instrument with share of 5% in four of efficient portfolios. Its participation is enhanced by its negligible correlation with bank deposits, global infrastructure and US treasury bonds. The correlation with shares and real estate on the Bulgarian market is also weak (with a coefficient of up to 0.3). FIW has a better coefficient of variation and shows very low to low correlation with the pointed markets, as for bank deposits and treasury bonds it is negative. Green CISs from the second half of the efficient frontier are characterized by a very good coefficient of variation (iShares Global Water) or a very high returns (QCLN, SMOG, TAN), exhibiting respectively a very high and moderate correlation only with S&P500, while the dependence with the others indices in the portfolio is weak.

reality, however, the currency risk is reduced. In addition, maintaining a portfolio with a significant share in a foreign currency implies costs for currency risk hedging transactions, which would have a negative impact on the reported return of pension funds. Maintaining resources on current accounts and bank deposits is low-risk, however, at the same time, it has also been characterised by negative yield during the last years of the analysed period. For this reason, efficient portfolios with a minimal share of liquid assets achieve lower returns and also lower risk at the second half of the efficient frontier.

Defining a certain pension fund as a “green” or “sustainable and responsible” one depends on the incorporation of ESG policy into the corporate culture and the overall activity of the investor. No quantitative criteria for the minimum weight of green assets in the investment portfolio have been introduced (and there is no need to do so) for an investor to be identified as “green”. However, in the event that pension funds have an explicit policy for applying environmental criteria in the investment process, the share of green assets in their portfolios should be higher and more stable than the one reported by pension funds that use a conventional investment approach. In this case, what would be the scale of the changes in risk and return of pension funds that could occur and could this compromise the financial interests of their beneficiaries?

The formation of environmental portfolios of different shades makes it possible to reveal their different risk profile at a given target return, to make a comparison to alternative options and to analyse the results in response to the questions formulated above.

The inclusion of green assets in investment portfolios has the potential to lower the risk at a given rate of return or increase the return at a given risk rate. Table 3 shows that VPFs in Bulgaria can form efficient “light green” portfolios at a lower risk and with a higher return potential as compared to conventional portfolios that lack green instruments. The efficiency frontier of the “medium green” portfolio lies above the conventional one at a target return higher than 10%, and the “dark green” portfolio outperforms it at a return higher than 12%.

Table 3. Green and benchmark portfolios’ risk under set rates of target return[11]

(annualized data, %)

	Portfolio	Target return									
		0.00	3.00	5.00	7.00	10.00	12.00	15.00	19.07	19.89	19.91
Standard deviation	Conventional	0.9737	1.6510	2.1720	2.9407	4.7354	6.0731	8.1588	12.7685		
	Benchmark	0.9497	1.6343	2.1586	2.9342	4.7159	6.0379	8.0899			13.6492
	Light green	0.9497	1.6343	2.1586	2.9381	4.7159	6.0379	8.0899			13.6492
	Medium green	0.9665	1.6564	2.1795	2.9683	4.7168	6.0379	8.0899			13.6492
	Dark green	1.3621*	1.7749	2.2676	3.0504	4.7571	6.0573	8.0930		13.6338	

Source: Authors’ calculations.

In practice, the “light green” portfolio is almost identical to the benchmark. The only difference in the risk of the two portfolios occurs at a target return between 6-8%, where the standard deviation of the “light green” portfolio is higher, but the difference is minimal (from

[11] The conventional portfolio is formed from an investment pool, which does not include green instruments. The portfolio with the basic quantitative limits (test 1) is defined as a benchmark.

* The value is obtained at a higher target return (0.2331%) as this is the lower bound for the portfolio return under the specified model constraints.

0.0001% to 0.0084%). The risk of the "medium green" portfolio becomes equal to the one of the benchmark at 12% target return, while the "dark green" portfolio turns out to be riskier at all values of the return.

VPFs' choice to be "light green" investors *would not harm the financial interests of their clients*; on the contrary, it would contribute to better results as compared to choosing the conventional approach. Portfolios where the weight of green instruments is at least 10% would not "sacrifice" their return either, if the portfolio yield achieved was a two-digit number. Unfortunately, however, the Bulgarian VPFs report such values very rarely. The "dark green" portfolio generates higher risk, however, the differences decrease as the risk-return profile increases. This is even more easily observed by calculating the portfolio return at a given target risk (table 4). In the low-risk range, the "medium green" portfolio would demonstrate up to 0.09% lower return than the benchmark, while the "dark green" portfolio would lag behind by up to 0.61%. At first sight, based on the results it could be concluded that portfolios with a higher weight of green instruments are unable to generate sufficiently attractive profiles at a low risk.

***Table 4. Green and benchmark portfolios' return under set rates of target risk*[12]**
(annualized data, %)

Portfolio	Target risk							
	0.97	1.65	2.17	2.94	4.74	6.07	8.16	12.77
	Portfolio return							
Conventional	0.00	3.00	4.99	6.99	9.99	12.00	14.99	19.07
Benchmark	0.13	3.07	5.04	7.01	10.02	12.05	15.09	19.51
Light green	0.13	3.07	5.04	7.00	10.02	12.05	15.09	19.51
Medium green	0.04	2.98	4.97	6.94	10.02	12.05	15.09	19.51
Dark green	0.23*	2.46	4.62	6.77	9.96	12.03	15.09	19.50

Source: Authors' calculations.

Considering that portfolios positioned at the far left of the efficient frontier usually comprise of predominantly low-risk bonds and bank deposits, the reasons for the higher risk of the "medium green" and "dark green" portfolios may be several:

- Lack of low-risk green assets on the financial markets. In such a case, if we apply a "floor" of 10-20% for green assets, even the first efficient portfolios include higher-risk bonds, stocks and CISs that follow the equity market;
- Faults of the investment pool used in the model. There are 7 indices for conventional bonds and only 1 for green bonds among the 35 investment instruments selected. It may turn out that green sovereign or municipal bonds, as well as investment-grade green corporate bonds, are excellent investment opportunities. Their "boom" however has only been observed over the past few years, so the short period of existence prevents us from making a thorough assessment of their financial performance;

[12] Risk is measured by standard deviation of portfolio return.
* The value is obtained at a higher standard deviation (1.3621%) as this is the lower bound for portfolio risk under the specified model constraints.

- The investment constraints of Bulgarian VPFs. When quantitative limits are added to the limited variety of types of green indices for bonds and CISs investing in debt, the optimization model is "forced" to look for green opportunities among other types of investment instruments.

In practice, investment constraints aimed at reducing the risk also reduce the possibility of VPFs to compile highly environmental portfolios, without causing a reduction in return. It should be noted, however, that VPFs operating in the Bulgarian market are still far from constructing "light green" or "medium green" portfolios with profitability that is fully competitive to the one of conventional portfolios. The lack of opportunity for persons who pay voluntary pension insurance contributions to choose from multiple alternatives reflecting their different investment preferences can be considered a significant drawback of the system.

3.2. Efficient frontiers based on 6-year "rolling" periods

How does the composition of efficient portfolios formed under the basic quantitative limits change as a result of a change in the studied 6-year period? Based on the data from the first time interval "A" (2012-2017) – stocks of the most capitalised companies in Bulgaria, the USA, India and Japan take part in optimal portfolios with a significant weight. Bond portfolios are formed by debt securities (sovereign, municipal and corporate) of countries from the Eurozone and the USA. The investment property market is a good choice during the period, as the limit of 10% of the assets in the portfolio is reached both for the Bulgarian market through REITs and the European real estate market. The conventional infrastructure also finds its place through debt instruments and all bank deposits should be in Bulgaria. *Green assets take part in 9 out of the 10 efficient portfolios with a weight between 6.34% and 20.14%* (table 5). The selection focuses on several indices, where the tracking stocks are characterised by high average return and those that reflect green debt – by low volatility. The efficient portfolios formed based on the data for the period 2013-2018 *do not differ significantly*. The decline in the weight of green assets after the eighth efficient portfolio is notable. Conventional stocks on the US market are being "displaced" by their equivalent low-carbon equity securities.

Table 5. Green assets weight in the optimized "rolling" portfolios[13] *(%)*

Period	Type of constraints	Number of efficient portfolio*									
		1	2	3	4	5	6	7	8	9	10
A	basic	10.40	6.34	7.37	8.44	10.70	8.08	10.36	12.38	20.14	0.00
	additional	10.40	6.28	6.86	7.07	8.87	11.54	8.36	11.11	14.52	0.00
B	basic	10.43	6.37	6.48	7.15	9.07	8.24	5.50	5.01	0.00	0.00
	additional	10.43	6.37	6.36	5.87	7.91	8.94	5.43	4.93	0.00	0.00
C	basic	10.70	7.13	7.23	7.46	7.79	2.27	0.36	0.02	0.00	0.00
	additional	10.70	7.13	6.60	6.00	5.00	0.00	0.00	0.00	0.00	0.00
D	basic	0.53	6.35	7.38	8.28	10.68	19.12	29.29	38.84	55.48	100.00
	additional	0.53	6.24	7.95	11.00	15.02	17.18	23.14	22.59	26.31	35.00
E	basic	0.27	2.35	5.95	7.23	6.38	17.35	24.18	34.60	52.04	30.00
	additional	0.28	2.19	6.91	7.99	9.71	12.60	16.76	20.19	17.41	30.00

Source: Authors' calculations.

[13] *The matching number of efficient portfolios does not mean the same risk and return.

Significant changes were observed in investment instruments that determine the efficient frontier calculated for time interval "C" (2014-2019). There is a sharp decline in the weight of Bulgarian stocks and sovereign and corporate debt from the Eurozone, which is offset by similar instruments representing the US market. Equity instruments are making their way into conventional infrastructure. In addition, in low-risk portfolios, the weight of bank deposits in the European Union is increasing at the expense of those in Bulgaria. Green investment instruments *are "visible" in the top 5 efficient portfolios* with a relative share of 7.13% to 10.70%. However, the share declines significantly in the second half of the efficient frontier and is reduced to zero in the options with the highest risk.

The efficient frontiers over the last 2 periods (D (2015-2020) and E (2016-2021)) also differ from the previous ones, but are similar to each other. The absence of European in-vestment properties and stocks of infrastructure companies is noticeable. Bulgarian stocks maintain low weights. Investments in US stocks are also reduced. The weights of green assets in optimised portfolios grow noticeably in the second half of the efficient frontier and reach values above 50%. There is significant participation of stocks of Asian fossil fuel-free companies (tracked by the S&P Asia 50 Fossil Fuel Free index) and a record number of ETFs (seven-eight).

It needs to be noted that during the initial periods (B and C) green instruments participated more significantly in the portfolios from the beginning of the efficient frontier, while during the last time intervals (D and E) – at the end of that frontier and, furthermore, the number (variety) of the selected green assets in efficient portfolios doubled over time. In fact, the risk-return profile of green CISs forming the investment pool *has been improving significantly over the last years of analysis*. About half of them are characterised by high volatility, however, during the period after 2014-2015, it was also accompanied by high returns, so they turned from initially unprofitable investment alternatives into good choice. Low-risk efficient portfolios during the last time intervals were almost entirely formed by the index reflecting the interbank money market in EUR (EONIA), reaching a weight of up to 99% generated by the unprecedented low interest rates and the low volatility of the index. Thus, the efficient frontiers originate from very low-risk levels and negative return, which further provides argument for the insignificant weight of green assets in the lower segment of the efficient frontiers of the last two "rolling" periods.

Compared to the optimizations where we have 10-year data, which demonstrate that the recommended weight of green assets should be above 5% in almost all cases, the results based on the 6-year periods are more contradictory. Based on Table 5, it becomes clear that the relative share of green investment instruments should be limited to under 1% or even completely absent in some time intervals. However, this affects the portfolios located high on the efficient frontier (at A, B and C) or those at the low-risk register (the first 1-2 efficient portfolios at D and E). The targets with respect to return typically position Bulgarian VPFs between the 3rd and the 5th portfolio of the efficient frontier. In this segment, all efficient portfolios contain a minimum of 5% green assets, and some portfolios allocate up to 10-11% to those assets. The thesis about *better financial performance of the "light green" investors is also valid* in the case of annual rebalancing of the portfolios.

The optimizations of the “rolling” portfolios made with “additional assets” deserve special attention, insofar as the efficient frontiers are located above those formed under the basic quantitative limits (Figure 6). The inclusion of Bulgarian government securities in the investment pool immediately assigns them high weights in the efficient portfolios, reaching up to 44% during the 2012-2017 period. In fact, this instrument is neither characterised by the highest return, nor by the lowest volatility, however, the risk-return ratio is the best among the entire range of investment assets. During the next two time intervals, their relative share at the efficient frontier decreases, mainly due to the presence of profitable green bonds (their coefficient of variation is even lower), however, the weights remain high (up to 25%) in the middle range. The impact of green bonds issued in RMB on the potential performance of investment portfolios is substantial and noticeable for all 4 periods (from B to E) they are included in. Although the risk and return of the “end” efficient portfolios coincide, the effect of the additional assets is visible precisely in the positioning range of VPFs in Bulgaria.

The return of the China Bond CIB Green Bond Index during periods B and C correlates very weakly (although, in most cases positively) with all assets in the investment pool. A weak dependence is reported only with EONIA for the period 2014-2019 (correlation coefficient is 0.35). During the next two time intervals, the correlation coefficients reveal weak dependence again, however, this time the direction is negative. The solutions of optimization problems “return” impressive values for the weight of this tool in efficient portfolios (often exceeding 15-20% and reaching up to 65-68% in some cases). Thus, the optimal weight of green assets increases dramatically. It is evident that even “dark green” portfolios lie on the efficient frontier. Bulgarian VPFs have the real opportunity to invest in these green bonds, certainly not in such large volumes, insofar as the portfolios are well-diversified and the costs for transactions for hedging currency risk are taken into account.

The index reflecting the performance of green bonds on the global market (Solactive Green Bond Index) does not perform sufficiently competitively compared to the other assets in the investment pool. It was included in just one efficient portfolio (the first one) for the period 2015-2020 and its weight was 0.58%. Interestingly, even in the absence of the Chinese green bonds, the weight of the global index remained unchanged and it was completely absent in the efficient choice during the last period. The results obtained correspond to the conclusions found by Mascelluti (2018).

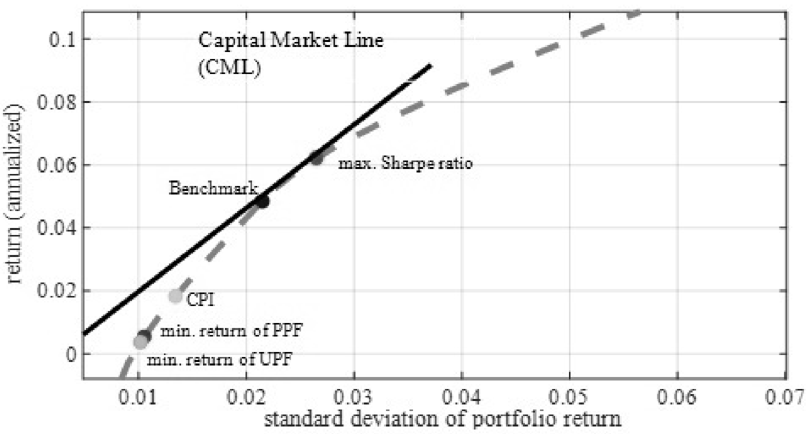
The funds that reflect the performance of the US energy companies MLP and Yield-Co are among the instruments with a high coefficient of variation. During the periods 2014-2019 and 2015-2020, the MLP fund (Invesco Morningstar MPL) reported a negative average return and very high volatility, which ranked this instrument in one of the last places among the investment pool. Even though a positive return has been reported over the last 6-year period, the fund maintained its weak ranking. Both funds reported positive dependence (with a moderate to significant degree) on the performance of the selected investment assets, which further hindered their choice for the optimised options. The fact that the Bulgarian VPFs face significant constraints to invest in energy companies with specific structure *does not have a negative effect at this stage* considering the results from the optimizations.

The analysis of “rolling” portfolios showed that the investment results and the optimal investment choices can vary significantly. Not all green investments are successful business cases, but some of them certainly have an excellent risk-return profile or bring diversification benefits to broad portfolios. Even seemingly small changes ("introducing" one or two new investment instruments) could have a significant effect on the investment performance of VPFs. Now the most important question is the one about asset allocation.

4. A critical overview of MV optimization solutions

The decision of pension funds which of the efficient portfolios to pursue is inevitably determined by the pre-formulated risk and return objectives. To complement the analysed picture, several portfolios were built, where the target return – performance is equal to the consumer price index (CPI), the benchmark (VOLIDEX index) and the minimum performance of universal (UPF) and professional pension funds (PPF) (by taking into account that there is no statutory requirement for the VPFs to calculate and guarantee minimum performance and that insurance is voluntary and the restrictions are more liberal, so it is recommended that VPFs realise return above the minimum one realised by the mandatory funds), which are illustrated on fig. 2.

Figure 2. Efficient frontier, capital market line and target portfolios[14]



Source: Authors' calculations.

The target portfolios with the minimum return of UPF and PPF are a fairly low "bar" for voluntary funds. The efficient portfolio yielding a return equal to inflation is generated at an annual standard deviation of 1.34%, and the one providing the return of the benchmark – of 2.16%. The share of green assets in the target portfolios falls in the range of 5.81% to 6.11% and is only formed by investments in two of the CISs (TGF and FIW) in all four cases. According to Tobin (1958), even though all portfolios lying on the efficient frontier offer the best return for the respective risk level (or the lowest risk for the respective return), the choice

[14] The illustrated efficient frontier (dashed line) is a segment of the graphical representation of the efficient portfolios under the basic quantitative limits. Calculations are based on 10-year period.

of a portfolio with the best expected risk-return ratio can be only made after including risk-free return in the analysis. The portfolios with the best characteristics are those that form the Capital Market Line (CML). It is constructed as a tangent to the efficient frontier through the risk-free return point (from the ordinate axis) and its slope is equal to the Sharpe ratio. Given that short selling is not allowed for VPFs, they are left with the option of choosing a portfolio that either coincides with the tangency portfolio or is positioned below that portfolio. The specific choice of the proportion of the risk portfolio and the risk-free asset the pension fund will invest in will depend on the fund's degree of risk aversion.

The portfolio maximising the Sharpe ratio is positioned higher than the target portfolios considered. Its return and risk indicators are 6.40% and 2.65%, respectively, on an annual basis. The optimization conducted found that the tangent portfolio was formed at a rather low level of green instruments – 1.72%.[15] If VPFs actually stopped at a combination involving a certain ratio between the risk portfolio and the risk-free asset, then the share of green assets in their portfolio should be lower than 2% under the constraints discussed. If VPFs aim for a nominal return of more than 7.67%, which is given by the tangency portfolio, they will position themselves to the right of it on the efficient frontier, thus significantly increasing the weight of green instruments (up to 20%). This practically means that green assets have the potential to improve the return on pension investments, but they can do so at a lower Sharpe ratio.

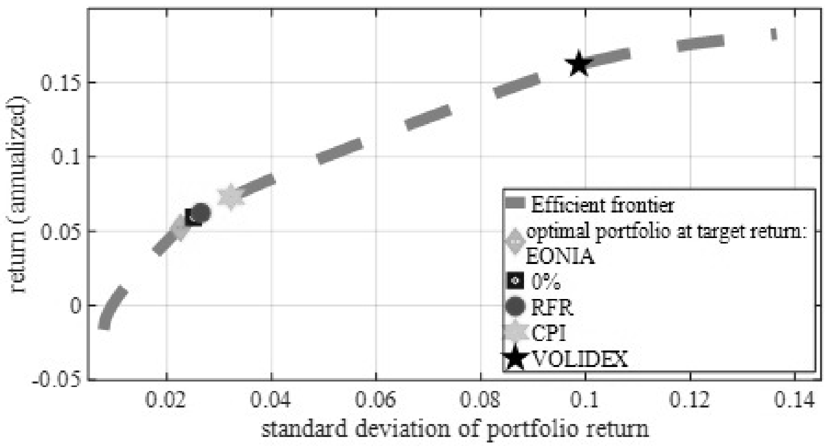
Since modelling risk sensitivity through a utility function is a difficult task, Roy (1952) considered that all investors are united by their willingness to minimise the likelihood of realising a "disaster", i.e. they should apply the "safety first" principle when choosing between investment alternatives. The disaster level is practically the investor's target return. The optimal portfolios are the ones that minimise the risk of return below the target and, respectively, maximise the value of the "safety first ratio" (SFR). If the risk-free return is defined as the "disaster level", then the SFR coincides with the Sharpe ratio.

The change in the optimal investment choice resulting from the choice of target return is illustrated in Fig. 3. Due to the close values of the EONIA index, 0 and the risk-free rate (RFR) for the period 2012-2021, the portfolios based on the "safety first" principles are positioned next to each other on the efficient frontier. The optimal portfolio with a target return equal to inflation is a little but farther away. If minimisation of the risk that the portfolio return will be lower than VOLIDEX is desired, the investor should position himself quite "high" along the efficient frontier.

What is the optimal investment choice in the presence of risk-free investment opportunities during the "rolling" periods? The share of green assets in the tangency portfolios (those with a maximum Sharpe ratio) varies significantly both during the 5 periods under consideration and with respect to the investment pool (Table 6).

[15] By default, optimization solutions show the results of compiling 10 efficient portfolios. It turns out that if the efficient frontier is formed by 30 portfolios, the lowest share of green assets is formed in the interval between 6-7% annual return and those green assets form only 1.5-2.5% of the investment portfolio.

Figure 3. Optimal investment choice according to the "safety first" criterion[16]



Source: Authors' calculations.

Table 6. Risk and return indicators of the portfolios with a maximum Sharpe ratio[17]

Indicator \ Period	Investment pool of 35 assets					Investment pool with additional assets				
	A	B	C	D	E	A	B	C	D	E
Standard deviation, % (annualized data)	3.43	3.32	2.35	2.66	2.72	3.22	1.92	1.54	1.70	1.51
Return, % (annualized data)	9.80	6.95	5.93	5.35	6.95	9.81	5.95	5.68	5.25	5.37
Coefficient of variation	0.3502	0.4767	0.3970	0.4968	0.3916	0.3280	0.3221	0.2718	0.3236	0.2802
Sharpe ratio	2.3249	1.8402	2.3062	1.9409	2.5404	2.4827	2.6594	3.3554	2.9773	3.5447
Share of green assets in the portfolio, %	12.43	9.57	7.72	7.30	8.78	7.26	45.02	43.16	61.65	71.51

Source: Authors' calculations.

The optimizations with 35 assets show that when faced with the choice between risk-free assets and a risk portfolio, VPFs should invest up to 12.43% of their investment resources in green instruments based on the results for the first period (A), and below 7.30% for the fourth period (D). On the other hand, extending the investment choice with just one type of assets *has the potential to significantly change the situation*. The inclusion of the Chinese green

[16] Five different target return values are assumed. Calculations are based on a 10-year period under basic quantitative limits. The data of net asset value, used for VOLIDEX calculation, is published by the Financial Supervisory Commission (eis.fsc.bg/pension-share/498/), RFR and EONIA index – by the European Central Bank (www.ecb.europa.eu/stats), CPI – by the National Statistical Institute (www.nsi.bg/en/content/779/macroeconomic-statistics).

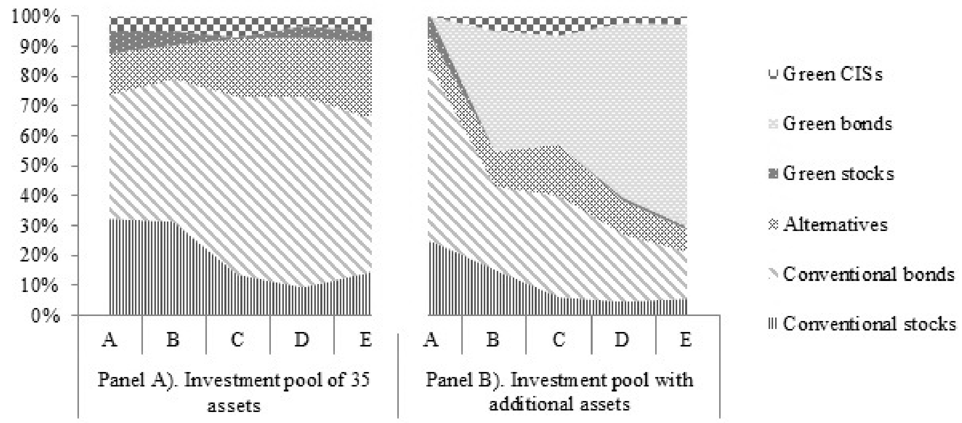
[17] Optimizations are performed under basic quantitative limits.

bonds shows that, based on the degree of risk aversion, VPFs should invest almost half or up to 2/3 of their resources in green instruments.

It needs to be noted that during the five periods considered, the portfolios with a broader investment pool were distinguished by a higher Sharpe ratio and a lower coefficient of variation as compared to their counterparts. Furthermore, these metrics were generated at lower returns and lower risk.

The significant dynamics in the relative shares of investment instruments in "rolling" portfolios demonstrate *the need for rebalancing by VPFs* (fig. 4). Based on the tracking of the optimised portfolios with the best risk-return ratio, it is observed that the share of conventional stocks changes from 9% to 32% and the share of traditional bonds varies between 41% and 64%. Investments in infrastructure and real estate should demonstrate smaller changes in weight.

Figure 4. Dynamics in the structure of "rolling" portfolios with a maximum Sharpe ratio[18]



Source: Authors' calculations.

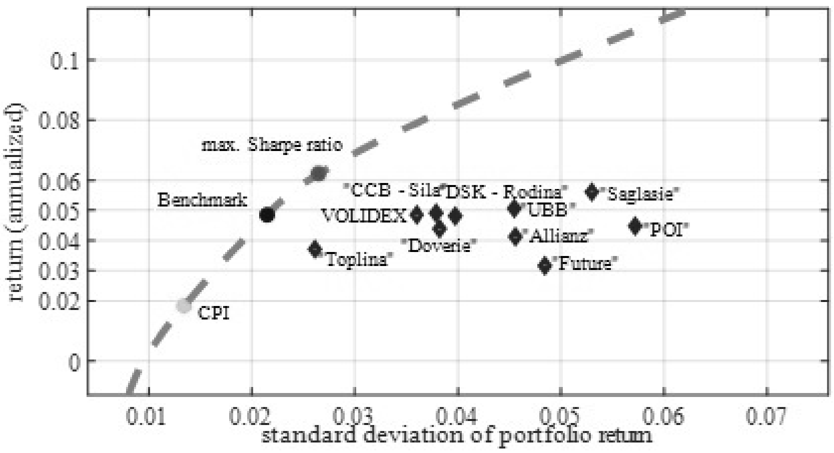
Optimizations with additional assets also lead to major structural changes. If the share of traditional shares in the portfolio was almost 25% during the first period, it was 5.56% during the last one. Conventional bonds are giving up their share in the portfolio to green bonds. At the same time, the group of alternative investments remains relatively constant.

After the numerous optimizations conducted, it is reasonable to raise the question about where the portfolios of VPFs are positioned based on the performance and risk achieved with respect to the efficient frontier. The results reported by VPFs for the period 2012 – 2021 place their portfolios below the optimal opportunity curve. The closest portfolio is the one of

[18] Optimizations are performed under basic quantitative limits. Investments in property (including through REITs), infrastructure and private equity are marked as "alternatives".

VPF "Toplina" and the farthest ones from the efficient frontier are VPF "Future" and VPF "POI" (Pensionnoosiguritelen Institute).

Figure 5. Efficient Frontier, target portfolios and VPFs investment portfolios [19]



Source: Authors' calculations.

How inefficient, however, are VPF portfolios? According to Michaud and Michaud (2008) and Statman and Clark (2013), it should be admitted that the efficient frontier is practically unattainable. These authors discuss an efficient area (range) of efficient portfolios that may turn out to be more reasonable and practical investment choices. Efficient portfolios derived by MV optimization are often defined as counterintuitive, inconsistent with the investors' preferences, and highly sensitive to inaccurate (imprecise) estimates. This view is valid, especially if short selling is allowed, since some of the optimal portfolios are practically unrealisable.

MV optimization is an "error maximiser" because using it leads to significant over-estimation of securities with high expected return, negative correlation and low variance, assigning them substantial weights. These securities are, of course, most likely also characterised by major estimation errors (Michaud and Michaud, 2008: 35). The studies of Jobson and Korki (1981), Fisher and Statman (1997), and Michaud and Michaud (2008) illustrate the extreme sensitivity of optimised MV portfolios to small changes in the estimates of parameters. Minor differences in baseline data result in major differences in the optimal allocation. Kritzman (2011) does not deny this fact, but notes that while changes in resource allocation are significant, the performance of optimal portfolios is largely similar, while Green and

[19] The illustrated efficient frontier (dashed line) is a segment of the graphical representation of the efficient portfolios under the basic quantitative limits. The portfolios marked with a rhombus represent the investment portfolios of VPF. The VPFs' portfolios return is calculated of net asset value, published by the Financial Supervisory Commission (www.https://eis.fsc.bg/pension-share/498). Calculations are based on 10-year period.

Hollifield (1992) recommend investors to give up their ideas of attractive portfolios and accept optimised portfolios, even if unattractive, as the best portfolios (Cite by: Statman and Clark, 2013). However, as noted by Statman and Clark (2013), investors have ignored and will likely continue to ignore this advice. In fact, investors do not always prefer portfolios lying on the efficient frontier because they are interested in additional characteristics (besides risk and return), such as considerations related to their values. Furthermore, MV optimizations "neglect" investors' consumption objectives (e.g., retirement, education, charity, etc.), particularly the fact that portfolios are the means for achieving these objectives (Statman and Clark, 2013: 2).

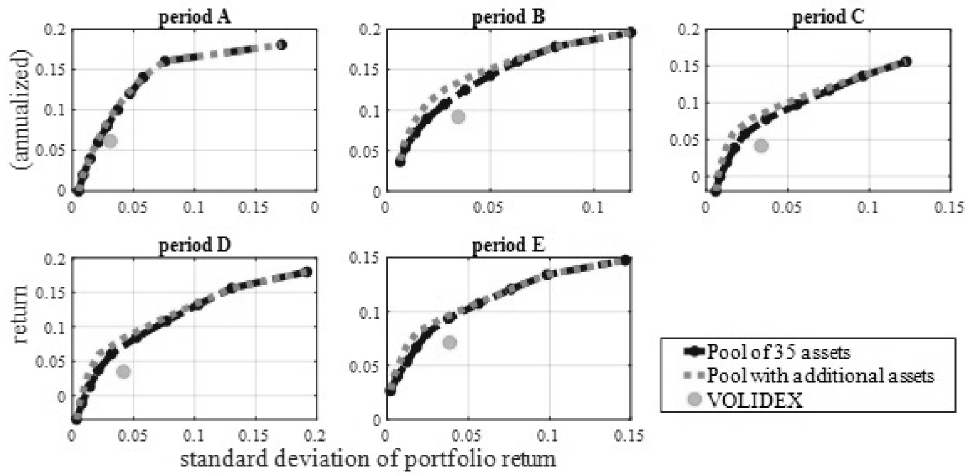
If the investor gives up seeking the "true efficient frontier" and puts their stake in the efficient range, they should use their judgement to determine the reasonable frontiers. In fact, judgment is an essential point in the proper application of MV optimization. In this context, conscious and desired socially responsible investments (e.g. green portfolios of different shades – light, medium or dark green) may fall within the efficient range, even if they do not lie on the efficient frontier, because such portfolios meet the public preferences of conscious investors (see further: Statman and Clark, 2013: 7-10).

From a holistic perspective, the expected utility of beneficiaries is a subjective quantity, beyond its traditional understanding as a function of risk and return, so ESG investments could contribute to its maximization. Beal, Goyen and Phillips (2005), for example, suggest adding one more factor to the utility – ethics. Thus, the indifference curve is transformed into a plane of three dimensions. In this line of thought, pension schemes should be designed to reflect the risk tolerance, investment objectives, values and philosophies of the insured persons, especially in cases where they are not only beneficiaries but bear the entire investment risk (like it is in Bulgaria).

The graphics in Fig. 5 and Fig. 6 reflect the efficient frontiers under the basic quantitative limits. As demonstrated by the previous experiments, however, adding more constraints inevitably leads to a lower positioning of the efficient frontier. VPFs often apply additional constraints (including, but not limited to, liquid assets and currency) to ensure better diversification and to meet the strategic allocation by investment assets specified in their investment policies.

The graph shows the position of the benchmark for the performance of Bulgarian VPFs (VOLIDEX index) against the efficient frontiers at different investment pools for the "rolling" portfolios. The index is positioned below the efficient frontiers of the 5 portfolios discussed, being farther positioned from the frontier reflecting the inclusion of additional assets.

Figure 6. Efficient frontiers and investment performance of VOLIDEX in the "rolling" periods[20]



Source: Authors' calculations.

5. Conclusion

The implementation of ESG determinants in the investment process of private pension funds in Bulgaria has the potential to not only improve the consideration of risk factors for portfolios, but to also generate better investment opportunities. The empirical research is an ample example that *green assets have found their place on the efficient frontiers*. Depending on the portfolio constraints applied and the risk profile of the efficient portfolios, MV optimizations demonstrate green asset weights between 2.54 and 20.25% (in 10-year data). The illustration of green portfolios by "light" and "medium" shades of green under the basic quantitative limits *is fully competitive to conventional portfolios*. In practice, the pursuit of green investments within the range of 5-10% of VPFs assets would not compromise the financial interests of the beneficiaries. This is also confirmed by the optimizations for the "rolling" periods. These demonstrate a positive shift in the risk-return profile of green instruments and highlight the allocation of assets.

References

Akpanibah, E., Osu, O. B. (2018). Optimal Portfolio Selection for a Defined Contribution Pension Fund with Return Clauses of Premium with Predetermined Interest Rate under Mean-Variance Utility. – Asian Journal of Mathematical Sciences, 2 (2), pp. 19-29. Available at: https://ssrn.com/abstract=3812223.

Beal, D., Goyen, M., Phillips, P. (2005). Why Do We Invest Ethically?. – The Journal of Investing, Fall 2005, 14 (3), pp. 66-78. DOI: 10.3905/JOI.2005.580551.

[20] Optimizations are performed under basic quantitative limits. Results are based on 6-year rolling periods.

Beneva, M. (2023a). Pensionnite fondove vav fokusa na zelenite investitsii. – In: Sbornik dokladi ot sedemnadeseta mezhdunarodna nauchna konferentsia na mladite ucheni „Ikonomikata na Bulgaria i Evropeyskia sayuz: inovatsii i transformatsia“, 2022, pp. 307-316.
Beneva, M. (2023b). Portfeylna optimizatsia s UPM/LPM algoritam za chastnite pensionni fondove. – Ikonomicheski i sotsialni alternativi, 29(1), pp. 90-106. Available at: https://doi.org/10.37075/ISA.2023.1.07.
Boulier, J., Huang, S., Taillard, G. (2001). Optimal management under stochastic interest rates: the case of a protected defined contribution pension fund. – Insurance: Mathematics and Economics, 28 (2), pp. 173-189. Available at: https://doi.org/10.1016/S0167-6687(00)00073-1.
Dechant, T., Finkenzeller, K. (2011). Infrastructure Investments: New Evidence from Dynamic Asset Allocation. pp. 1-46. Available at: https://ssrn.com/abstract=1903662. [Accessed 12 September 2022].
Deelstra, G., Grasselli, M., Koehl, P. F. (2003). Optimal investment strategies in the presence of a minimum guarantee. – Insurance: Mathematics and Economics, 33 (1), pp. 189-207. Available at: https://doi.org/10.1016/S0167-6687(03)00153-7.
Deelstra, G., Grasselli, M., Koehl, P. F. (2004). Optimal design of the guarantee for defined contribution funds. – Journal of Economic Dynamics and Control, 28(11), pp. 2239-2260. Available at: https://doi.org/ 10.1016/j.jedc.2003.10.003.
Fisher, K., Statman, M. (1997). The mean-variance optimization puzzle: Securities portfolios and food Portfolios. – Financial Analysts Journal, 53(4), pp. 41-50. Available at: https://www.jstor.org/ stable/4480007.
Green, R., Hollifield, B. (1992). When will mean-variance efficient portfolios be well diversified. – Journal of Finance, 47(5), pp. 1785-1810. Available at: https://doi.org/10.2307/2328996.
Haberman, S., Vigna, E. (2002). Optimal investment strategies and risk measures in defined contribution pension schemes. – Insurance Mathematics and Economics, 31(1), pp. 35-69. Available at: https://doi.org/10.1016/S0167-6687(02)00128-2.
He, C., Cai, L. (2012). Investing in Dynamic Green Portfolios. – Journal of Environmental Investing, 3(2), pp. 23-42. Available at: https://www.thejei.com/wp-content/uploads/2015/01/174-581-1-PB.pdf.
Herzel, S., Nicolosi, M., Starička, C. (2012). The Cost of Sustainability in Optimal. – The European Journal of Finance, 18(3-4), pp. 333-349. DOI:10.1080/1351847X.2011.587521.
Hollenwaeger, S. (2017). Challenging Pension Funds Model Portfolios with Listed Private Equity. – AIAR, 6(1), pp. 49-66. Available at: https://caia.org/sites/default/files/11_aiar_vol-6_issue-1_challenging_pension_funds_7-19-17.pdf.
Jobson, J. D., Korkie, B. (1981). Putting Markowitz Theory to Work. – The Journal of Portfolio Management, 7(4), pp. 70-74. Available at: https://doi.org/10.3905/jpm.1981.408816.
Konicz, A., Mulvey, J. (2015). Optimal savings management for individuals with defined contribution pension plans. – European Journal of Operational Research, 243(1), pp. 233-247. Available at: https://doi.org/10.1016/j.ejor.2014.11.016.
Kritzman, M. (2011). The Graceful Aging of Mean-Variance Optimization. – Journal of Portfolio Management, Winter, 37(2), pp. 3-5. DOI:10.3905/jpm.2011.37.2.003.
Liu, Z. et al. (2023). Optimal portfolios for the DC pension fund with mispricing under the HARA utility framework. – Journal of Industrial and Management Optimization, 19(2), pp. 1262-1281. Available at: https://doi.org/10.3934/jimo.2021228.
Markowitz, H. M. (1952). Portfolio Selection. – Journal of Finance, 7 (1), pp. 77-91. Available at: https://www.jstor.org/stable/2975974.
Markowitz, H. M. (1959). Portfolio selection: Efficient Diversification of Investment. Yale University Press. Available at: https://www.jstor.org/stable/j.ctt1bh4c8h.
Mascelluti, E. (2018). Going Green: Analysis of a Sustainable Portfolio. Libera Universita Internazionale Degli Studi Sociali: Department of Economics and Finance. Available at: https://tesi.luiss.it/21432/1/676751_MASCELLUTI_ELEONORA.pdf. [Accessed 23 May 2021].
MathWorks (2021). Quadratic Programming Algorithms (R2021b). Interior-point-convex quadprog Algorithm. Available at: https://uk.mathworks.com/help/optim/ug/quadratic-programming-algorithms.html# bsqspm_. [Accessed 6 June 2021]
Menoncin, F., Scaillet., O. (2006). Optimal asset management for pension funds. – Managerial Finance, 32(4), pp. 347-374. Available at: https://doi.org/10.1108/ 03074350610652260.
Michaud, R., Michaud, R. (2008). Efficient asset management. A Practical Guide to Stock Portfolio Optimization and Asset Allocation. Second Edition, Oxford University Press. Available at: https://img-cdn.tinkoffjournal.ru/-/efficient_asset_management_michaud_2008_VgLyRYC.pdf.

Pedersen, L., Fitzgibbons, S., Pomorski, L. (2021). Responsible investing: The ESG-efficient frontier. – Journal of Financial Economics, 142(2), pp. 572-597. Available at: https://doi.org/10.1016/j.jfineco.2020.11.001.

Porage, C. (2020). Sustainability in Portfolio Optimization. – Independent Project I, ST433A, Örebro University School of Business. Available at: https://www.diva-portal.org/smash/get/diva2:1549036/FULLTEXT01.pdf. [Accessed 20 February 2022].

Roy, A. D. (1952). Safety First and the Holding of Assets. – Econometrica, 20(3), pp. 431-449. Available at: https://doi.org/10.2307/1907413.

Shen, S., LaPlante, A. (2019). Strategic Asset Allocation with Climate Change. – Netspar Academic Series, DP 01/2019-003. Available at: https://www.netspar.nl/assets/uploads/P20190124_DP003_Shen.pdf. [Accessed 28 April 2022].

Statman, M., Clark, J. (2013). End the Charade: Replacing the Efficient Frontier with the Efficient Range. Available at: http://dx.doi.org/10.2139/ssrn.2230548. [Accessed 5 May 2021].

Tobin, J. (1958). Liquidity preference as behavior towards risk. – The Review of Economic Studies, 25(2), pp. 65-86. Available at: https://doi.org/10.2307/2296205.

Vigna, E. (2009). Mean-variance inefficiency of CRRA and CARA utility functions for portfolio selection in defined contribution pension schemes. – Carlo Alberto Notebooks, Working Paper N 108. Available at: https://www.carloalberto.org/wp-content/uploads/2018/11/no.108.pdf. [Accessed 8 June 2021].

Vigna, E. (2011). On Efficiency of Mean-Variance Based Portfolio Selection in DC Pension Schemes. – Carlo Alberto Notebooks, Working Paper N 154. Available at: http://dx.doi.org/10.2139/ssrn.1775806.

Vigna, E., Haberman, S. (2001). Optimal investment strategy for defined contribution pension schemes. – Insurance: Mathematics and Economics, 28(2), pp. 233-262. Available at: https://doi.org/10.1016/ S0167-6687(00)00077-9.

Walker, E. (2005). Optimal Portfolios in Defined Contribution Pension System. Available at: https://www.oecd.org/finance/private-pensions/36278542.pdf. [Accessed 17 March 2021].

Wojt, A. (2010). A Portfolio Selection and Lower Partial Moments. – KTH Royal Institute of Technology: Department of Mathematics, Sweden. Available at: https://www.math.kth.se/matstat/seminarier/reports/M-exjobb09/091214b.pdf. [Accessed 14 November 2021].

Wong, V. (2020). Statistical Aspects of Sustainability in Optimal Portfolio Theory. Master Thesis 2020: 4, Mathematical Statistics, Stockholm University. Available at: https://kurser.math.su.se/pluginfile.php/20130/mod_folder/content/0/Master/2020/2020_4_report.pdf. [Accessed 21 November 2021].

Zhu, S., Dong, Y., Wu, S. (2022). Optimal investment of DC pension plan with two VaR constraints. – Communications in Statistics, Theory and Methods, 51(6), pp. 1745-1764. Available at: https://doi.org/10.1080/03610926.2020.1767141.