Kamen Mirkovich, Professor, Dr. Ec. Scs.

GENERAL THEORY OF UTILITY AND VALUE

In this article the author justifies the necessity of creating a General theory of utility and value as categories of the value individual economic behaviour. They are constructed on the basis of the economics, and therefore, any comprehensive and fundamental economic exposition should be based on them. Together with the known in the literature theories of value and marginal utility, introduced and justified are notions of value and marginal utility (which at the individual level are subjective value and subjective marginal utility). A theory of the marginal value could not have, for the economic science, a lesser meaning than the theory of marginal utility (constructed by the Austrian subjective school). The creation of a relative theory of utility and value is suggested, in which the relative utility and value are comparable with each other and allow the analysis of the value economic behaviour of the individual. With the help of those relative categories it is possible to construct categories of added utility, added value and added product (as well as marginal added utility, marginal added value and marginal added product), which are totally compatible with the contemporary economics, deriving also the individual curves of product's demand and supply, and the individual supply of labour.

The contemporary economics owes to the classical political economy for the fact that it almost abandoned the theory of value, considering the latter contradictory to the utility theory. On the other hand, substantial part of the proponents of the notion of value as a key economic phenomenon, interpret it mainly or entirely as public value, without attempting to grasp its multi-dimensional essence, which comes out at the different levels of economic organisation of production, starting from the individual economic agent, continuing with the firm and market, and ending with the society as a whole. According to the author, the economic science should be modified so as to extend on equal footing both the problems of economic essence and economic phenomena. *Here we support and justify the idea that in such an economic science of essence and phenomena, central place should be given to the general theory of utility and value, which have equal importance for the economic knowledge, including the building of market categories, especially the product prices and production factors.* Object of the research here are the subjective utility of products used by the individual and the subjective (individual) value of these products. To facilitate the analysis and the article's length limitations, we assume that the individual satisfies his/her needs through the use of only one product and produces the same kind of product by employing only one factor (labour). Therefore, we have one-product consumption, one-product production and one-factor production. In reality, the individual satisfies

his/her needs through the use of many kinds of products, which s/he creates by employing several factors of production.[1]

Total utility and marginal utility of the product

The assessed, by the individual, ability of a certain product, in its capacity of consumer value of certain kind, to satisfy with certain intensity his/her needs is *utility* U_p of that product. The utility is an expression of the subjective preferences of the individual, and therefore, it is also subjective. However, the preferences and utility are formed also under the influence and complex mixture of various economic, social, psychological, biological, ideological and natural factors since the individual expresses him/herself as owner and consumer of goods under the influence of the surrounding environment.

G. Debreaux proved[2] the existence of *a function of total utility of products* (often only called *product utility function*), $TU_p(q_u)^2$, defined in the space of products (goods) Q. *The total products' utility*, TU_p, is the utility of the combination of products (in their entire quantity, q_u), owned and consumed by certain consumer. When the combination of products contains only one kind of product (as is the case here), then the term function of the total product utility, $TU_p(q_u)$, is used, as well as the term total product utility, TU_p.

The economic theory proves that all first derivatives of the function of the total utility $TU_p(q_u)$ are positive:

$$(1) \qquad MU_p(q_u) = \frac{\partial TU_p(q_u)}{\partial q_u} =$$

$$= \frac{\partial TU_p(q_u)}{\partial q_{u1}}, \frac{\partial TU_p(q_u)}{\partial q_{u2}}, \ldots, \frac{\partial TU_p(q_u)}{\partial q_{un}} \, .$$

The first derivatives

$$(2) \qquad MU_p(q_u) = (MU_{p1}(q_u), MU_{p2}(q_u), \ldots, MU_{pn}(q_u)),$$

are the so called marginal utilities of products. *The marginal utility of the product* MU_p (of certain kind of product) is the first derivative of the total utility of the set of owned and consumed products with respect to the accumulated volume of this product, i.e., the relation between the marginal increase of the function of total utility of products and marginal increase of the volume of certain product, included in the vector of consumption. In every point of the space Q, the increase in the consumption of a certain product from the vector q_u, *without changing the consumption of the other kinds of products from the same vector*, leads to increase (growth) of the total utility $TU_p(q_u)$ of the set of consumed products.

---

[1] *Дебрьо, Ж.* Теория на стойността (Аксиоматичен анализ на икономическото равновесие). Mirkovich, K., N. Boginov (eds.). Sofia, 1999.

[2] According to the case considered in the text, the index symbol p is used for marking the phrase "of one type of product" (a product as a single concept) or "of many types of products" (a product as a complex concept).

For all rational numbers, the total utility function $TU_p(q_u)$ is quazi-concave. It is assumed that the function is twice differentiable, i.e. has continuous second derivatives (and therefore smooth). Then we can introduce a *product-utility matrix of Hase,* Hu, which consists of those second derivatives and is entirely negatively defined:

$$(3) \qquad H_u \; = \; \frac{\partial^2 TU_p(q_u)}{\partial q_u^2} \; < \; 0 \; ,$$

where $\partial q_u^2 = \{\partial q_{ui}\, \partial q_{uj}\}$ (i,j = 1, 2, …, n)

The included in the main diagonal of the product-utility matrix of Hase elements with negative value

$$(4) \qquad \frac{\partial^2 TU_p(q_u)}{\partial q_{ui}^2} \; < \; 0 \qquad (i = 1, 2, …, n)$$

show that the marginal utility of each product decreases when the level (volume) of its consumption increases. That is, the so called, *law of the decreasing marginal product utility*, introduced by Herman Gosen and meaning *the first law of H. Gosen*.

The decreasing growth of the total utility, when the consumer set contains only one kind of products (i.e. with one-product consumption needs), is illustrated in the upper part of Figure 1 showing the *curve of the total product utility* TU_p, where q_u is a scale variable and represents the quantity of the individually consumed product. The curve TU_p is increasing and has a positive slope towards the axes. Under one-product consumption needs, it represents a set of points of the relationship between the quantities of individually owned and consumed product (in natural units) and the levels of the product's total utility in utility units. With certain conditionality, it is possible to say that the curve TU_p starts from zero. From there on, the total product utility starts to increase with the rise in q_u, but with decreasing speed, which follows the law of decreasing marginal utility since the initial levels of $MU_p(q_u)$ of the marginal utility are relatively high.

Thus, it is expected that the marginal utility MU_p measure in natural units, which is contained in one natural unit of certain kind of products, decreases with the rise in the volume qu of the consumption of that product. As a result, the total utility TU_p measured in utility units, contained in the whole volume of the product measured in natural units, increases with decreasing returns to scale. The reason is that with the increasing volume of consumption the degree of satiation of the individual also increases and as a result his/her utility value for a unit of product decreases. *Here, however, a fundamental, even though at a first glance simple question arises, which have almost not been discussed in the economics, general and specialised, literature.* More specifically, after reaching a certain degree of satiation of his/her needs, the individual interrupts the consumption process, due to

biological, household, production or public reasons, as well as the limitations to his/her incomes, and after the effect of needs satisfaction has already passed, the individual again starts the process of consumption from the initial point, the initial highest value of the marginal utility of the first unit of consumed product.

Total Product Utility
(utility units)

$TU_p(q_u)$

$TU_p(q^*)$ - ○ A

TU_p

0   q^* q_u

Total Product Value
(value units)

$TV_p(q_v)$

TV_p

$TV_p(q^*)$ - ○ B

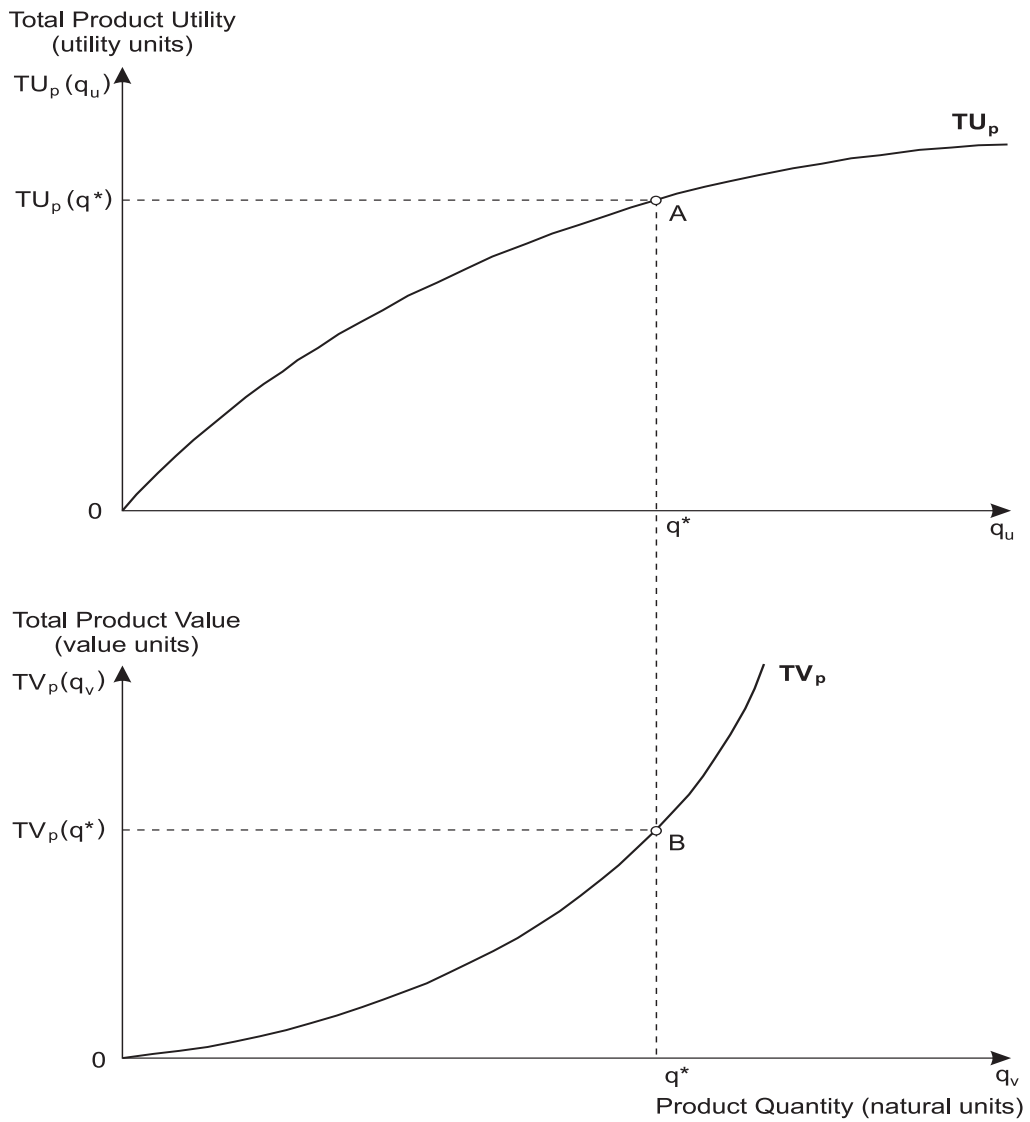0 q^* q_v

Product Quantity (natural units)

Figure 1

It is clear that the individual consumption is a cyclical process as after the end of one cycle starts a new one, which, other conditions being equal, repeats the same quantitative relations of decreasing marginal product utility and decreasing returns of the total utility of the whole volume of the product. This cycle should not be confused with the so-called life cycle of the product. The specific *cycle of individual consumption*, which the author calls *consumer cycle*, has certain longevity and represents the *period of consumer cycle*. Of course, in reality the individual needs have a much more complex structure, since the individual needs both non-durable and durable goods with different length of use. Therefore, the whole process of production combines a multitude of cycles with different periodicity, cycle periods. For the derivation of the main relations in the theory of utility (later also in the theory of value), however, suffices for the analysis to consider only a one period consumption, i.e. to be reckoned that the consumption consists of cycles, having equal length, i.e. periodicity. During each subsequent cycle, the utility relations would repeat. There is sufficient ground to assume that under one-period consumption, this is the consumption cycle lasting for 24 hours, which is assumed to be a unit of time in this analysis. If the considered process is related to the individual's participation in the product production, then within a 24 hours, the individual has reasonably used his/her production capacities (capacities to produce a product) as in the process of consumption s/he has recuperated and reproduced these capacities (as a labour force) and is again able to participate in production. After the end of the 24-hour cycle of consumption, begins again the highest possible level of marginal utility of product.

Depending on his/her individual status (subjective consumer preferences, personal income and other limitations) the individual is able to consume within the assumed unit of time or other quantity of the product, i.e. to satisfy his/her needs to a certain degree. So, the consumption is a multi-choice process. In that sense the individual could form (induce) within one consumer cycle (in this case – within 24 hours) certain value of the total and marginal utility. That means that within one unit of time, certain intensity of consumption could be formed, which is measured by the total utility of the consumed product volume during that unit of time. In other words, the *intensity of consumption* IC represents and measures the total utility, formed (induced) of a unit of astronomical time, given that the latter is the period of consumption cycle.

Total value and marginal product value

During production, the labour force uses its *production abilities* (abilities to create goods, labour abilities), which nature is specific for each individual. *The nature of the production abilities is subjective*. Their level is a result of the complex interaction of a multitude of personal and society factors (including general and professional education, accumulated work experience, professional

environment, technological level of production and management). The production abilities, even though subjective by nature, are influenced by the environment surrounding the individual including the fact that production is an organized and managed process, in which the individual follows the technological and other requirements as well as economic necessity. Ultimately though, the use and realization of the production abilities, while the individual created the product, is determined by the subjective attitude, subjective wishes, subjective perceptions and subjective judgment of the individual regarding those possibilities, before taking the decision of whether to work and produce goods.

The attributes of a good in the process of its creation as a product of labour to contain, in certain extent, to become an object in itself and to carry production abilities of the labour force, while the individual as a worker spends his/her energy and exhausts his/her body, is its (product and value and individual's) *value*. In other words, *the value of the product V_p. Just as the utility is subjective the value (individual value) is subjective*, as the latter is ultimately determined by the subjective production abilities, subjective attitude, subjective perceptions and subjective production preferences of the individual. This, however, does not mean that the subjective value of the product (like the subjective utility) is not at all objectively determined. It is objectively determined only as far as the subjective behaviour of the individual as producer is motivated and determined under the influence of the surrounding objective economic production and social environment.

The power of this subjective influence has different degree, which depends on *the forms and level of organization of social production*, under which the individual works, i.e. realizes him/herself as a worker. *Under the subsistence economy*, this relation is weak: the individual decided almost on his/her own how and how much to work, considering his/her production abilities. *Under the market economy* this relation is strong. *Under continuous production of goods* (as a form of market economy, under which the individual organizes and realizes on his/her own production and does not sell his/her labour), the reached level of product value depends also on the market price of the product (since the value is a function of the supplied product, and the volume of optimal production and supply depends on the price). *Under the developed market economy* (as a form of market economy, under which the individuals sells his/her labour) the level of product value, to a large extent, depends on the firm's economic and technologic conditions of labour and labour legislation. This dependence gets different modifications under perfect and imperfect competition.

According to the author, the product value and utility form the group of *valued economic ingredients* (V_p, U_p), shortly – the product *value ingredients*. Using this term later on will use simply the expression *product values* (V_p, U_p) (in plural) meaning the economic ingredients attributed to the product. Instead,

the value of the product W_p (in singular) is the general term of product value or utility, which are its private cases (i.e. its different kinds).

It was already mentioned that the utility is an assessment, which people give to the product ability in its aspect of a consumer value, to satisfy with certain intensity their needs. In that respect the product could be, for example, very useful, of little use or even useless. In the economic theory, the value (of the product) is usually defined as the assessment, which people assign to utility in relation to the degree of scarcity of the product. For example, the air has almost no value, even though extremely useful, since it is supplied in abundance (still). The bread has a relatively low value, even though it is very useful, since it is in relatively satisfactory quantities. Some unique objects, which have small utility, are of great value, since single and therefore scarce. However, if having the same degree of scarcity, the product with higher use has higher value. In the theories, there are also claims that the above relationships are applicable also to the value. Perceiving the *worth* of a product as equal to only the *value* is unjustified, as well as perceiving it as equal to *utility*.

In this exposition, the author suggests another interpretation of value. This is the *ingredient theory of value*, which is based on the production and consumption (generally the system of reproduction). *The value is the assessment, which people give to their possessions*, which is conducted in two ways: *first*, from the point of view of production of the product, which uses production factors (in this case – labour) and *second*, from the point of view of the possession and consumption of the product, satisfying human needs and recuperating working force (and with that, even though elementary, the circle of reproduction is closed). In that sense, the value of the product has two relatively independent, but at the same time, in certain degree of interdependent, ingredient input-output kinds: product value and product utility. *Therefore, the value and utility are the values of the product, but neither one entirely exhaust its worth.*

The value of the product is *an input value economic ingredient*, since it represents in the product the use of labour in its capacity as a production factor (which is an input economic ingredient), where the product is a result of the production. On the other hand, the product utility is *an output value economic ingredient*, since it represents in the product (which is an output economic ingredient), where the product is subject of consumption.

The ingredient theory fits as a subsystem within the more general *ingredient theory of wealth*, which the author thinks, has to be developed in the economic theory. Depending on whether it is perceived as phenomenal or actual level, the wealth is both object and value.

As a phenomenon, the level of wealth exists and is perceived in its direct objective form – *objective wealth*. Conditionally it could also be called *perceived wealth*. Therefore, this is a phenomenon economic theory. Naturally, the phenomenon expresses the inside nature of the object. In the system of social

production, the objective wealth has two kinds – *input object wealth*, represented by the production factors (labour, capital and endowments) and *output object wealth*, represented by the production products. In this sense the object wealth is composed of *objects of economic ingredients*, which are divided into input and output. The production factors are *input object economic ingredie**nts**, while the products – *output object economic ingredients*. The object economic ingredients are not being studied in this exposition.

At the same level, wealth exists and is being perceived in its value form – this is *the valued wealth* or also *the value of the wealth*. Conditionally, it could be called *the transcendental wealth*. Therefore, it is a core economic category. The valued wealth is the assessment, which people assign to the object wealth. Within the system of social production, the valued wealth has two kinds – *input valued wealth*, represented by the value of the products and *output valued wealth*, represented by the utilities of the products. In that sense, the valued wealth is composed of *value economic ingredients*, divided also into input and output ones. *The object of this exposition is studying the value economic ingredients (value and utility).*

Similarly to the function of general utility, according to us, could be introduced also *the function of total value of products* (of the combination of products) $TV_p(q_v)$ (i.e. the combination of produced products, defined in the space of products Q). When the combination of products includes only one kind of product, then we will use the term *function of the total product value* (of the given type of product) $TV_p(q_v)$. *The total product value* TV_p is the value of the combination of products (in their entirety), created by a certain producer. When the combination of products (goods) includes only one kind of product, then will be used the term *total product value* TV_p (of certain kind of product).

It could be proved that all first partial derivatives of the function of the total value $TV_p(q_v)$ are positive:

$$(5) \qquad MV_p(q_v) = \frac{\partial TV_p(q_v)}{\partial q_v} =$$

$$= \frac{\partial \, TV_p(q_v)}{\partial q_{v1}} \, , \frac{\partial TV_p(q_v)}{\partial q_{v2}} \, ,..., \frac{\partial TV_p(q_v)}{\partial q_{vn}} > 0.$$

The first partial derivatives

$$(6) \qquad MV_p(q_v) = [MV_{p1}(q_{v1}), MV_{p2}(q_{v2}), ..., MV_{pn}(q_{vn})],$$

the author calls marginal values of the products. *The marginal values are positive. The marginal value of the product* TV_p (of a certain product) is the first derivative of the total value of all produced products with respect to the volume of that product, i.e. it is the marginal ratio between the differential growth of the

function of the total product value and the differential growth of the product volume, included in the set (vector) of produced products.

At every point of the space Q, the increase of the production of a certain product from the vector q_v, *without changing the production of the other products of the same vector*, leads to increase (growth) of the total value of the set of produced products. The changes in the marginal value are determined by *the function of the marginal product value* $MV_p(q_v)$, which depends on the volume of produced products (goods) q_v.

For all real numbers, the function of the total value $TV_p(q_v)$ is quazi-concave. It is assumed that the function is twice differentiable, i.e. has continuous second derivatives (and therefore, smooth). Then we can introduce *the value Hasian matrix* H_v, which consists of the second derivatives and is entirely defined:

$$(7) \qquad H_v \; = \; \frac{\partial^2 TV_p(qv)}{\partial q_v{}^2} \; > 0,$$

where $\partial q_v{}^2 = \{\partial q_{vi}\,\partial q_{vj}\}$ (i,j = 1, 2, …, n).

The included in the main diagonal of the value Hasian matrix elements with positive value

$$(8) \qquad \frac{\partial^2 TU_p(q_v)}{\partial q_{vi}{}^2} \; > 0 \qquad (i = 1, 2, …, n)$$

shows that the marginal value of each product is increasing with the growth of the level (volume) of its production. We will define this relation as *the law of increasing marginal value of the product*. It signifies *the scarcity of the production possibilities* with the increase of the production level (volume). According to this law, with the rise of the quantity of the products, which the individual produces, the total value increases, but with increasing pace, i.e. the marginal levels increase.

The speeding increase of the total value, when the producer set contains only one kind of product (i.e. one-product individual abilities), is illustrated on the lower part of Figure 1 as *the curve total value of the product* **TV_p**, where q_v is the scale value and represents the quantity of produced by the individual product. The curve **TV_p** is increasingly growing and has a positive slope towards its coordination axes. It represents a set of points representing the relationship between the quantity of produced by the individual product in natural units and the level of its (of the product) individual total value (in value units). With certain conditionality, it could be said that the curve **TV_p** begins at the coordination axes. After that point (after the production process begins (when $q_v>0$), the total value begins to increasingly grow while the initial levels $MU_p(q_u)$ of the marginal value are relatively lower.

The above exposition shows that, it is a regularity that the marginal value MV_p (measured in utility units), contained in one natural unit of a certain kind of product increases with the increase of the volume q_v of the production of that product by the individual. As a result, the total value TV_p (measured in value units) contained in the whole volume of the product (measured in natural units) grows increasingly with the increase of this volume. The main reason for the regularity in question is that, as motioned earlier, with the increase of the production volume, increases the degree of satiation and scarcity of the production abilities of the individual and subsequently its value assessment for a unit of product increases.

Analogically to the case of consumption, here also arises one fundamental (even though on the face of it elementary) question. After reaching a certain degree of applying his/her production abilities, the individual interrupts his/her production process (due to biological, domestic, production, legal or social reasons) and after certain time, when s/he has recovered his/her capabilities, resumes the labour process, so to speak from the original point, from the first (lowest) level of marginal value of the first unit of produced product. It is clear that the individual's labour is a cyclical process, after the end of one cycle begins a new one, under which other things being equal, the same quantitative relationships repeat - increasing marginal value and increasing growth of the total value. The single cycle of individual's labour, which will be called *labour cycle*, has certain length, which represents *the period of the labour cycle*. For the derivation of the main relationships in the theory of value (as before in the utility theory), it is enough the analysis to rest only on the one-period labour process, i.e. to consider that labour is composed of cycles, having equal length, i.e. periodicity. *Therefore, further it will be assumed that the individual applies his/her production capabilities in labour cycles with equal length as the attention over the formation of the total value TV_p and the marginal value MV_p is only within one cycle.* During the next cycle, the relationships will repeat. There is sufficient ground to assume that the labour cycle has the length of 24 hours.

If this process is related to the participation of the individual in the product consumption, then within 24 hours this individual (as this was clarified also before) will use in reasonable degree his/her production capabilities, but in the process of consumption, s/he has recovered his/her capabilities and again is able to take part in production. After the end of the 24 hours, the cycle of labour begins again with lowest amount of marginal product value.

Depending on the individual's status (subjective production preferences, firm, technological and normative requirements and other limitations) the individual has the possibility to produce within the assumed unit of time certain quantity of the product, i.e. to apply his/her capabilities in certain degree. Therefore, labour is a multi-possibility process. In that sense, s/he (the

individual) could formulate (induce) within one production cycle (in this case – 24 hours) certain level of the total and individual's marginal value. This means that within the assumed unit of time, there could be formulated certain labour intensity, which is measured by the individual total value of the produced during the unit of time product, i.e. *the labour intensity* IL is equal and measured by the total value, formed (induced) during one astronomical time, when the latter is the periodicity of consumption.

Relativistic theory of utility and value

On Figure 1 with q* is depicted that quantity of the product for which creation the individual uses production capabilities which could be recovered through individual consumption of the same product quantity q*. We will denote that as *the value equilibrium product*. After its production and consumption there is neither excess of the product (over the necessary for the recuperation of the spent production abilities consumption), nor deficiency. Then TU*=TU(q*) represents the total utility of the value equilibrium product (the upper part of Figure 1), while TV*=TV(q*) represents the total value of the value equilibrium product (lower part of Figure 1).

Let us divide the function of the total product utility TU(q) of the total utility of the value equilibrium product TU(q*). The new function RTU(q)=TU(q): TU(q*) the author calls *the function of the relative total product utility*. To it corresponds *the curve of the relative total product utility RTU$_p$*, depicted at Figure 2. Also, let us divide the function of the total product value TV(q) of the total value of the value equilibrium product TV(q*). The new resulting function RTV(q)=TV(q):TV(q*) the author calls function of the relative total product value. To it, corresponds *the curve of the relative total value of the product RTV$_p$*, depicted also on Figure 2. The relative utility RTU and the relative value RTV of the product are principal terms in the suggested and constructed by the author *relativistic theory of utility and value*, which consists of the relativistic theory of the utility and relativistic theory of value.

The relativistic theory of utility is the product utility (including the total and marginal) in relative units and studies the relationships between them, as well as them and the other economic categories, phenomena and processes. From that point of view, the considered until now product utilities appear as their absolute utilities. In the relativistic theory the relative total utility of the value equilibrium product of each individual is always equal to unity, i.e. RTU(q*)=TU(q*):TU(q*)=1. *The relative utility of the products* is expressed as relative utility units of those products. The relative utility units are constructed through the relationship between different kinds of absolute utilities, on one hand, and the absolute utilities of the value equilibrium product of the individual. *From that follows that the relative utilities are the product of their corresponding absolute utilities with the relative utility units.*
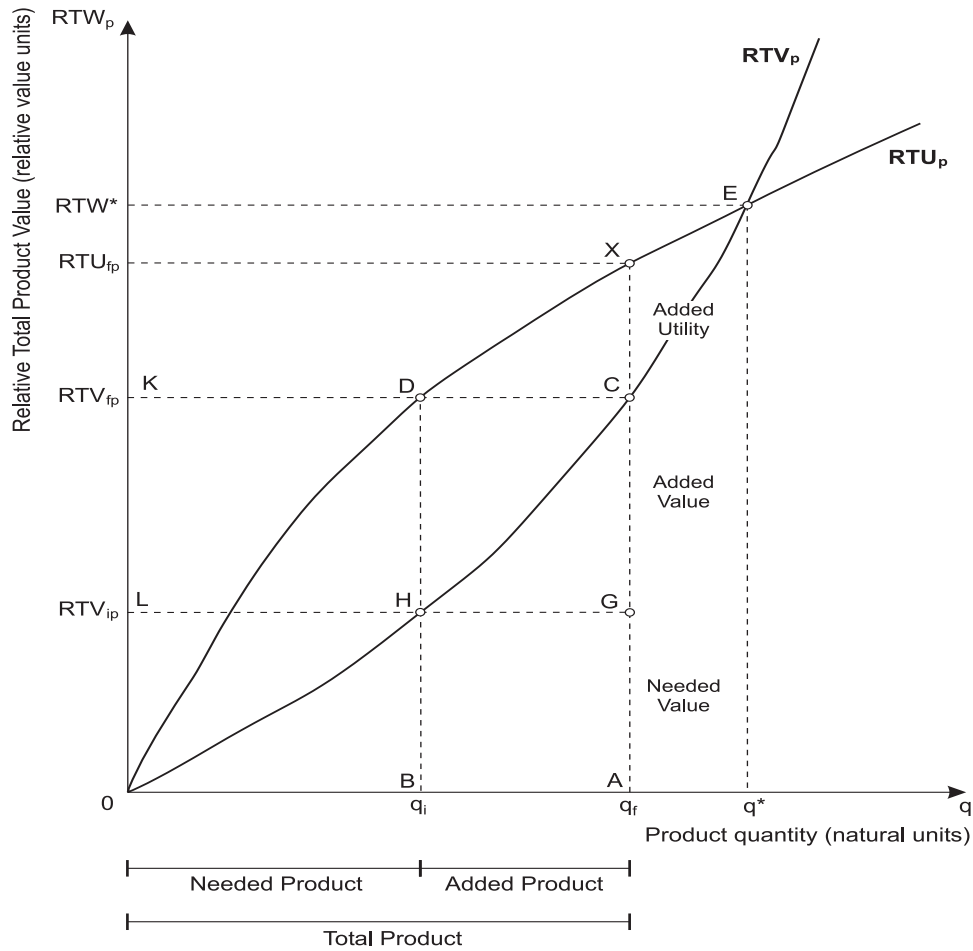
Figure 2

Types of *relative product utility* $RTU_p=RTU_p(q_u)$ and the *relative marginal product utility* $RMU_p=RMU_p(q_u)$. For them the following statements are valid:

1. The total utility is the relationship between the absolute total product utility and the size of the absolute total utility of the value equilibrium product or, which is the same, is the product of the absolute total utility with the relative utility units. The relative total utility $RTU_p=RTU_p(q_u)$ is the total size of the contained relative individual utility in combination with q_u of possessed and consumed by the consumer products (in their total quantities). Similarly to the absolute individual

total utility ATU_p, the relative total utility RTU_p is also growing decreasingly – a fact, which is due to the law of the decreasing marginal utility.

2. The relative marginal utility $RMU_p=RMU_p(q_u)$ is the relationship between the absolute marginal product utility and the size of the absolute utility of this product or, which is the same, is the product of the absolute marginal utility with the relative utility units. The relative marginal product utility is the growth of the relative total utility of the whole combination of possessed and consumed by the consumer products, induced by the increase with one natural unit of the considered product. Or also, it is the relationship between the differential growth of the relative total utility of the combination of products and the differential growth of the quantity of the considered product. In general the relative marginal utility is the first differential of the function of the relative total utility. Similarly to the absolute marginal utility AMU_p, the relative marginal utility RMU_p is also decreasing with the growth of the product quantity q_u.

Since other things being equal, the absolute total utility of the value equilibrium product is constant (for the specific individual), then *the relationships between the different types relative utilities (total and marginal) are equal to the relationships between the different kinds of absolute utilities (total and marginal).* For the relative utilities the same laws are valid, which are valid for the absolute utilities, including the law of decreasing marginal utility. Therefore, the law of decreasing marginal utility is a law both for the decreasing absolute and the decreasing relative marginal utility.

The relativistic theory of value is the product value (including total and marginal) in relative units and studies the relationships between them and them and the other economic categories, phenomena, processes. From that viewpoint, the considered until now product values appear as their absolute values. In the relativistic theory, the relative total value of the value equilibrium product of each individual is always equal to unity, i.e. $RTV(q^*)=TV(q^*):TV(q^*)=1$. *The relative product value* is expressed in relative value units of those products. The relative value units are constructed through the relationship between the different types of absolute values, on one hand, and the absolute values of the value equilibrium product of the individual. *From that follows that the relative values are the product of their respective absolute values with the relative value units.*

The types of relative units are: *the relative total product value* $RTV=RTV(q_v)$ and *the relative marginal product value* $RMV=RMV(q_v)$. For them the following statements are valid:

1. The relative total value is the relationship between the absolute total product value and the size of the absolute total value of the value equilibrium product or, which is the same, is the product of the absolute total value with the relative value units. The relative total value $RTV=RTV(q_v)$ is the total size of the contained relative individual value in the combination qu of possessed and consumed by the consumer products (their whole quantities). Similarly to the

absolute individual total value ATV, the relative total value RTV is also increasingly growing – a fact due to the law of increasing marginal value.

2. The relative marginal value $RMV=RMV(q_v)$ is the relationship between the absolute marginal product value and the size of the absolute total product value, or, which is the same, is the product of the absolute marginal value with the relative value units. The relative marginal value of e certain product is the growth of the relative total value of the whole combination of the created by the producer products, induced by the growth with one natural unit of the considered product. Or also: it is the relationship between the differential growth of the relative total value of the combination of products and the differential growth of the quantity of that product. In general, the relative marginal value is the first differential of the function of the relative total value. Similarly to the absolute marginal value AMV, the relative marginal value RMV is also growing with the rise of the product quantity qu.

Since, other things being equal, the absolute total value of the value equilibrium product is constant (for the given individual), then *the relationship between the different kinds of relative values (total and marginal) are equal to the relationship between the different kinds of absolute values (total and marginal)*. For the relative terms, valid are the same laws, which are valid for the absolute values, including the law of increasing marginal value. Therefore, the law for the increasing marginal value is law both for the increasing absolute and the increasing relative marginal value.

Value economic equilibrium of the individual

Through the possession and consumption of a certain quantity of a product (respectively products) depending on his/her subjective consumer preferences *the individual forms* (other things being equal) certain level of the absolute and relative total and marginal utility $ATU_p(q_u)$, $AMU_p(q_u)$, $RTU_p(q_u)$ and $RMU_p(q_u)$ of that product (respectively products). Then his/her actions we will call *individual's utility behaviour. Under the one-product analysis*, it is described by the curves of the absolute and relative total product utility $AITU_p$, AMU_p, RTU_p and RMU_p. *The utility behaviour of the individual is subjective utility behaviour, which is motivated and consumed in certain social and natural environment.* Through the production of a certain product quantity (respectively products) depending on his/her subjective production preferences *the individual forms* (other things being equal) certain level of absolute and relative added value $ATV_p(q_v)$, $AMV_p(q_v)$, $RTV_p(q_v)$ and $RMV_p(q_v)$ of that product (respectively products). That action we will call *individual's value behaviour. Under one-product analysis,* it is described by the curves of the absolute and relative total marginal product value **ATV_p, AMV_p, RTV_p** and **RMV_p**. *The value individual behaviour is subjective value behaviour, which is also motivated and consumed under certain social and natural environment*. In its entirety, the utility and value behaviour form *value economic behavour of the individual.* It is a general term for the utility or value behaviour of the individual, which are its special cases (i.e. its types).

On Figure 2 are plotted simultaneously the curve of relative total utility RTU_p [which represents the function $RTU_p(q_u)$ and the curve of total value RTV_p [which represents the function $RTV_p(q_v)$] of the product of a certain economic agent. As it was already noted, the curve RMU_p is ascending with a positive slope but with decreasing marginal utility [the second derivative of the function $RTU_p(q_u)$ is negative], but the curve RMV_p is ascending with a positive slope, but increasing growth, since the law for the increasing marginal utility applies [the second derivative of the function $RTV_p(q_v)$ is positive]. The derived figure, the author will call *general scheme of the value economic behaviour of the individual*.

In the total scheme of the value economic behavoiour of the individual we distinguish the point E where the curve of the relative total utility RTU_p crosses the curve of the relative value RTV_p. This is the only point where (if we exclude the center of the coordination system) at which there is a coincidental equilisation, *first*, between the manufactured product and that consumed by the individual, and, *second*, between the relative total utility and the relative total value of the product. In other words, there is both equilised product and equilised relative total value of the product. Therefore, this is *the point of the value economic equilibrium of the individual.*

The abscise of the point E is the value equilibrium product. The ordinate of the point E is equal to a unity and this is the relative value of the value equilibrium product, both equal to its relative total utility and relative total value.

The analysis of the economic behaviour of the individual based on the explained above regularities represents special interest in the microeconomic theory, since it shows the possibility to define terms such as added product and added value (compatible with the contemporary economics), as well as to derive the individual curves of demand and supply on the goods market and the individual curves of demand and supply on the labour market.

Added value, added utility and added value in the relativistic theory of utility and value

Further the attention in the current analysis turns mainly to the part of Figure 2 that depicts the value economic behavour of the individual, limited by the curves of the relative total value of the product RTV_p and the relative total utility of the product RTU_p on one hand, and the point E of the value economic equilibrium of the individual.

Let us assume that during a unit of time (i.e. during the period of one cycle of the value economic behaviour) the individual had produced a product of quantity q_f natural units ($0<q_f<q^*$). His/her volume is equal to the length of the line OA. This product volume has relative total utility of RTU_{fp} relative utility units [where $RTU_{fp}=RTU_p(q_f)$] equal to the length of the line AX (where the corresponding q_f point X lies on the curve RTU_p), and the relative total value of size RTV_{fp} relative value units [where $RTV_{fp}=RTV_p(q_f)$], equals the length of the line AC (where the corresponding point of q_f point C lies on the curve RTV_p).

It should be noted that when $0<q_f<q*$, *the relative total utility is larger than the relative total value of the product*. This statement is simple, but has a fundamental meaning for the economy. *It means that the part of the necessary abilities, which the individual spends to produce certain quantity of the product, is smaller than the part of the needs, which the individual could satisfy with the same quantity of that product. There is an excess utility, which could be used for the recuperation (and reproduction) of large quantities of the abilities, which subsequently to be applied for the production of larger product quantity with respect to the initial*. This excess of utility the author calls *relative total added utility of the product* $RTSU_p(q_f)$. *The added utility is utility in excess, i.e. over that quantity of utility, which is enough for the recuperation of capabilities necessary for production of the initial product quantity (product q_f)*. The relative total added utility $RTSU_p(q_f)$ is equal to the length of the line CX, i.e. the difference between the relative total utility $RTU_p(q_f)$ and the relative total value $RTV_p(q_f)$ of the same quantity of the product.

If from the relative total utility $RTU_p(q_f)$ of the product qf is subtracted by its relative total added utility $RTSU_p(q_f)$, the difference could be called *relative total needed product utility* $RTIU_p(q_f)$ (of the product q_f). This is the length of AC. *The relative total needed utility of a certain product quantity is that part of its total utility, which is necessary for recovering and reproduction of those individual capabilities which use could produce the same product quantity (product q_f)*.

The above statements show that the relative total utility RTU_p of a certain product is equal to the sum of the relative total necessary utility $RTIU_p$ and the relative total added utility of that product. In that the relative total utility RTU_p represents (appears as) *the relative total full utility of the product* $RTFU_p(q_f)=RTU_p(q_f)$ (of the product q_f). Generally, $RTFU_p=RTIU_p+RTSU_p$. *For the reader it is clear that the relative total needed utility $RTIU_p$ of the product (the length of AC) which is only a part of its relative total full utility $RTFU_p$ (the length of AE) is equal to the total size of the relative total value RTV_p of that product (again the length of AC)*.

The next step of this analysis is for the reader to look at the points C, D and K, which lie on the same horizontal line (Figure 2). This line shows the ordinate RTV_{fp} (the length of AC, equal to the line OK) which is the relative total value RTV_{fp} of the product q_f (according to point C lying on the curve **RTV_p**). However, the same line crosses also the curve **RTU_p** in the point D. This means that the ordinate RTV_{ip} is equal to the line OK) which is the relative total utility RTU_{ip} of a product of another size, denoted with qi in the point B. The term $RTV_{fp}=RTU_{ip}$ represents the relative total value, but *related to different by volume products*: relative total value of the product qf and relative total utility of the product q_i. *The meaning of $RTV_{fp}=RTU_{ip}$ is that the part of the used (spent) capabilities is equal to the part of the satisfied needs. This means that the individual economically reproduces himself: the satisfied needs RTU_{ip} are enough for the individual to recover his/her used capabilities RTU_{ip}, and the opposite, the used capabilities RTU_{ip} are enough for the product reproduction which consumption satisfies the needs RTU_{ip}*.

It is important that the product q_f is larger than the product qi. More precisely, the product qf which has the relative total value RTV_{fp}, has a larger than the product q_i, which has the relative total utility RTV_{ip} equal to RTV_{fp}. From that viewpoint, the product q_f is the *individual full product*. So, the relative individual total utility RTU_{ip} of the product with volume q_i natural units is enough (through the consumption of q_i) for recovering the capabilities RTFV, which are needed for the production of larger volume full product of q_fi natural units. Therefore, the product q_i could be called *needed product* (needed for the production of the full product q_f).

From the above, it follows that the indicated above relative total value of the product RTV_{fp} (the length of AC) now looks like *the relative total value of the full product* RTV_{fp}. The indicated above relative total full utility of the product $RTFU_{fp}$ (the length of AX) now looks like the *relative total full utility of the full product* $RTFU_{fp}$. Also, in the light of the considered relative total added utility of the product $RTSU_{fp}$ (the length of CX) now looks like *the relative total added utility of the full product* $RTSU_{fp}$. The considered relative total needed utility $RTIU_{fp}$ (the length of AC) now looks like *the relative total needed utility of the full product* $RTIU_{fp}$. The conclusion above, that the relative total needed utility of the product is equal to its relative total value, now looks like: *the relative total needed utility of the full product $RTIU_{fp}$ (the length of AC) is equal to its relative total value $RTIV_{fp}$ (the length of AC).*

Further the analysis continues with considering the types of total utilities of the needed product and its total value, first considering the basic term for total utility of the needed product. The relative individual total utility RTU_{ip} (the length of BD) is *the relative total utility of the needed product* RTU_{ip}. Therefore the comparison with the above relationships leads to the following important for the economic theory conclusion: *the relative total value of the full product RTV_{fp} (length of AC) is equal to the relative total utility of the needed product RTU_{ip} (the length of BD).* The same way, we could draw the following conclusion: *the relative total needed utility of the full product $RTIU_{fp}$ (the length of AC) is equal to the relative total utility of the needed RTU_{ip} (the length of BD). It is also true that the absolute needed utility of the full product $ATIU_{fp}$ is equal to the absolute total utility of the needed product ATU_{ip} (the length of BD).*

The relative total utility RTU_{ip} of the needed product q_i is larger than its relative total value RTV_{ip}, i.e. its relative total full value $RTFV_{ip}$. The difference between them, i.e. RTU_{ip}-RTV_{ip}=RTU_{ip} − $RTFV_{ip}$, determined by the length of HD we will call *the relative total added value of the needed product* $RTSU_{ip}$. The difference between the relative total utility RTU_{ip} and the relative total added utility $RTSU_{ip}$ of the needed product q_i, determined by the length of BH, could be called *the relative total needed utility of the needed product* $RTIU_{ip}$. *From the above exposition it follows that there is an equilibrium between the relative total needed utility $RTIU_{ip}$ of the needed product qi and its relative total value RTV_{ip} (the length of BH) or, which is the same, there is equilibrium between the relative total needed utility $RTIU_{ip}$ of the needed product qi and its relative total full value $RTFV_{ip}$.*

There is place for the statement that in the light of the written so far, the relative total utility RTU_{ip} of the needed product qi looks like *the relative total full utility of the needed product* $RTFU_{ip}$. It is evident that the relative total full utility $RTFU_{ip}$ of the needed product q_i (the length of BD)is equal to the sum of the relative total needed utility $RTIU_{ip}$ (the length of BH) and the relative total added utility of the needed product $RTSU_{ip}$ (the length of HD).

The comparison between the considered relationships show that *the relative total added utility of the full product $RTSU_{fp}$ (the length of CX) is equal to the difference between the relative total full utility of the full product $RTFU_{fp}$ (the length of AX) and the relative total full utility of the needed product $RTFU_{ip}$ (the length of BD).*

The relative total value of the full product RTV_{fp} contains a certain part, which is necessary for the self-sufficiency of the capabilities of the individual and for the self-reproduction of the full product. That part of RTV_{fp} value units we will call *the relative total needed value of the full product* $RTIV_{fp}$ (the length of AG). The value of $RTIV_{fp}$ shows the part of the relative total value of the full product which is necessary for the reproduction of the individual capabilities which application is enough to reproduce the needed product which consumption recovers the individual capabilities enough for the reproduction of the full product. *The above analysis leads to an important conclusion for the economics, that the relative total needed value of the full product $RTIV_{fp}$ (the length of AG) is equal to the relative total full value of the needed product $RTFV_{ip}$ (the length of BH).*

The difference between the relative total value of the full product RTV_{fp} (the length of AC) and its relative total needed value $RTIV_{fp}$ (the length of AG) the author calls *the relative total added value of the full product* $RTSV_{fp}$ (the length of GC). The level of $RTSV_{fp}$ shows the part of the relative total value of the full product which is above the needed for the reproduction of the individual capabilities which application is enough for the reproduction of the needed product which consumption recovers the individual capabilities, enough for the reproduction of the full product. *In other words, the added value is a value in excess of what its owner could spend for the production of the product, which consumption exceeds the necessary for the production consumption, or for expansion if this production.*

The above statements show that the relative total value RTV_{fp} of the full product q_f is equal to the sum of the relative total needed value $RTIV_f$ and the relative added value $RTSV_f$ of the full product q_f. In that respect, the relative total value represents (looks like) *the relative total full value of the full product* $RTFV_{fp}$. Generally $RTFV_{fp}=RTIV_{fp} + RTSV_{fp}$. *For the writer it is clear that the relative total needed utility $RTIU_{fp}$ of the full product (the length of AC) which is only a part of its relative total full utility $RTFU_{fp}$ (the length of AX), is equal to the whole value of the relative total full value $RTFV_{fp}$ of that product (again the length AC). The difference between the considered relationships also shows that the relative total added value of the full product $RTSV_{fp}$ (the length of GC) is equal to the difference between the relative total full value of the full product $RTFV_{fp}$ (the length of AC) and the relative total full value of the needed product $RTFV_{ip}$ (the length of BH).* From the analysis made so far, we can make also the conclusion, most important for the economic

science, that *the relative total added value of the full product RTSV$_{fp}$ (the length of GC) is equal to the relative total added utility of the needed product RTSU$_{ip}$ (the length of HD)*.

The difference q$_f$-q$_i$ between the individual full and individual needed product represents *the individual added product* q$_s$. The individual added product is the product in excess, the product over the individually needed product. *The added product is that produced by the individual part of the full product, which exceeds the product which consumption is necessary for the reproduction of the capabilities sufficient for production of the full product. In that sense, the added product is not necessary for the reproduction of the full product. It could be used for consumption, exceeding the needed as the increased with this value capabilities could be used for accumulation or not to be used for expanding production. The added product could also be partially or entirely owned by the buyer of labour (as a production factor) when the individual sells his/her labour.*

The right answer to the question to what is equal *the relative total value of the added product* RTV$_{sp}$ could be given only if the volume of the individual added product begins to be accounted for only above the volume of the individual one needed product and if the level of the relative total value of the added product begins to be accounted for over the level of the relative total value of the needed product. This means that point H should be used, in a way, as a beginning of the coordination system (Figure 2). Then it is clear that the relative total value of the added product RTV$_{sp}$ is equal to the length of GC. The conclusion is that *the relative total value of the added product RTV$_{sp}$ is equal to the relative total added value of the full product RTSV$_{fp}$ and, therefore, is equal also to the relative total added utility of the needed product RTSU$_{ip}$.*

This way, analogically to the used above, the right answer of the question to what is equal *the relative total utility of the added product* RTU$_{sp}$ could be given only if the volume of the individual added product begins to be accounted for above the volume of the individual needed product and if the level of the relative total utility of the added product begins to be accounted for above the level of the relative total utility of the needed product. This means that the point D should be used, in a way, as a beginning of the coordination system (Figure 2). Then it is clear that the relative total utility of the added product RTU$_{sp}$ is equal to the length of CX. The conclusion is that *the relative total utility of the added product RTU$_{sp}$ is equal to the relative total added utility of the full product RTSU$_{fp}$.*

The current analysis could be further enriched at least in two directions: *first*, as the marginal analysis is applied to the terms added product, added utility and added value (i.e. introducing also the terms marginal added product, marginal added utility and marginal added value), and *second*, as in the scope of the production factors apart from labour is also included capital. Then it will be possible to construct a more complete microeconomic theory for utility and value (where both capital and labour play a role) and the income distribution according to the marginal products of the production factors in the conditions of perfect competition and imperfect competition on the product, labour and capital market.

Attachment

Value Theory of the Austrian School of Thought
of the Subjective Political Economy

The theory of value is originally constructed within the theory of the Subjective Political Economy. This is a theory for the subjective value, which is based on the theory of the subjective marginal utility. *Here I will focus on two notions of value constructed by the Austrian School of Subjective Political Economy* (mainly represented by Karl Menger, Fridrich von Wizer and E. Borm-Bawerk). According to them, the essence of the subjective value (or, in other words, the essence of the subjective assessment of the individual) is placed the marginal utility. There is no disagreement in the Austrian School on the idea that the value (i.e. the subjective value) of the last unit of a certain good of certain quantity (store) equals the marginal utility (i.e. the subjective marginal utility) of that good. Yet, there are differences on whether the total value of that quantity of a certain kind of good equals the total utility of that quantity.

According to one of the theories supported by Fridrich von Wizer, the total value of a certain quantity of a given good is equal to the product of its marginal utility with the quantity of that good.[3] Now we know that under perfect competition the equilibrium market price of the product is equal to its marginal utility (if we assume that a unit utility is measured by one currency unit; in the more general case the ratio of the equilibrium market prices of two kinds of products under perfect competition is equal to the ratio of their marginal utilities). Therefore, the theory of F. von Viser claims that the total utility is equal to the product of the equilibrium market price with the quantity of the product an that more precisely, the maximum value the market price could take is the subjective value. However, the total utility of the total quantity of the good (that is the integral of the function of its marginal utility, which is decreasing) is larger than the product of the marginal utility with the quantity of the good (the difference between these two total values is called consumer surplus). Therefore, according to F. von Viser, the total value of the total quantity of the good is smaller than the total utility of that quantity (the difference between the two being the consumer surplus). F. Von Viser considers the value only as a form of utility measurement. s/he writes: "Measured is not the utility, but the value: the value is a form through which utility could be measured. In terms of computation this represents considerable relief. To calculate the utility of the store [of goods] is too hard, to calculate the value is simple".[4]

According to I. G. Blyumin, the theories of F. von Viser show that the principle of marginality has a commodity [market] character [the marginal utility

---

[3] *Viser, F.* Der naturliche Wert. Wien, 1889, p. 25.
[4] Ibid., p. 32-33.

equals the equilibrium market price]. "In relation to that the utility has a new meaning. The utility equals the maximum price, which the buyer agrees to pay for a certain commodity".[5] Similar statement later makes also A. Marshal. By the way, it is no coincidence that the Russian economist-mathematician N. N. Shaposhnikov writes, that "…the higher is the utility assigned to a product, the more I agree to pay for it. From two products, I will pay more for that which has higher utility for me. If I intend to acquire a unit of a certain good, my assessment will be determined by the utility of the given good; however, if I rely on acquiring several units of a given good, my assessment will be determined by the magnitude of the utility of that unit of good, which satisfies my smallest daily needs"[6]. According to F. von Viser, as well as other representatives of the Austrian political economy, the value of the commodity (its subjective assessment) is determined by the method of the decreasing quantity of the good (i.e. through deprivation): the loss of certain unit of a good causes dissatisfaction of a necessity of the smallest intensity, or similarly, deprivation of marginal utility.

According to the other theory, the total value of a certain quantity of a good is larger than the product of that quantity with the marginal utility of the good. E. Bom Baverk writes: "Our assessment of one or other material good at the same time and the same conditions could only differ depending on whether we assess certain units or considerable quantities of those material goods, perceived as a whole unit".[7] G. Casel supports a more compromising position. According to him, if it is up to the individual to choose the quantity of the good, then the total value of that quantity [in German Gesammtwert] is the product of the quantity with the marginal utility; if, however, the individual has the alternative either to buy the whole quantity of the good or entirely refuse it, then the total value of that quantity is the sum of the marginal utilities of all units of the good graded in descending order of their magnitudes.[8]

The essence of the difference between value and utility in the Austrian School (in her entirety) precisely reveals I. G. Byumin: "The analysis of the theory of utility gives the opportunity to clarify the reason for the breakup between value and utility. The Austrians, as is well known, assign great importance to the differentiation, which they make between utility and value. According to them, the major deficiency of the old theory of utility (e. g. Say, Condiliac) is in that they equalize these two notions. Due to that in their theories there was a deep contradiction between theory and reality. Rather valuable goods could have too low subjective value. From the point of view of

---

[5] *Блюмин, И. Г.* Критика буржоазной политической экономии. Т. I. Субективная школа в буржоазной политической экономии. Издательство Академии наук СССР. Moskow, p. 103.

[6] *Шапошников, Н. Н.* Теория ценности и распределения. Moskow, 1912, p. 5.

[7] *Бем-Баверк, Е.* Основы теории ценности хозяственных благ. Leningrad, 1929, p. 23.

[8] *Cassel, G.* Grundriss einer elementaren Preislehre. Zs. gesamte Staatswissenschaft, 1899, Bd. 55.

the Austrians, the subjective assessment represent independent phenomenon. The order of the subjective assessments does not coincide with the order of utility. *The utility of the good is determined by the most intensive necessity, which it is possible to satisfy. The subjective assessment is determined by the smallest necessity, which could be satisfied by certain quantity [of the good].* As a result, the two orders have independent meanings. The two orders cross at only one point. This point is the last point, i.e. the last unit of the order. For the last product, the utility is equal to the subjective value. Therefore, the subjective value is determined ultimately by the utility but is equal to it only under exceptional circumstances. Therefore, the subjective assessment of one bread is considerably smaller of its utility; but that subjective assessment is determined by the utility of another bread, which could have been used last and could have satisfied a necessity of the smallest intensity."[9]

The suggested in this theory ingredient theory of value (with its two kinds of value and utility) is not based on the value perceptions of the Austrian School and the subjective political economy, but has its own independent logic. In order to make a comparison, however, it is useful for the reader to know other points of view related to the use of the different economic notions (in the closer of more distant history of the economic science), in this case – the notions of the value of the product (good).

4.X.2004

---

[9] *Блюмин, И. Г.* Op. cit., p. 111-112.