Emil Hristov, Senior Research Fellow, Dr. Ec. Scs.

A COMPLEX MODEL FOR MEASUREMENT AND FACTOR ANALYSIS OF POPULATION AGEING

A model for measurement and analysis of population ageing has been developed. The already published various methods are synthesized in a complex model for measurement and factor analysis of ageing. The novice here is the further development of factor model. In that method a hipothetical population with preserved intensities of the three processes - age-specific fertility rates of women, mortality and net migration by sex and age from a previous period – is established. At these conditions two tasks of factor analysis are solved. In the first one, ageing is divided in two parts – by the three processes' preserved intensities and by their changes in surveyed period. In the second task ageing is subdivided in three parts - by change in the number of live births and by the different intensities of mortality and net migration by age in accounted period. An analysis of the persistent population ageing in Bulgaria in 1990-2000 has been conducted.

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The author has already published methods for measurement of ageing and a model for factor analysis of that phenomenon.[1] The new aspect here is the further development of the factor model and its joining with the method for measurement of ageing in a common or complex model. On that ground an exact analytical relation between ageing of population and influences of separate factors is established. Another major change in the factor model is the dropping out of the equity between total number of hipothetical and the real (actual) population in the end of the surveyed period of ageing. In the old model, the number of hipothetical population by sex and age in the end of surveyed period was established with preserved intensities of mortality and net migration of previous period. That hipothetical population had the same age structure of real population from the beginning, used to assess also the hipothetical number of live births. Then, the difference between real and hipothetical population in the end of surveyed period results solely to the difference between their age structures. In the new model, hipothetical population is estimated again but with preserved intensities of all three processes - age-specific fertility rates of women, mortality and net migration by sex and age of previous period. In that way, ageing is subdivided in two parts: its size only by the preserved intensities of the three processes and by the changes in these intensities in the surveyed (accounted) compared to previous (basic) period. The idea behind preserving intensities of all three processes of the previous period

[1] *Hristov, E.* Ageing of Bulgarian population in the past century – age-specific effects and empirical analysis – Population, 2003, Issue 1-2, page 18-29; The Factor Model of Population Ageing from Changes in Live Births, Deaths and Net Migration Growth. – Economic Thought, 2003, Issue 2, page 91-108.

was presented for the first time in Bulgaria at the International Expert Workshop on the issues of ageing.[2]

Of these changes in factor model two closely related but different tasks of analysis can be pointed. The first one can provide an exact solution of ageing at three processes' preserved intensities and separately from their changes. In the second task, the total size of ageing can be subdivided in three components or effects: ageing by the change in number of live births (estimated in the first task) and two separate ageings by the different intensities of mortality and net migration by age in surveyed period. Other prominent models for factor analysis of ageing are not reviewed since they are, in our opinion, hypothetical and limited. The reviewed article, for example, reviews a model of stable population.[3] It is noted that stable populations are hipothetical since they reflect only certain modes of age mortality rates and changes in live births without taking account of the population migration movement. In current circumstances, however, migration can have a large and diverse influence on population number and age structure. Ageing of real population requires not only comparison with stable population but also with real population and even lees comparisons solely between stable populations.

Complex Model for Measurement and Factor Analysis of Population Ageing

That model represents the successive differences between population average ages. Each difference measures a certain effect (ageing in years) of common, separate and joined influences of the mentioned factors. The statistical model for analyzing the difference between two mean levels (average ages) is applied, where only age structure of population varies, while indiators (successive ages) are constants.

In the first stage, total ageing of population in the end of five-year period is measured, compared to population in the same period beginning. The five-year period has two main advantages. Firstly, it is long enough to account for some ageing that is insignificant in shorter periods and is affected by other factors. The second advantage allows dealing with five-year age groups of population, as there is more sustainable data for them than for one-year periods. The common wisdom that each five-year age group in the beginning of a period transits after five-years in the next older age group is used here. On that ground the surveyed period of ageing can be larger, encompassing for example two and more five-year periods. In fact, ten-year periods are most suitable as they are most indicative in time and usually represent intervals between two successive censuses of the population. Having in mind that present statistics does not account for the emigration of persistent population, ten-year periods provide

[2] *Hristov, E.* Ageing of Bulgarian Population in the Period 1991-2001 (Methodological Solutions and Analysis), report. Workshop "Understanding the Drivers of Population Ageing in Central and Eastern Europe – Fertility, Mortality and Migrations". University of Oxford, September 19-21, 2007.

[3] *Hristov, E.* Factor Model for Ageing of Population..., p. 92.

opportunity for indirect assessment of that migration. In that relation, the complex model is applied for measurement and analysis of Bulgarian population ageing in 1990-2000 on data from the last two censuses - 4.12.1992 and 1.03.2001. Undoubtedly, that was a very dramatic period in the demographic history of Bulgaria in the end of past century, since it is marked by severely dropping natality, growing mortality and continuing emigration of persistent population after the exile of Bulgarian Muslims and Turks in the late 1980's. Generally speaking, those processes were intrinsic part and consequences of the huge socio-economic changes in Bulgaria during the transition to market economy and democracy.

The complex model analysis begins by measurement of population ageing by 31.12.2000 compared to that measurement by 31.12.1990. It is conducted separately for men and women as there are differences in the number of live births, mortality and net migration by age of the two genders. First, on the ground of known number of population by sex and age by 31.12.1985, results of the two censuses in 1992 and 2000, annually published number of deaths by the so-called elementary aggregates in 1986 - 2000 and annual data for total emigration balance in that period, are estimated by sex and age by 31.12.1990, 31.12.1995 and 31.12.2000[4]. For the five-year age groups of those populations the symbols ${}_0P_{x_1x+5}$ and $r1P_{x,x+5}$ and $r2P_{x,x+5}$ are used. Also, conventional populations by 31.12.1995 and 31.12.2000 derived from preserved intensities of the three processes in the past period 1981-1990, are represented by $c1P_{x,x+5}$ and $c2P_{x,x+5}$.

From those populations, the following structures by sex and age are estimated: ${}_0S_{x,x+5}$ of 31.12. 1990, $r2S_{x,x+5}$ and $c2S_{x,x+5}$ of 31.12.2000. Then, through the popular method for measurement of ageing average ages $\overline{X}_0, \overline{X}_{r2}$ and $\overline{X}_{c2}$ are calculated from the expression C_iS_i, where C_i is the middle of five-year age intervals (i= 1,2,…, 19), while S_i is the relative share of respective population in the same brackets.[5] Averages C_i are conventional for the average ages of population in those intervals.[6] For the last age interval of 90 and more years (I = 19) the average ages by

[4] Results of Population Censuses in 1985, 1992 and 2000 (see Demographic and Socio-economic Profile of Population. Vol. I. Sofia, CSM, 1988; Demographic Characteristics. Vol. I. Sofia, NSI, 1994; Population, Vol. I. Demographic and Social Characteristics of Population. Sofia, NSI, 2004, N 1; annual publication Population for the period 1986 – 2001. Sofia, CSM, CSI).

Annual data for net migration balance are cited from *Kalchev, I.* Emigration in Bulgaria. Sofia: Dunav Press, 2001, p. 128, table V.I.

[5] See *Hristov, E.* Ageing of Bulgarian Population..., p. 18-21.

[6] The i index is introduced for convenience and substitutes the expression $(x_1 x + 5)$ for five-year age intervals.

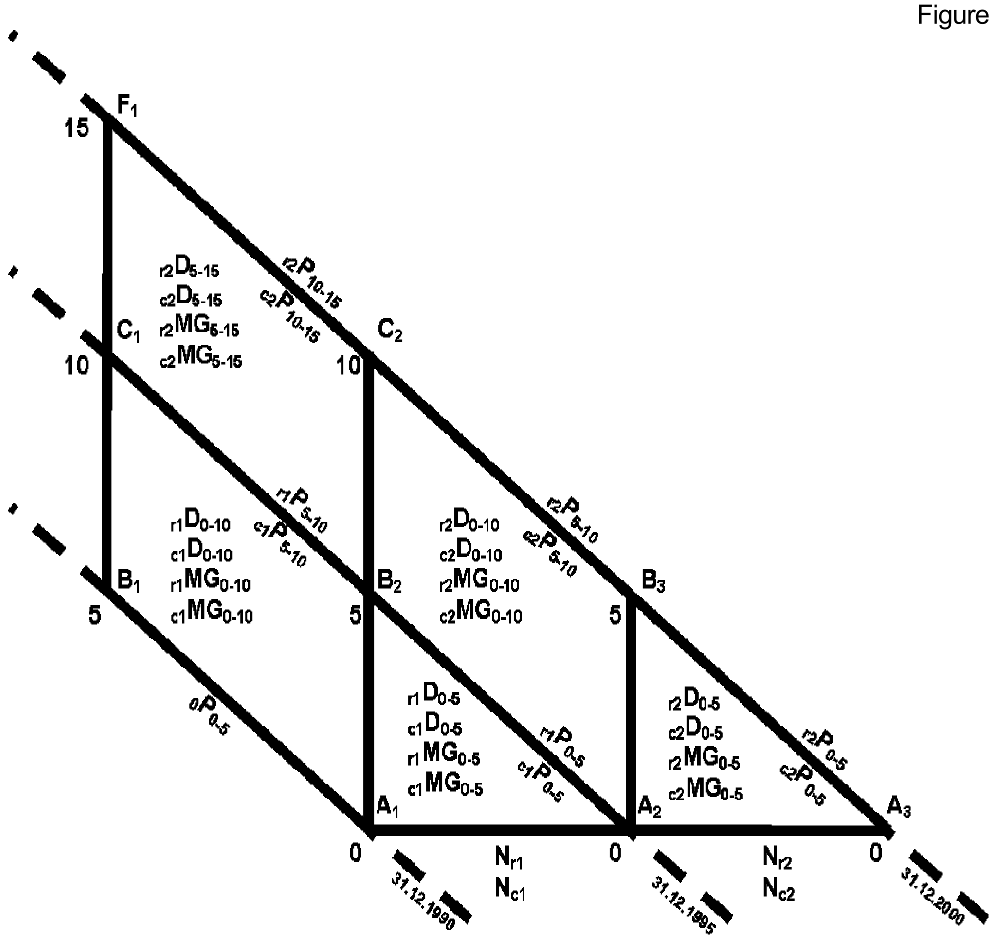
sex are calculated by relative shares for one-year ages of men and women from the Population census of 4.12.1992, that are most accurate for the beginning of the surveyed period. For men and women C_{19} they are respectively 92.45 and 92.52 years. Then, the difference $\Delta\overline{X}_{r20} = \overline{X}_{r2} - \overline{X}_0$ measures the total ageing of population in the surveyed period. It is also presented as an algebraic sum of products $\sum_{i=1}^{19}\left(C_i\overline{X}_0\right)\times\left(S_{r2i} - S_{0i}\right)$, where each product represents the effect or contribution of each i-age on ageing. According to the suggested model development, the difference $\Delta\overline{X}_{r20}$ can be also presented as an algebraic sum of two differences: $\Delta\overline{X}_{r20} = \Delta\overline{X}_{c20} + \Delta\overline{X}_{r2c2} = \left(\overline{X}_{c2} - \overline{X}_0\right) + \left(\overline{X}_{r2} - \overline{X}_{c2}\right)$.

The first difference $\Delta\overline{X}_{c20} = \overline{X}_{c2} - \overline{X}_0$ presents the size of population conventional ageing in the end of surveyed (reported) ten-year period at preserved intensities of the three processes from a past (basic) ten-years period. That difference can be presented by similar to the suggested sum of effects by age: $\Delta\overline{X}_{c20} = \sum_{i=1}^{19}\left(C_i - \overline{X}_0\right)\times\left(S_{c2i} - S_{0i}\right)$.

The second difference $\Delta\overline{X}_{r2c2} = \left(\overline{X}_{r2} - \overline{X}_{c2}\right)$ measures additional ageing or rejuvenating of population in the surveyed period. Algebraic symbol of that difference is estimated from the inequalities between average ages with two possible cases: if $\overline{X}_0 < \overline{X}_{c2} < \overline{X}_{r2}$, the difference $\left(\overline{X}_{r2} - \overline{X}_{c2}\right) > 0$ and shows additional ageing. If $\overline{X}_0 < \overline{X}_{c2} < \overline{X}_{r2}$, $\left(\overline{X}_{r2} - \overline{X}_{c2}\right) < 0$ and demonstrates population rejuvenating only due to the changes in intensities of the three processes. Like the previous differences between average ages, the difference $\Delta\overline{X}_{r2c2}$ can be presented as an algebraic sum of effects by age: $\sum_{i=1}^{19}\left(C_i - \overline{X_0}\right)\times\left(S_{r2i} - S_{c2i}\right)$. That difference is the grounds for the suggested development of factor analysis. To demonstrate the analysis, part of the popular Lexis diagram is used (see the Figure).[7] It clearly shows the relations between data for population and certain demographic events, accounted simultaneously on the three indiators - year of event, year of birth and age.

[7] *Sugarev, Z.* Sofia: “Nauka I Izkustvo”, 1975, p. 113-122.

Figure



All data on that Figure relates to five-year age intervals. Segments A_1A_2 and A_2A_3 represent real (actual) live births N_{r1} and N_{r2} respectively in 1991-1995 and 1996-2000. The same segments represent also conventional live births N_{c1} and N_{c2} for those periods through preserved age-specific fertility rates of women in the periods 1981-1985 and 1986-1990. Then, the changes in number of live births $\Delta N_1 = N_{r1} - N_{c1}$ and $\Delta N_2 = N_{r2} - N_{c2}$ are due only to the changes in age-specific fertility rates in 1991-1995 compared to 1981-1985 and in 1996-2000 compared to 1986-1990. Of all real live births N_{r2} real (actual) deaths $r1D_{0-5}$ and $r2D_{0-10}$ result, while from conventional live births N_{c1}, conventional deaths $c1D_{0-5}$ and $c2D_{0-10}$ result. The number of real deaths results from data of the annually published elementary aggregates, while that of conventional deaths is estimated by cohorts probability of dying $c1q'_{D0-5}$ and $c2q'_{D0-10}$ for the two five-year periods 1981-1985 and 1986-1990. The number of quoted real and conventional deaths is presented by the triangle $A_1A_2B_2$ for $r1D_{0-5}$ and $c1D_{0-5}$, and parallel segment $A_2B_3C_2B_2$ for $r2D_{0-10}$ and $c2D_{0-10}$. Similarly, from real live births N_{r2} in 1996-2000 real deaths $r2D_{0-5}$ result, while from the conventional live births N_{c2} - conventional deaths $c2D_{0-5}$ result. The latter are assessed through cohort probability of dying $c2q'_{D0-5}$ in 1986-1990. The two groups of deaths – real and conventional are graphically presented with the triangle $A_2A_3B_3$.

After deaths, the real or actual net migration by age is estimated. First, the differences $r\Delta P_{0-5} = r2P_{0-5} - N_{r2}$ and $r\Delta P_{5-10} = r2P_{5-10} - N_{r1}$ are found, whose algebraic symbols reflect real deaths and migration growths. Those growths are determined through deaths: $r2MG_{0-5} = r\Delta P_{0-5} - r2D_{0-5}$ and $r1MG_{0-5} + r2MG_{0-10} = r\Delta P_{0-10} - \left(r1D_{0-5} + r2D_{0-10}\right)$. Then, as we said, conventional net migration is derived from the cohort's probabilities for that growth from 1981-1985 and 1986-1990 periods. In the same way conventional deaths are determined from the number of conventional population by age by 31.12.2000. For the two initial age intervals 0-5 and 5-10 years conventional population is determined from the following sequences: $c2P_{0-5} = N_{c2} - c2D_{0-5} + c2MG_{0-5}$ and $c2P_{5-10} = N_{c1} - \left(c1D_{0-5} + c2D_{0-10}\right) + \left(c1MG_{0-5} + c2MG_{0-10}\right)$.

Following the same logic, real and conventional data for deaths and migration growths in the rest age intervals are estimated. For example, the sum $r1D_{0-10} + r2D_{5-15}$ of real deaths derived from the initial population ${}_oP_{0-5}$ by 31.12.1990 (segment A_1B_1), is calculated from the sum of respective elementary aggregates. Then, conventional deaths $c1D_{0-10}$ and $c2D_{5-15}$ result from the cohort's probability of dying $r1q'_{D0-10}$ in 1981-1985 and $r2q'_{D5-15}$ in 1986-1990. The two groups of deaths are represented by the same parallel sources $A_1B_2C_1B_1$ and $B_2C_2F_1C_1$. On its side, real migration growths $r1MG_{0-10}$ and $r2MG_{5-15}$ are determined through the known differences: $r1MG_{0-10}$ + $r2MG_{5-15}$ = $r\Delta P_{10-15} - \left(r1D_{0-10} + r2D_{5-15}\right)$, where population growth is $r\Delta P_{10-15} = r2P_{10-15} - {}_0P_{0-5}$.

Other conventional migration growths are estimated from the respective cohort's probabilities $r1q'_{mg_{0-10}}$ from 1981-1985 and $r2q'_{mg_{5-15}}$ from 1986-1990. Through those growths and conventional deaths the number of conventional population is estimated $c2P_{10-15} = {}_0P_{0-5} - \left(c1D_{0-10} + c2D_{5-15}\right) + \left(c1MG_{0-10} + c2MG_{5-15}\right)$. The same proceedings are used for the rest age intervals. It must be noted that we can speak of net migration probabilities only when that growth is a negative quantity, i.e. emigrants from the persistent population outnumber the immigrants. Then the difference between them represents emigrants from the native or initial population. In positive net migration where immigrants outnumber emigrants, it is considered that the difference between them does not result from the initial population. In that case the indicator index should be used instead of probability. Aside from this difficulty there are other challenges as well, when interpreting migration and mortality of persistent population. They won't be discussed here, however, since they are actually neglected.

In the factor analysis' second task total ageing can be subdivided in three effects: $\Delta\overline{X}_N$ - due to the changes in number of live births in surveyed (reported) period 1991-2000 compared to previous (basic) period 1981-1990, $\Delta\overline{X}_D$ - from the total number of deaths and its distribution by age in surveyed period 1991-2000 and $\Delta\overline{X}_{MG}$ - of the total net migration (balance) and its distribution by age in surveyed period 1991-2000. According to that task, only the first effect $\Delta\overline{X}_N$ results from changes in the process of age-specific fertility rates in the surveyed period 1991-2000 compared to previous 1981-1990. The other two effects depend solely on processes' intensities in surveyed period. That task is clearly presented in the figure showing that ultimate population by 31.12.2000 results from the initial population of 31.12.1990 only for the ages over the age of 10 years. The two initial age groups of 0-5 and 5-10 years have no relation with population by 31.12.1990 as they result from the live births in 1991-2000. Therefore, influence of natality, live births or age-specific fertility rates of women on ageing need to be measured through those indicators' changes in the surveyed period compared to another past period. Those three effects are estimated through the cohort's approach in demographic analysis. They are comparable since deaths and net migration result from the same initial population of a certain age and are reflected in the same ultimate population of that age.

The second task of factor analysis begins with an assessment of the first effect $\Delta\bar{X}_N$. It is determined by intensities' changes in the three processes, i.e. of the analysis' first task. The total effect of those changes was determined through $\Delta\bar{X}_{r2c2}$ = $\bar{X}_{r_2} - \bar{X}_{c_2} = \sum_{i=1}^{19}\left(C_i - \bar{X}_0\right)\times\left(S_{r2i} - S_{c2i}\right)$. Furthermore, it can be divided in three independent effects: $\Delta\bar{X}_{N0}$ - only due to the change of live births, $\Delta\bar{X}_{drc}$ - only due to the changes of deaths by age; $\Delta\bar{X}_{mgrc}$ - only due to the changes of net migration by age. The three effects is measured by the following differences: $\Delta P_i = P_{r2i} - P_{c2i}$ or the differences by age between the real and conventional population by 31.12.2000; $\Delta N_i = \left(N_{r_1} + N_{r_2}\right) - \left(N_{c_1} + N_{c_2}\right)$, or the differences between real and conventional live births for the two five-year periods 1991-1995 and 1996-2000; $\Delta D_{rci} = D_{ri} - D_{ci} = \left(D_{r1i} + D_{r2i}\right) - \left(D_{c1i} + D_{c2i}\right)$, or differences by age between real and conventional deaths in the two five-year periods; $\Delta MG_{rci} = MG_{ri} - MG_{ci} = \left(MG_{r1i} + MG_{r2i}\right) - \left(MG_{c1i} + MG_{c2i}\right)$ or the differences by age between real and conventional migration growths in the two five-year periods.

The relation between all differences is $\Delta P_i = \Delta N_i + \Delta D_i + \Delta MG_i$. Through it the joined structure difference $\Delta S_i = \left(S_{r2i} - S_{c2i}\right)$ is subdivided proportionally to the following factor differences with their algebraic expressions: ΔS_{Ni}, due only to the changes in live births, ΔS_{di} - only to the changes in deaths and ΔS_{mgi} - only to the changes in net migration by age. The connection between structural differences is simmilar: $\Delta S_i = \Delta S_{Ni} + \Delta S_{di} + \Delta S_{mgi}$. Then, applying common methods for measurement of ageing, the three effects are estimated: $\Delta\bar{X}_{N0} = \sum_{i=1}^{19}\left(C_i - \bar{X}_0\right)\Delta S_{Ni}$, $\Delta\bar{X}_{drc} = \sum_{i=1}^{19}\left(C_i - \bar{X}_0\right)\Delta S_{di}$ and $\Delta\bar{X}_{mgrc} = \sum_{i=1}^{19}\left(C_i - \bar{X}_0\right)\Delta S_{mgi}$. For the ultimate solution of the second problem, however, only the effect $\Delta\bar{X}_{N0}$ is needed. It is determined from the sum of the first two age intervals 0-5 and 5-10: $\Delta\overline{X}_{N0} = \sum_{i=1}^{2}\left(C_i - \overline{X}_0\right)\Delta S_{Ni}$,

where $\Delta S_{Ni} = \frac{\Delta Nj}{\Delta P_i} \times \Delta S_i$ at i=1, j=2; i=2, j=1.

Then, separate influences of deaths and net migration in surveyed period 1991-2000 have to be estimated. At that stage, they are not separately measured as there is also a joined effect of the three processes' influence. Therefore, first the effect of the total influence of deaths and net migration is calculated and then the joined effect is estimated. For the total influence, new population by sex and age $P_{1i}^{'}$ and $P_{2i}^{'}$ by 31.12.1995 and 31.12.2000 is calculated. It is conventional only for the first two age intervals 0-5 and 5-10 years, as the two age groups $P_{c21}^{'}$ and $P_{c22}^{'}$ result from the conventional live births N_{c2} and N_{c1} at preserved age-specific fertility rates of women of the previous period 1981-1990 and mortality and migration intensities by age in the surveyed period 1991-2000. For the rest age intervals of over 10 years, real populations P_{r1i} and P_{r2i} by 31.12. 1995 and 31.12.2000 are used. In these circumstances, observed population does not reflect changes in live births, as age-specific fertility rates from the past period remain intact but it does reflect simultaneous influences of deaths and net migration by age in the surveyed period. From there, the respective age structure $S_{2i}^{'}$ results. Then, the new conventional mean age $\overline{X}_2^{'} = \sum_{i=1}^{2} C_i S_{2i}^{'} + \sum_{i=3}^{19} C_i S_{2i}^{'}$ is calculated, where for the first two age intervals 0-5 and 5-10, the relative shares $S_{21}^{'}$ and $S_{22}^{'}$ result from the first two age groups $P_{21}^{'}$ and $P_{22}^{'}$. For the rest age intervals of 10 years and over (i-3,4,...,19), the relative shares $S_{2i}^{'}$ are estimated from the real population by age P_{r2i} by 31.12.2000 compared to the total population $P_2^{'} = \sum_{i=1}^{19} P_{2i}^{'}$.

The effect of deaths and net migration net influence in the surveyed period is found from the difference between the new mean age $\overline{X}_2^{'}$ and mean age $\overline{X}_0$ of initial population by 31.12.1990.

$$\Delta\overline{X}_{dmg0} = \overline{X}_2^{'} - \overline{X}_0 = \sum_{i=1}^{19}\left(C_i - \overline{X}_0\right)\left(S_{2i}^{'} - S_{0i}\right).$$

Aside the effects $\Delta\overline{X}_{N0}$ and $\Delta\overline{X}_{dmg0}$ there is also a joined effect from the simultaneous influences of the three processes. It is determined by the difference

$$\Delta\overline{X}_{ndmg} = \Delta\overline{X}_{r20} - \Delta\overline{X}_{N0} - \Delta\overline{X}_{dmg0}.$$

At the next stage, the joined effect can be divided between the separate influences of the three processes. First, it is divided proportionally to the two net effects: $\Delta\overline{X}_{N0}$ - only from the change of live births, and $\Delta\overline{X}_{dmg0}$ - only from total influence of deaths and net migration in surveyed period. Additional ageing of the change in live births is $\Delta\overline{X}_N^{'} = \dfrac{\Delta\overline{X}_{N0} \times \Delta\overline{X}_{ndmg}}{\Delta\overline{X}_{N0} + \Delta\overline{X}_{dmg0}}$, while the additional ageing of deaths and net migration $\Delta\overline{X}_{dmg}^{'} = \dfrac{\Delta\overline{X}_{dmg0} \times \Delta\overline{X}_{ndmg}}{\Delta X_{N0} + \Delta\overline{X}_{dmg0}}$.

The total or gross ageing from the change in live births is the sum $\Delta\overline{X}_N = \Delta\overline{X}_{N0} + \Delta\overline{X}_N^{'}$. The other ultimate effect of the total (gross) influence of deaths and net migration is the sum $\Delta\overline{X}_{dmg} = \Delta\overline{X}_{dmg0} + \Delta\overline{X}_{dmg}^{'}$

With the additional effect $\Delta\overline{X}_{dmg}^{'}$ conventional mean age $\overline{X}_2^{'}$ grows, reflecting the total net influence of deaths and net migration on ageing in the surveyed period. The new mean age $\overline{X}_2^{''} = \overline{X}_2^{'} + \Delta\overline{X}_{dmg}$ reflects the gross influence of deaths and net migration on ageing.

The ultimate solution of the problem is found through distribution of the total (gross) influence of deaths and net migration $\Delta\overline{X}_{dmg}$ of two effects. The first $\Delta\overline{X}_d$ is of deaths' total influence, while the second $\overline{X}_{mg}$ - from the bulk influence of net migration. The two effects are due to different total number of deaths and crude growth of total population, as well as to the different intensities of mortality and net migration by age. For that aim, the positive effect of deaths and net migration additional influences $\Delta\overline{X}_{dmg}^{'}$ is distributed between positive age effects of ageing, stated with the results $\left(C_i - \overline{X}_0\right)\times\left(S_{2i}^{'} - S_{oi}\right) > 0$. From the resulting growths of teose effects, the respective structure differences $\Delta S_i^{'} = \left(S_{2i}^{'} - S_{0i}\right)$ grow too. The sum of increased negative structure differences at $\left(C_i - \overline{X}\right) < 0$ must be equal to the sum of increased positive structure differences at $\left(C_i - \overline{X}_0\right) > 0$. From those new increased structure differences $\Delta S_i^{''} = \left(S_{2i}^{''} - S_{0i}\right)$, new diminished relative shares $S_{2i}^{''}$ are estimated at $\left(C_i - \overline{X}_0\right) < 0$ and new increased relative shares $S_{2i}^{''}$ at $\left(C_i - \overline{X}_0\right) > 0$. According to that conditional sum of decreases of new relative shares at $\left(C_i - \overline{X}_0\right) < 0$ must be equal to the sum of yincreases of the new shares at $\left(C_i - \overline{X}_0\right) > 0$.

Then, the differences $\Delta P_i^{'} = P_{2i}^{'} - P_{0i}$, $D_i^{'} = D_{1i}^{'} + D_{2i}^{'}$ and $MG_i^{'} = MG_{1i}^{'} + MG_{2i}^{'}$ are presented. As we said, the two age groups $P_{21}^{'}$ and $P_{22}^{'}$ for the first two age intervals 0-5 and 5-10 years are derived from conventional live births N_{c2} and N_{c1} with mortality and net migration intensities in the two five-year periods 1991-1995 and 1996-2000. Similarly, the number of deaths $D_1^{'}$ and $D_2^{'}$ and net migration $MG_1^{'}$ and $MG_2^{'}$ in the two age intervals are conventional, since they result from conventional live births N_{c2} and N_{c1}, even though they are measured by mortality and net migration intensities for the two periods 1991-1995 and 1996-2000. For the rest ages of over 10 years the population $P_{2i}^{'}$ is the real P_{r2i} by 31.12.2000. For those ages, deaths and net migration are real figures as they are derived from the initial population P_{0i} by 31.12.1990 from the two processes' intensities in the surveyed period. Here the relation between populations and demographic events is $\Delta P_i^{'} = D_i^{'} + MG_i^{'}$. From it, structural differences $\Delta S_i^{''}$ are fairly distributed in two factor differences: $\Delta S_{di}^{''} = \frac{D_i^{'}}{\Delta P_i^{'}} \times \Delta S_i^{''}$ only from the influence of deaths by age and $\Delta S_{mgi}^{''} = \frac{MG_i^{'}}{\Delta P_i^{'}} \times \Delta S_i^{''}$ only from influence of net migration by age. From the resulting factor differences ultimate age effects $\left(C_i - \overline{X}_0\right) \times \Delta S_{di}^{''}$ only from influence of deaths and $\left(C_i - \overline{X}_0\right) \times \Delta S_{mgi}^{''}$ only from influence of net migration are estimated. The two joined effects for all ages are the respective sums:

$$\sum_{i=1}^{19}\left(C_i - \overline{X}_0\right) \times \Delta S_{di}^{''} \text{ and } \sum_{i=1}^{19}\left(C_i - \overline{X}_0\right) \times \Delta S_{mgi}^{''} .$$

Those effects equal the two factor differences $\Delta\overline{X}_d$ and $\Delta\overline{X}_{mg}$. The first one is $\Delta\overline{X}_d = \left(\overline{X}_2^{'''} - \overline{X}_0\right)$, where conventional mean $\overline{X}_2^{'''}$ is derived from structural differences $\Delta S_{di}^{''}$, while the second difference is $\Delta\overline{X}_{mg} = \left(\overline{X}_2^{IV} - \overline{X}_0\right)$, where conventional mean $\overline{X}_2^{IV}$ is derived through other structure differences $\Delta S_{mgi}^{''}$. Here we cannot represent analytically the two averages $\overline{X}_2^{'''}$ and $\overline{X}_2^{IV}$ due to space constrictions. Finally, according to the second problem's solution, total ageing is represented with an algebraic sum of the three effects - $\Delta\overline{X}_{r20} = \Delta\overline{X}_N + \Delta\overline{X}_d + \Delta\overline{X}_{mg}$.

Applications of the Complex Model and Analysis of the Results

The suggested methodology has it that net ageing of men amounts to $\Delta\overline{X}_{r20} = 2,28$ years, while net ageing of women is $\Delta\overline{X}_{r20} = 3,01$ years.[8] Stronger ageing of women is due to their survivability, evidenced from the analysis' results for broad age intervals in Table 1.

Table 1

Ageing of population by sex and major age groups in 1990 - 2000 (in years)

Gender	0 – 15 years	15 - 65 years	65+ years	Total
Men	+1.42	- 0.08	+ 0.94	+ 2.28
Women	+ 1.49	+ 0.08	+ 1.44	+ 3.01

According to Table 1, ageing is most prominent in the youngest ages 0-15 years, respectively 1.42 years for men and 1.49 for women. That phenomenon results

[8] For precision, all results show second decimal figure.

from the significant decline of live births – for men it amounts to 608 365 boys in previous period 1981-1990 of 395 310 in 1991-2000, while for women respectively, 576 083 of 373 187 girls. For the next largest age interval - 15-65 years, for men there is a slight rejuvenating by 0.08 years, while for women there is visible ageing. Even more notable is the difference by sex in the last interval of oldest ages, where ageing of men is by 0.94 years, while that of women is by 1.44 years. Interesting result for rejuvenating of men in the middle interval of 15-65 years is visible. Although small, it is objective, as it is confirmed by the two average ages of men in that interval by 31.12. 2000 and 31.12.1990. They are respectively $r2\bar{X}_{15-65} = 38,95$ years and $_0\bar{X}_{15-65} = 39,14$ years, therefore their difference of $\Delta\bar{X}_{15-65} = 38,95 - 39,14 = -0,19$ years demonstrates even stronger rejuvenating. That result is not accurate, however, as it represents a difference of two averaged distributions by age only in the 15-65 years interval. The suggested methodology measures rejuvenating more precisely, as it accounts the changes in relative shares of all ages. Meanwhile, if only the difference between joined relative shares of men in the reviewed interval is taken into account, as the usual practice is, the result is an insignificant but positive number 0.02, signifying ageing. In women, slight ageing in the 15-65 years interval by 0.08 years is evidenced by slightly stronger ageing from the difference between their average ages: $\Delta\bar{X}_{15-65} = 39,81 - 39,69 = 0,12$ years. Thus, objectivity and sensitivity of the applied methodology is proved again.

The effects of net ageing in Table 1 represent summary results of the effects at preserved intensities of three processes and at their changes. Separate results for them are represented of Table 2.

Table 2

Ageing of population by sex and broad age groups at preserved and changed intensities of demographic processes in 1991-2000 (in years)

Broad age groups	At preserved intensities of processes from 1981-1990		From changes in intensities of processes in 1991-2000		Total ageing in 1990-2000	
	men	women	men	women	men	women
0-15	+0.35	+0.43	+1.07	+1.06	+1.42	+1.49
15-65	- 0.02	+0.03	- 0.06	+0.05	- 0.08	+0.08
65 +	+0.82	+1.02	+0.12	+0.42	+0.94	+1.44
Total	+1.15	+1.48	+1.13	+1.53	+2.28	+3.01

Effects of net ageing at preserved intensities of three processes of the 1981-1990 period are shown in the last line of Table 2. For men it is $\Delta\bar{X}_{C20} = \bar{X}_{C2} - \bar{X}_0 = 37,88 - 36,73 = 1,15$ years, while for women that ageing is stronger because $\Delta\overline{X}_{c20} = 40{,}12 - 38{,}64 = 1{,}48$ years. Apparently, there is ageing only from those processes' intensities from the previous period as well. On the next stage, broad age intervals analysis ascertains that ageing is strongest in the last interval for older ages of 65 and over, lesser in the first interval of childhood ages under 15 and insignificant for women by 0.03 years in the largest mean interval. For men in the same interval, a slight rejuvenating is evident, of only 0.02 years. Whether that rejuvenating is real can be certified with the known difference between two average ages of men for the same interval in the end and beginning of surveyed period. First, in preserved intensities of the previous period $\Delta\overline{X}_{15-65} = c2\overline{X}_{15-65} -_0 \overline{X}_{15-65} = 39{,}1395 - 39{,}1398 = -0{,}0003$ years. In fact, the difference does not show change, since it appears only in the fourth decimal symbol. As an inaccurate indicator, however, it can speak of a slight rejuvenating. In its turn, the difference between relative shares of men in average age interval is very small too – 0.6630-0.6711= -00083 and demonstrates rejuvenating.

The next ageing, presented in Table 2, reflects changes of processes' intensities in 1991-2000 compared to the previous period 1981-1990. The bulk effect of that ageing on men is estimated by $\Delta\overline{X}_{r2c2} = \overline{X}_{r2} - \overline{X}_{c2} = 39{,}01 - 37{,}88 = 1{,}13$ years, while for women that difference is 41.65 – 40.12=1.53 years. In fact, those results are almost equal to the results of processes' preserved intensities notwithstanding of the slightly stronger ageing of women at changed intensities.

In preserved intensities of processes and their changes alike, ageing of women is stronger in the broad age intervals. However, for the difference of effects at preserved intensities, ageing is most prominent in the first age interval of 0-15. It also determines the higher levels of ageing for two genders in the same interval. The main reason for that ageing is the abovementioned prominent decline of live births during the surveyed period 1991-2000 compared to the previous period 1981-1990. In the middle largest interval of 15-65 years, ageing of women is least significant. Together with the even smaller ageing at preserved intensities of processes, a slight net ageing of women in that interval. Opposite to women, men show some rejuvenating in the mean age interval by 0.06 years. It is confirmed from the difference between their average ages $\Delta\overline{X}_{15-65} = r2\overline{X}_{15-65} - c2\overline{X}_{15-65} = 38{,}95 - 39{,}14 = -0{,}19$ years. There are two main reasons for that even small rejuvenating but they won't be presented with data due to space constrictions. The first one is that mortality intensities in the period 1991-2000 are not higher than those of negative net migration. Unlike the initial period, 1981-1985 is marked by slight but positive net migration in most ages of the 15-65 years interval. The second reason is that in middle and older ages of that interval, mortality has increased in 1991-2000 compared to 1981-1990. That increase leads to decline of relative shares in observed ages and therefore to a slight diminishing of average ages or rejuvenating. If only the known difference between relative shares of men 0.6926 – 0.6630 = 0.0296 is used, it demonstrates a slight ageing. In conclusion, those comparisons verify the conclusion that the applied methodology is more precise and accurate than the other methods.

The figures in Table 2 represent summarized results from the final solution of the first problem of ageing. Table 3 presents summarized results of the second problem's solution, i.e. the final objective of factor analysis. It also presents the influenced effects by sex of the three factors of ageing in the surveyed period (1991-2000).

Table 3

Population ageing by sex due to changes in live births and different intensities of mortality and emigration by age in 1990-2000 (in years)

Sex	Factors			Total ageing
	Changes in live births	Deaths by age	Net migration by age	
Men	1.31	0.61	0.36	2.28
Women	1.26	1.12	0.63	3.01

From Table 3 it is visible the first effect of live births' changes in the period 1991-2000 compared to 1981-1990. In men it is slightly stronger than the one for women. The crucial effects are derived from the following data on live births and their changes in the two reviewed periods:

For men, real live births in 1991-2000 are $N_1 = 216284$ and $N_2 = 179026$ in 1996-2000 (see the Figure). The latter number demonstrates a strong decline of live births in the second period. Even stronger is the decline if real live births compared to conventional live births for those periods at preserved age-specific fertility rates of women from the previous two periods 1981-1985 and 1986-1990. Conventional live births of boys are $N_{c1} = 294170$ and $N_{c2} = 275615$ (see the Figure). The difference between real and conventional live births boys for the first period 1991-1995 is $\Delta N_1 = 216284 - 294170 = -77886$ less live births, while for the second period 1996-2000 it is $\Delta N_2 = 179026 - 275615 = -96589$ diminished live births only due to the changes of age-specific fertility rate of women. Those changes have led to significant decline in the real number of boys compared to their conventional number in the first two age intervals of 0-5 and 5-10 years by 31.12.2000. Respective differences between their real and conventional number by 31.12.2000 are $\Delta P_1 = P_{r21} - P_{c21} = 166086 - 256437 = -90351$ diminished boys of age 0-5 years and $\Delta P_2 = P_{r22} - P_{c22} = 197693 - 271717 = -74024$ of age 5-10. Respective structure differences for the two age intervals 0-5 and 5-10 years are $\Delta S_1 = S_{r21} - S_{c21} = 0,0429 - 0,0632 = -0,0203$ and $\Delta S_2 = S_{r22} - S_{c22} = 0,0511 - 0,0670 = -0,0159$. From them and the figures for ΔN_j and ΔP_i, factor structure differences ΔS_{Ni} only due to the changes in live births are calculated. More specifically, for the first age interval 0-5 years we have $\Delta S_{N1} = \frac{\Delta N_2}{\Delta P_1} \times \Delta S_1 = \frac{-96589}{-90351} \times (-0,0203) = -0,0217$, while for the second 5-10 years - $\Delta S_{N2} = \frac{\Delta N_1}{\Delta P_2} \times \Delta S_2 = \frac{-77886}{-74024} \times (-0,0159) = -0,0167$. From the calculated factor differences the first net effect of men's ageing is estimated only from the changes in live births:

$$\Delta \overline{X}_{N0} = (2,5 - 36,73) \times (-0,0217) + (7,5 - 36,73) \times (-0,0167) = 1,23 \text{ years.}$$

Similarly, the effect $\Delta \overline{X}_{N0}$ for women is determined. The number of real live born girls in 1991-1995 is $N_{r1} = 204569$ and $N_{r2} = 168618$ in 1996-2000. Similarly to boys, a definite decline in the number of real live born girls in the second five-year period can be observed. Respective conventional live born girls are $N_{c1} = 277825$ and $N_{c2} = 261728$, therefore $\Delta N_1 = N_{r1} - N_{c1} = -73256$ and $\Delta N_2 = N_{r2} - N_{c2} = -93110$ less live born girls only from the changes in age-specific fertility rates of women. Differences between number of real and conventional girls by 31.12.2000 for the first two age intervals are $\Delta P_1 = P_{r21} - P_{c21} = 157122 - 244395 = -87273$ diminished girls in the interval 0-5 years and $\Delta P_2 = 187876 - 257118 = -69242$ diminished girls in the next interval 5-10 years. Structural differences for the two age intervals are $\Delta S_1 = 0,0386 - 0,0575 = -0,0189$ and $\Delta S_2 = 0,0461 - 0,0605 = -0,0144$. Respective factor structure differences only due to the changes in live births girls are $\Delta S_{N1} = \frac{-93110}{-87273} \times (-0,0189) = -0,0202$ for the first age interval - 0-5 years, and $\Delta S_{N2} = \frac{-73256}{-69242} \times (-0,0144) = -0,0152$ for the second one - 5-10 years. Therefore, the net effect $\Delta \overline{X}_{N0}$ on girls' ageing only due to changes in live births is $\Delta \overline{X}_{N0} = (2,5 - 38,64) \times (-0,0202) + (7,5 - 38,64) \times (-0,0152) = 1,21$ years. or net effect $\Delta \overline{X}_{N0}$ on girls is almost equal to the effect on boys.

The next effect $\Delta\bar{X}_{dmgo}$ presents the total influence of deaths and net migration in the surveyed period without the joined influenced effect of all three processes. For men we have $\Delta\bar{X}_{dmgo} = \bar{X}_2^{'} - \bar{X}_0 = 37{,}65 - 36{,}73 = 0{,}92$ years of ageing. Then, the joint effect is $\Delta\bar{X}_{ndmg} = \Delta\bar{X}_{r20} - \Delta\bar{X}_{N0} - \Delta\bar{X}_{dmg0} = 2{,}28 - 1{,}23 - 0{,}92 = 0{,}13$ years of ageing. That joint result is proportionally distributed in the two net effects: $\Delta\bar{X}_{N0} = 1{,}23$ years ageing only due to the change in live born boys and $\Delta\bar{X}_{dmgo} = 0{,}92$ - only due to total influence of deaths and net migration of men in the surveyed period 1991-2000. From that data, additional ageing of men due to the change in live births is $\Delta\bar{X}_N^{'} = \frac{1{,}23 \times 0{,}13}{1{,}23 + 0{,}92} = \frac{0{,}16}{2{,}15} = \approx 0{,}08$ years of ageing. The other additional ageing due to deaths and net migration is $\Delta\bar{X}_{dmg}^{'} = \frac{0{,}92 \times 0{,}13}{1{,}23 + 0{,}92} = \frac{0{,}12}{2{,}15} \approx 0{,}05$ years of ageing. Therefore, the total (gross) ageing of men due to change in live births is $\Delta\bar{X}_N = \Delta\bar{X}_{N0} + \Delta\bar{X}_N^{'} = 1{,}23 + 0{,}08 = 1{,}31$ years (see Table 3). The other ultimate effect of deaths and net migration net (gross) influence in the surveyed period is $\Delta\bar{X}_{dmg} = \Delta\bar{X}_{dmgo} + \Delta\bar{X}_{dmg}^{'} = 0{,}92 + 0{,}05 = 0{,}97$ years of ageing. Further, the positive effect of the additional influence of deaths and net migration $\Delta\bar{X}_{dmg}^{'} = 0{,}05$ years (more precisely 0.054264 years) is distributed between positive age effects of ageing $\left(C_i - \bar{X}_0\right) \times \left(S_{2i}^{'} - S_{oi}\right) > 0$. Through those growing effects the increased structure differences $\Delta S_i^{'} = \left(S_{2i}^{'} - S_{oi}\right)$ are estimated, that turn into structural differences $\Delta S_i^{''} = \left(S_{2i}^{''} - S_{oi}\right)$. Then, the ascertained differences between the conventional population $P_{2i}^{'}$ and the real population P_{oi} of men or $\Delta P_i^{'} = P_{2i}^{'} - P_{oi}$ are presented as algebraic sums of the respective deaths $D_i^{'}$ and migration growths $MG_i^{'}$ by age, or $\Delta P_i^{'} = D_i^{'} + MG_i^{'}$. From them the structural differences $\Delta S_i^{''}$ are subdivided in two factor differences $\Delta S_{di}^{''} = \frac{D_i^{'}}{\Delta P_i^{'}} \times \Delta S_i^{''}$ and

$\Delta S^{"}_{mgi} = \frac{MG^{"}_{i}}{\Delta P^{'}_{i}} \times \Delta S^{"}_{i}$. With the known expressions $\left(C_i - \overline{X}_0\right) \times \Delta S^{"}_{di}$ and $\left(C_i - \overline{X}_0\right) \times \Delta S^{"}_{mgi}$ age and total effects only from influence of deaths and only from influence of net migration in the surveyed period are estimated. For men, the total effect of deaths' influence is $\Delta\overline{X}_d = 0{,}61$ years of ageing, while the effect of net migration influence is smaller since $\Delta\overline{X}_{mg} = 0{,}36$ years. The two results are presented in Table 3. The sum of three factor effects equals the total ageing of 2.28 years.

Similarly, after going through all described steps, the result for women is $\Delta\overline{X}_d = 1{,}12$ years ageing, which is larger than the ageing of men. Stronger is the effect of women's ageing only from net migration because $\Delta\overline{X}_{mg} = 0{,}63$ years. The sum of the three factor effects is also equal of total ageing of women 3.01 years (see Table 3). Notably, the influence of deaths on ageing is stronger than that of net migration in both genders. Therefore, we can conclude that the analysis through based on comparison between real population in the beginning and the end of the surveyed period is more accurate than the one based on pre-determined hipothetical populations used in the other models. Stronger influence of deaths can be explained with the fact that their total number for men during the whole surveyed period 1991-2000 is 621 948, while the total loss of men from emigration is only 175 092 or deaths outnumber over 3.5 times the total migration balance. Another attribute of those two different influences is that number of deaths for the two distinct five-year periods is almost the same - respectively 306 294 men in 1991-1995 and 315 654 in 1996-2000. Unlike, negative migration balance of men in 1991-1995 is –129 796 and only –45 296 for 1996-2000. Those figures show lower migration in the second period 1996-2000, amounting to about - 9000 men per year compared to approximately - 26 000 in the first period 1991-1995.

Similar differences between deaths and net migration can be observed in women as well. The number of women's deaths for the whole period 1991-2000 is 516 451, while their negative migration balance is only –184 659 or approximately 2.8 times less than deaths. Compared to the negative balance of men, the balance of women is bigger by –9567 and demonstrates slightly larger migration in the surveyed period. Probably following the higher male migration in the previous period 1986-1990 with emigration balance of –162 084 men compared to –156 192 women the latter have emigrated mostly after men and therefore have slightly larger migration rates in the following periods. In spite of that relation between women and men, however, there is probably a stronger tendency of independent movement of women. That statement is confirmed from the data of female migration in the two periods - 1991-1995 and 1996-2000. The emigration balance for the first period is –137 605 women and only –47 054 women for the second one, or annual average of about –27 500 and –9400 women. Those numbers are slightly larger than the figures for men. In conclusion, we can add that the complex model can be applied for measurement and analysis of ageing not only of the total population but also of any groups such as economically active, workforce, employed, ethnic minority groups, etc. measurement and analysis of the influences of separate demographic processes on ageing in population prognosis, especially the effects of future migration policies. The method can be applied on macro level and locally alike. For example, without the influence of live births it can be applied on micro-level for measuring the ageing of company human resources. In that case the analysis takes into account only the influence of growths (recruited – relinquished) and the deaths of employed by age.

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