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COMPLETE MARKETS OF ARROW AND DEBREU AND THE DYNAMIC DISEQUILIBRIUM

This article discusses the two main models of complete markets, the criticism to them that form the early Arrow-Debreu theory, as well as the supplements and the shortcomings of the so-called revised version of the theory. We underline the most important consequences from this theory and present generalized comments.

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Theory of intertemporal equilibrium, more familiar as theory of complete markets, generalizes analyses of economic equilibrium under uncertain economic conditions of K. Arrow (1953) and Gerard Debreu (1959) and the following criticism and additions to the models deduced by them. With spot markets closed in the first phase and comprehensive trade transactions these theory is a good starting point for analysis of structural components of dynamic disequilibrium. On the one hand it describes the dynamic equilibrium path around which equilibrium trajectories converge, and on the other it depicts the transition to the model of sequent markets and those of different long term contracts.

Usually the uncertainty in economy is due to random processes in the surrounding physical world, provoked from the changes in the economic fundamentals like endowments, technologies, and preferences. Following the terminology of contemporary theory of sunspot equilibrium,[1] uncertainty, which is associated with objective probability for occurrence of a particular state of economic nature, is named intrinsic uncertainty. But even when the fundamental parameters were not random variables, the economic process can generate random results. The economy is a social system, and economic agents don't always dispose of certain information about the behavior of other participants (Shell, 2007, p. 1.). The lack of coordination between individual plans leads to uncertainty for which the term extrinsic or behavioral uncertainty is accepted.[2]

To incorporate time in the static equilibrium the theory of Arrow and Debreu considers the intrinsic uncertainty, accepting implicitly, that the extrinsic uncertainty is not revealed. Both authors analyzed the problems, related to time and uncertainty, on the specific institutional base, that was named by Hicks (1939) futures economy. In a similar way as in the productive model of Walras the perfect foresight traders contract with the markets. The markets work only in the initial trade phase, t=0, therefore the traders sign contracts to deliver commodities at

[1] In this theory the sunset equilibrium of distribution of resources doesn't depend on the economic fundamentals, but it is an outcome of coordinated decisions of agents based on signals by external uncertain variable (Cass and Shell, 1983, p. 194).

[2] The term market uncertainty is also used.

each and every future phase, t>0. The specification (description) of the commodity to be delivered includes not only its physical characteristics, but also the place and conditions (state of nature) of the delivery. By the way, this is the reason for the difference between the price of one good, traded in January in Sofia, and the price of another good, traded in July in Varna. The futures economy guarantees the carrying out of future transactions through the construction of complete (from where comes the name of this theory) set of insurance and futures markets (Debreu, 1959). It settles the whole uncertainty in the economy.

However, as claimed by Grandmont, a similar structure conceals the time (1987). If in the initial moment the equilibrium is achieved, production and sale are ordered consistently as in a calendar schedule. Because the coordination of decisions is reached in the initial phase through the complete set of insurance and futures markets, the existence of a set of sequential developing in time markets is not assumed, and the expectations, money, financial assets as well as share markets, don't play a significant role. Moreover, the complete system of insurance and futures markets can assure a defense against all unforeseeable events on the market, only if it is enough complex, detailed and precise (Arrow, 1965).[3]

Although the theory of complete markets is designed as analytic instrument of competitive equilibrium, the aspects of efficiency, related to contingent trade, highlight the role of long tem contracts in the insurance of risk. The comprehensive contract Arrow-Debreu shows how the trade agreement would look like in an ideal and a foreseeable world, and it is a prototype model, with which the other contract models are compared.

Presenting the economy as a combination from spot markets and long-term contracts, the theory Arrow-Debreu don't describe only the intertemporal equilibrium or the dynamic equilibrium path. It suggests the most realistic scenario for dynamic development of economy, based on the concept of dynamic disequilibrium.

The criticism to the early theory resulted in the appearance of the rational expectations concept, and the criticism to the revised version motivated the differentiation of the theories of sequence markets. The impossibility for insurance of extrinsic uncertainty sheds light on the role of long-term contracts in the dynamic trade relations.

The framework of theory Arrow-Debreu presented here is borrowed from Guth (1994).

Essence and characteristics of the main Arrow-Debreu models

In spite of its flaws, the theory of Arrow and Debreu (1953, 1959) remains the main instrument in the process of modeling of economic decision under

[3] According to Arrow (1965) the phenomenon of a moral hazard is another serious challenge for construction of a complete system from the insurance market.

conditions of disequilibrium[4]. Uncertainty still is presented as disclosure of one state from the set of states of nature, although the interpretation of decision making by traders is changed repeatedly.

Arrow and Debreu used (Debreu extend the interferences of Arrow for longer periods of time) two main models for dynamic equilibrium: (1) Model with contingent (on the state of nature) claims; (2) Model with market securities. In the period, after 1975, in ambition to give an answer of the unceasing criticism in economic literature, the followers of Arrow and Debreu changed the basic idea of these models, incorporating the prices on the future spot markets in the states of nature. However, this new revised version inspired additional questions and contradictive assessments.

The concept of complete markets follows the two fundamental theorems for welfare economics: (1) In the absence of externalities, when the consumers and producers functions are not convex (so called non-convexity of economic activity), each competitive equilibrium is a Pareto optimal one; (2) Under appropriate redistribution, even if some reservations, each Pareto optimal distribution can be realized as a competitive equilibrium. These two theorems, equalizing the competitive equilibrium and optimal distribution of Pareto, indeed characterize a completely certain economy, and prompted how the perfect foresight traders have to overcome the revealing of intrinsic uncertainty.

The model “contingent claims” enabled the economist to define for the first time the interconnection between the optimal Pareto distribution and the competitive equilibrium under conditions of uncertainty. This model, which still extends its analytical territory toward the financial sector, used the theory of expected utility by von Neumann and Morgenstern (1944), substituting subjective for objective probabilities. Although in many aspects the approach of Arrow and Debreu is near to the theory of Savage (1954), both authors point out the interdependence among preferences and the states of nature and not the assessments of subjective probabilities. (The transition between two approaches to the expected utility is obvious). Whereas Arrow (1953) applied additively separable concave functions of expected utility, Debreu (1959) relied on convex set of preordering preferences.

Arrow (1953, 1964)[5] introduced uncertainty in the sphere of pure trade exchange in the following way. In an economy where *I* is the number of individuals, *C* of goods, and *S* of the states of nature, before realizing whatever of states, the individuals must buy and sale X_{SC} contingent claims, each of them giving property right of one unit of good *c,* when the state *s*[6] is occurred. Although in the first statement the state is identified only with the physical conditions, it soon accepts a

[4] For the meaning of this theory points the fact that both authors won the Nobel Prize in Economics.

[5] The first version of the model of Arrow dated from 1953, but a more popular one, reprinted with small corrections, is from 1964.

[6] Except for the “state of nature”, this random variable is also named the “state of world” (Arrow, 1954).

wider interpretation, reflecting other exogenous variables of uncertainty that traders face.

The so-called technical restriction of economy is the requirement for each different good in each possible state of nature, the sum of contingent claims to correspond to the total quantity of this good:

$$\sum_{i=1}^{I} X_{isc} = \overline{X}_{sc}, \forall s, c . \quad (1)$$

For Arrow with complete set of contingent claims, i.e. with one claim for each good in each possible state of nature, the competitive economy under uncertain conditions is similar to pure trade exchange, but with several exceptions. First, in economy with contingent claims the number of transactions is multiplied S times, as are the states of nature. While in a pure economy the number of traded goods are S, in the contingent claims economy the number of trade instruments (as are the contingent claims) is S×C. The second essential difference is that the individual instead of maximizing the utility from consumption of C goods he maximizes the expected utility, which is the sum of products of utility from good *c* in the state *s* and the subjective probability this state to come true. Since the utility from the contract with contingent claims is increasing with decreasing rate, i.e. concave function, the function of expected utility derived these characteristics. More precisely, it is non-decreasing and quasi-concave.

Although the maximization of expected utility at first glance looks like as an unimportant change, namely this change provoked later many contradictions about those aspects of uncertainty, which agents assess with subjective probabilities. The disputes over this question have continued until now.

The sequence of events starts to happen, after the signing of X_{SC} trade contracts with contingent claims. When the competitive equilibrium is achieved, the contracting stops is terminated, the state of nature s* is revealed, and the contingent claims for this state are used. With the given expected utility and the weight of total utility *(w_i)* of each individual *i*, the central planner, maximizing $\sum_{i=1}^{I} w_i V_i$ subject to restriction (1), reaches the optimal allocation X^*_{isc}.

On the other hand, with the set of money income of each individual i (Y_i) and prices for each claim to each unity of good *c* at the state *s*, the quantity X_{isc}, with which the individual maximizes his expected utility subject to constraint:

$$\sum_{s=1}^{S} \sum_{c=1}^{C} \overline{P}_{sc} X_{isc} = Y_i , \quad (2)$$

leads to the same optimal result X^*_{isc}. In this way the competitive equilibrium on the markets with contingent claims achieves the optimal allocation of Pareto. This outcome is well-known as the first theorem of Arrow (1964).

In the second part of his article Arrow (1964) formulates the security version of the model with contingent claims, using financial securities, which are paper claims to money. Previously, before the occurrence of each possible state of the nature *s*, the individuals buy financial securities (one for each state), which further (after occurrence of the state) convert in money to buy goods on the spot markets. So, algebraically the individuals need from S+C contracts (in comparison with S×C in the first model) to achieve competitive equilibrium, that is optimal to Pareto.

What is needed this mechanism to work is to bind the individual income (Y_i) simultaneously with the price of financial security *s* (q_s) and the future spot price (P_{sc}) of good *c* in the state *s*. Thus, the technical constraint is presented as:

(3) $$\sum_{s=1}^{S} q_s Y_{is} = Y_i \text{, and}$$

(4) $$\sum_{s=1}^{S} \sum_{c=1}^{C} P_{sc} X_{isc} = Y_i \text{ .}$$

The question is how to define q_s и P_{sc}? For Arrow the prices of securities must respond to the condition:

(5) $$q_s P_{sc} = \overline{P}_{sc} \text{ ,}$$

where $\overline{P}_{sc}$ are the prices of contingent claims in the first model. It really means that the individuals know the prices $\overline{P}_{sc}$ (!?), which is the guarantee that this version of the model, in a similar way as in the first one, allocates efficiently the goods, i.e. achieves X^*_{isc}.

The efficient allocation of the goods by both modes is the second theorem of Arrow.

The early theory Arrow-Debreu

In the period after 1955 the models of Arrow and Debreu provoked wide and complex wave of analysis of the general economic equilibrium and of the role of contracts with contingent claims.

The uniformity between these first critical publications, that together form the early theory Arrow-Debreu, is, that they accepted the state of nature as describing of one or more joint events about the external environment and the both versions of Arrow's model are absolutely equivalent (Guth, 1994, p.4).

The more notable criticisms and additions to the models of contingent economy belong to Radner (1968, 1970), who analyses the role of spot markets, information, and the learning from market prices, to Starr (1973) and Harris (1978) with their investigation of the relations between *ex ante* и *ex post* optimal allocation and efficiency, to Grossman (1981), who defined the equilibrium under admission of rational expectations, as well as to Coutinho (1986) with his formal models of the

concepts of Grossman. The beginning of the end of this early stage of the theory Arrow-Debreu placed Nagatani (1971) and Arrow (1975) with their attempts to change the definition of economic environment.

In spite of the different interpretations of the inferences of Arrow, the greater part of analysts agreed with the affirmation (known as *lemma*), that at the presence of complete set of markets with contingent claims all transactions would be realized in the first phase of trade relations. So, in spite of opportunities for next profitable transactions, in the absence of new information or a change in preferences or budget constraints, neither trader would demand new dealings, i.e. the next trade phases are useless.

Radner criticizes this lemma, because it works, only when all players ignored the meaning of future spot prices. However, if some participants expect some unknown factor to change the spot prices defined in advance, in the next trade phases they would take positions that will remove the prices from their starting equilibrium values. Even under the assumption that these processes are only temporal, the terms of trade in the next phases will change the initial positions. In this situation for achievement of intertemporal equilibrium the traders must undertake quite "paradoxical" strategies.

Radner (1968, 1970) established that the equilibrium Arrow-Debreu is possible, only when all the individuals have equal access to same information. Analyzing what information is needed in the first trade phase in the model with financial securities, Radner concluded that the prices, paid at the delivery of goods, can be interpreted as spot prices, but it is a mistake to think of these prices as predetermined and projected in time for the dates to which they refer. "To achieve the equilibrium when choosing their individual plans the traders must have access to the whole price system that includes guarantees for the future transactions besides to spot prices. Thus at the beginning all agents shall dispose with a common forecast for all equilibrium spot prices at every future date and event" (Radner, 1970, p.456).

Attempting to break up this framework, Radner (1968) widens the model with contingent claims, adding the assumption that agents dispose with differentiated information. The conclusion of Radner is that if under new conditions the information received is only about economic environment, the equilibrium with contingent claims may be optimal for the information structure chosen. However, if the agents receive information about the behavior of other participants, the generated externalities may change the preferences and in this way may divert the competitive equilibrium from its optimal position.[7] This is indeed the first attempt of Radner to reflect in the theory of Arrow-Debreu behavioral uncertainty. His formal model, nevertheless, presented cases, in which the agents have fixed information

[7] Especially the presumption, that the acquirement of information has a "set-up cost", which depends on the scale of production, proposes non-convexity of the set of production possibilities. Such characteristic contradicts to main assumptions in the optimality theorems.

structures, and only hinted at what may be happen, if someone possibly learned something new from prices or activities of others.

These supplements made by Radner were expressed in his theory of plans, prices and price expectations, which is successive analog of intertemporal equilibrium (1970, 1972). The idea of Radner (1970), that the spot price is the source of information, is borrowed from Hayek (1945) and attracts the attention of the Arrow-Debreu literature after 1975.

According to Radner the second reason for criticism to the models of Arrow and Debreu is the inadequate treatment of money, the stock markets and the activity on the markets in the dynamic perspective. To overcome these omissions Radner (1970, p.458) recommended future additions to the theory Arrow-Debreu to contain the following aspects: (1) There is uncertainty about prices on the future spot markets, which have to be considered equally with intrinsic uncertainty about market environment; (2) The producers would not have uniform and well-founded approach to compare the net revenues in different time interval and under different probability conditions. The stockholders principally are motivated to trade with stocks, because it is an opportunity for them to change the dependence of their future revenues from the states of nature. Instead of selling the shares of some firm, the stockholder may decide to influence the management of the firm and to change the producer plan in accordance with his subjective judgment and attitude to risk. This inference contradicts to the concept of Modigliani-Miller (1958) for neutrality of corporate management in the assessment of the values of assets. As it was affirmed by Jensen and Meckling (1976) the interrelations between the stockholders (principal), manager (agent) and the institutional structure of the firm (corporative governance) acquires a special meaning under asymmetry of information and different subjective expectations of the agents; (3) Consumers face not only one budget (discount) constraint, as it is assumed in the models of Arrow-Debreu, but the sequence of budget constraints, one for each time interval. Having in mind his unclear participation in the future profit of producers, consumer would succeed very hard to assess (discount) the whole his endowment to the initial time interval; (4) For construction of his individual plan the agent must take into consideration future market prices, therefore he anticipates these prices. As nothing may guarantees the correctness of the anticipation, most probably the individual plans would be revised over time; (5) If at some period of time the participants dispose with different information, the equilibrium prices reflect, even in very complicated way, the overall information of traders. This condition suggests, that the participants who can “read” the market process through their observations over prices, will guess at least a part of information of competitors.[8]

[8] With these inferences Radner (1982) generalizes his own analyses of different aspects of the Arrow-Debreu theory. Even under the condition of complete markets, if there is a great differentiation in agent’s information, some markets will be redundant, and no trading on them would be expected (1968); In a model with several time intervals the markets are not complete, and the agents don’t learn something more about economic environment from prices (1972); Even if in a model with two intervals

During the last two decades these generalizations of Radner were ones of the often cited in the economic literature. And this fact is not casual. They open the door for the concepts of rational expectations and complete contracting as alternatives of theory of intertemporal equilibrium and comprehensive contract Arrow-Debreu, as well as for their combining in the first approach of dynamic disequilibrium.

The third critical direction in early theory Arrow-Debreu brings in question the execution of criteria for efficiency. Starr for example looks for an answer of the question: Is the optimal *ex ante* allocation of contingent claims of Arrow a sufficient condition for the resulting (after occurrence of the event) *ex post* allocation of the real goods to be also optimal.[9] When the answer is no, we can talk only about optimal allocation of risk. If, nevertheless, on a later stage some trader would increase (with redistributions) his utility only if other trader will become worse, this is already *ex post* Pareto optimal allocation (Starr, 1973, p. 82).

Starr computed that in the pure trade exchange the necessary and enough condition an economy with contingent claims to achieve *ex post* optimal Pareto allocation is the agents to formulate identical probability distributions about states of nature or the so called "universally similar believes" (1973). In economy with production, however, this condition is not enough. The prices of contingent claims should also be consistent simultaneously with "universally similar believes" and profit maximization of producers. Starr asserts that in both economies for achievement of *ex post* Pareto equilibrium the most important role plays not the information about what state will occur, but the common aspects in expectations of traders (1973, p. 94).

Assuming that *ex post* efficient allocation exists, Harris (1978) tried to define such *ex ante* mechanism for allocation of recourses, which leads to *ex post* optimal equilibrium. He borrowed from Lindahl (1919) the concept for efficient allocation of public goods[10] and introduced the term "personalized price mechanism", for each state of nature which is product from market price of contingent claims and subjective probability this state to occur.

Further, if we assume, that all the states can be assessed with corresponding probability, and functions of utility are concave and strictly monotone (to abstract from the points of saturation) and besides that they are additively separable, it is possible due to personalized price mechanism in pure trade exchange to achieve an *ex post* efficient allocation for a given state *s*, an unique *ex post* efficient allocation for

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Radner investigates what happens, when the agents learn from prices (1979); Radner specifies the definition of equilibrium under different forms of differentiation of information (1979, 1982).

[9] The terms ex ante and ex post are introduced in economics by Gunner Mirdal (1928) who stated: "ex ante are the quantitative measurements of the planned at the beginning of the period actions, and ex post are quantitative measurements of the executed in the end of the period actions" (Dostaler, (1990).

[10] According to Lindhal in the allocation of each public good it is possible to achieve efficient equilibrium, whenever individuals dispose with specific price, corresponding to the received from this good utility.

all the states of nature, and an *ex ante* optimal allocation for each set of probability expectations of consumers (Harris, 1978, p. 430). If we add the conditions, that consumption is strictly positive and the functions of utility are differentiable, nevertheless, unique *ex post* efficient allocation for the markets is available (Guth, 1994, p. 7).

As an extension of last criticism toward economy with contingent claims Arrow-Debreu, Grossmann (1981) defined optimal Pareto allocation under imperfect foresight of agents and different distribution of information between them. In this way he revealed his outlook for the rational expectations equilibrium. With its specific interpretation of intertemporal equilibrium the Grossmann model is a good starting point for presenting the theory of rational expectations from the point of view of dynamic disequilibrium.

The idea of Grossmann is that the equilibrium in such an economy allocates recourses in different scenario, than in this with access of all agents to the whole information available on the market, i.e. with perfect foresight traders. During the time the traders learn how the equilibrium prices depend on the variables of underlying demand. They use this information to revise their consumer plans and to request recontracting of the prices. In the long term, however, the prices will be balanced at a level, which no one will desire to change. The last state is called by Grossmann the rational expectation equilibrium. The most important conclusion from Grossmann's analysis is, that in an economy with asymmetric information the rational expectations equilibrium can coincide with the equilibrium at open access to the whole information, but it is not certain. Grossmann gives a proof, that if the markets Arrow-Debreu are active and complete, in a sense that they embrace the whole space good-state of nature, and if utility functions of traders are additively separable, there are no zones of saturation, they are concave, and are differentiable in each time interval, rational expectation equilibrium exists, which is *ex post* Pareto optimal. For Grossmann this conclusion is: "A remarkable addition to the fundamental theorems of economic of wellbeing under conditions of differentiated information...Although it is possible to appear many other rational expectations equilibrium. (Grossmann, 1981, p. 555).

The reason for the last inference of Grossmann is the differentiation between fully revealing information equilibrium, which coincides with Arrow-Debreu equilibrium, and the partially revealing information equilibrium, when equilibrium prices are not completely transparent (Grossmann, 1977).

Coutinho (1986) extends the model of Grossmann (1981), formalizing rational expectations equilibrium under conditions, adopted by Grossmann. Following the ordinary model with contingent claims, Coutinho generalizes multitude attempts for modeling of rational expectations of traders in theoretic framework Arrow-Debreu. He illustrated example of partial revealing equilibrium, and equilibrium, which is *ex post* Pareto dominated.

In short, Continuo examined economy with contingent claims with two time intervals, two possible states of nature (J=1, 2) – s_1 и s_2, and two consumers

(i=1,2) with the same preferences, whose behavior is described by the utility functions of von Neumann-Morgenstern. The endowments of both consumers e_1 and $e_{2,}$ take values 1 or 0 depending on the state of nature *s*. Consumer *i* receives information signal y_i, and then trades on the complete set of markets with contingent claims Arrow-Debreu. The rational expectations equilibrium is defined by three vectors (one price vector and two vectors of demand of both consumers):

$$(6) \qquad p = \left[p^1(y), p^2(y)\right];\ x_1 = \left[x_1^1(y_1, p), x_1^2(y_1, p)\right];\ x_2 = \left[x_2^1(y_2, p), x_2^2(y_2, p)\right],$$

for values of x_i, which maximize expected utility:

$$(7) \qquad \max\ U\left(x_i^1\right)\Pr\left(s_1 \middle| y_i, p\right) + U\left(x_i^2\right)\Pr\left(s_2 \middle| y_i, p\right),$$

where utility function is increasing with decreasing growth, i.e. $U' > 0, U'' < 0$, and optimization is subject to constraints $px_i \le pe_i$, and $x_1^J + x_2^J \le e_1^J + e_2^J$, for i=1,2 и J=1,2.

Continuo assumed, that the utility functions are of Cobb-Douglas type, i.e. $U\left(x_i^J\right) = \ln x_i^J$, and under this condition "as it is well known the economy Arrow-Debreu has an unique equilibrium position" (Coutinho, 1986, c.884).

Further Coutinho demonstrated, that the price vector p(y)=(1,1), for each *y*, in combination with the demand functions x_1=(1/2,1/2) and x_2=(1/2,1/2) present rational expectations equilibrium, which reveals partially information in the economy. Since the price vector p(y)=(1,1) doesn't bring additional information and can't hint which of both states will occur, the consumers continue to assign probability with $\Pr\left[s_J \middle| y_i, p = (1,1)\right] = 1/2$ for J=1,2 и i=1,2. In this equilibrium all the consumers will insure against the uncertain conditions, allocating their initial endowments in equal parts (in this case the half) in each possible (in this case the both) state of nature. The price vector p(y)=(1,1) will not reveal information, although the economy as a whole receive certain signal for the state of nature. Therefore, even if the sequent markets with contingent claims were opened, no one (consumer) has incentive to change his plans.

In the above example *ex post* Pareto dominated fully revealing equilibrium (Grossmann, 1977) is presented by the price vector p=(1,0), when y_1=y_2, and the price vector p=(0,1), when $y_1 \neq y_2$, as in these equilibriums $\Pr\left[s_J \middle| y_i, p\right] = \Pr\left[s_J \middle| y_1, y_2\right]$, and the demand is equal to supply in each state of nature.

To prove *ex post* Pareto domination of fully transparent equilibrium over partially revealing information equilibrium under the condition of rational expectations, Coutinho assumed that the consumers choose the production technology. This allows them to allocate their initial endowments among three options: (1,0); (0,1); (1/2,1/2). At the price vector (1,1) the consumers will prefer to divide income and risk between two states of nature. Such strategy maximizes the

expected utility and leads to equilibrium. The central planner, nevertheless, may choose production technology in a more effective way. If the probability distribution definitely directs to one of the two states of nature, the central planner will chose a technology, which in the first case leads to endowments e_i=(1,0), and in the second case leads to e_i=(0,1) for both consumers. In both cases the allocation by the central planner dominates over the competitive equilibrium (1/2,1/2).

Interesting detail is that while *ex post* partially revealing equilibrium is dominated by fully transparent equilibrium *ex ante* the roles are exchanged. Partially revealing information equilibrium is Pareto dominated, since for *ex ante* information the price vector, p=(1,1), allocates income in such way, that the consumers insure each other (Coutinho, 1986, p. 884).

Although the conclusions of Grossmann and Coutihno are restricted in the framework of contingent economy Arrow-Debreu, the thesis for fully transparent equilibrium that dominates over all other equilibriums, is completely consistent with the typical for dynamic disequilibrium an equilibrium path and converging about it equilibrium trajectories.

The assumption for additive separable utility functions in the early theory Arrow-Debreu is subject of sharp criticism, because it means zero substitution among commodities. The last one provoked contradictive assessment on the base of the contingent claims models, especially in analysis of speculative behavior. For Guth, “To find a solution of the current modeling handicap may be adopted the concept of Debreu for preference preordering of consumers” (1994, p. 9).[11]

Revised version of theory Arrow-Debreu

More considerable changes in the theory Arrow-Debreu occurred, when Nagatani (1975) set the fundamental question about the way, in which the traders learned future spot prices in security version Arrow-Debreu. As we already demonstrate with equation (5), the product of prices of securities and future spot prices is equal to prices of contingent claims in the first version of the model. If we assume, that in the security model the individuals know $\overline{P}_{sc}$, it means, that they know prices of markets, which don’t exist!

The lack of information about $\overline{P}_{sc}$ in security model of Arrow indeed means uncertainty about future spot markets. Without knowing $\overline{P}_{sc}$, individuals can’t determine P_{sc}. This uncertainty is a source of risk and is a precondition for speculations. When the payments for the securities don’t correspond to the prices

[11] To Debreu preference preordering generalizes the tastes of consumers, their assessment of probability distributions, and also their attitude to the risk (1959, p.101). Disposing with their preference preordering and endowments, the individuals accept prices as given, and choose consumption, which is optimal to their preferences (Guth, 1994, p.9). The assumption of additive separability is preferred, as it simplifies finding of first derivatives.

in a given state of nature, the individuals could prefer inefficient allocation, which can compensate real or perceived risks from price uncertainty.

For Nagatani (1975) in the contingent claims model Arrow-Debreu the traders give up more information than in the security model. In the first model the trader reveals the whole S×C vector X_{isc}, whereas in the second only S vector Y_{is}. In the security model agent knows what income he would have at each state *s*, but when he buys security, he can't envisage what quantity of good *c* he will buy in this state.

Because of this uncertainty, the allocation of the incomes and goods on the markets is not efficient.[12]

Arrow (1975) suggested two possible decisions of this dilemma. The first one is the concept of repetition of so-called identical lotteries. After enough long time period as a result of repetition the individuals learned what prices would prevail in each state of nature. The logic is that all the states of nature were randomly chosen many times, so the individuals can make a comparative assessment of prices for each state. Apparently such a mechanism would work hard, but even when it is a fact, the traders will face the uncertainty from changing preferences and/or other characteristics of market over time.

The second answer is turned to a norm in the successive literature Arrow-Debreu. The prices are defined as a part of states of nature. In this way the states of nature describe all the sources of uncertainty on the markets. Incorporation of future spot markets in the state of nature is the revised version of theory Arrow-Debreu. It raised, nevertheless, new objections.

Predefined space of state of nature is a considerable step back from the initial model of intertemporal equilibrium. According to Radner this creates conditions for appearance of new sources of intrinsic uncertainty about environment, as well as extrinsic uncertainty, related to the behavior of the other participants (Radner, 1968, p. 32). As we have already pointed out, the early theory Arrow-Debreu describes only intrinsic uncertainty. This contradiction, that caused doubts in the idea for construction of complete set of markets and perfect foresight of traders, grows to suggestions for presenting the economy as a sequence of markets, none of them complete in the sense, given by Arrow and Debreu.

Arrow considered that when the prices are part from the state of nature, price uncertainty doesn't exist anymore. But since in the model implicitly is accepted, that the different kinds of uncertainty are exogenous for the economic system variables, and the prices are endogenous ones, complications occur regarding the interpretation of the model (Arrow, 1975, p. 487).

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[12] Nagatani thinks that the allocation of goods is inefficient, since it is impossible to allocate income in efficiently. In contrast to perfect information, in this case some individuals dispose of more money and others with less (Nagatani, 1975, p. 485). Under line Nagatani adds, that when utility functions belong to the type Cobb-Douglas, the allocation of income is as under perfect information. The speculative intentions about future spot prices, however, ordinary lead to suboptimal allocations on the secured markets Arrow-Debreu.

The first problem of the revised version is that the presenting of prices as exogenous variables contradicts to the general economic character of equilibrium Arrow-Debreu. If the focus of attention is directed to the price shocks, and not to shifts in the underlying demand and supply, than we are back in the realm of pre-modern partial equilibrium analysis (Guth, 1994, p.12). We must specify, however, that in the additions of Debreu (1959) the future prices are defined as a function of states of nature, not vise versa.

Second, identified by Nagatani problem of uncertainty about future spot market prices is only one potential source of intrinsic uncertainty, influencing the model Arrow-Debreu. Other possible sources of intrinsic uncertainty are the changing preferences, the changing expectations as a result of receiving new information, the effect of sunspot equilibrium over the general equilibrium and so on. Under so many uncertain factors the construction of complete set of contingent markets would be impossible.

Harris (1978) was the first who noted the problem with the changing preferences in his attempts to clarify *ex post* optimal allocation: "The conflict between *ex ante* and *ex post* Pareto efficiency in intertemporal allocation of recourses under conditions of uncertainty is an example for complications inspired by changing preferences. This problem may have serious consequences for the correct assessment of well-being of society, since significant differences between *ex ante* choice *ex post* preferences are possible. "This problem cast doubt on the validity of the principle of consumer sovereignty[13] as a means for evaluating the allocation of recourses". (Harris, 1978, c.427).

The third problem is widening of the states of nature, so that they can reflect (and eliminate) uncertainty about changing preferences. Suppose it is done, preconditions for development of moral hazard are available. Having in mind that the money, received by individuals from different securities, depends on their preferences, they are ready to change these preferences to receive additional benefit from the securities. They can save part of the payments on securities, they own, or can avoid part of their obligations when they sold the securities.

Arrow claims that the moral hazard is a special case of lack of information, and it comes forward when the insurance company can't delimit the different states of nature (Arrow, 1975, p.463). According to Radner the moral hazard is one of the reasons for the lack of some of contingent claims markets.

The fourth important problem is the effects from behavioral uncertainty. On the sequence of markets in the model of Debreu the agents trading with contingent claims would appraise what state of nature is occurred, only if they can prove the strategies of other participants. Therefore the prices make many short term moves on these markets before acquiring their final values. Radner comments these phenomena in the following way: "In each time interval spot prices depend on evolution of economy, including evolution of state, in two ways – directly, from observations over the environment, and indirectly, from the decisions, made until a given moment. But if the

[13] The consumer determines what producers have to supply.

agent would like to learn something more about the state of nature from new prices, he must know the strategies of other agents used to this moment. Unfortunately, the agents are not able to assess other strategies on the base of their expectations about future markets". (Radner, 1968, p. 35).

Burness and others (1980) present different interpretation of the same problem. In revised version each state of nature describes completely the physical conditions and equilibrium prices in each moment from the beginning to the end of the history of an economic system. And since the individuals learn what state actually has occurred only in the end of whole history, there is no way before this moment to accomplish the security payments on the base of states of nature. This means, that there is no way before this moment the consumers plans to be accomplished on the spot markets. The conclusion of the authors is, that in the context of behavioral uncertainty incorporation of spot prices in specification of states of nature leads to restriction of the model to a framework with two periods – today's market of financial securities and tomorrow's spot market and consumption (Burness and others, 1980, p. 15).

From theoretical point of view the construction of the revised economy Arrow-Debreu on itself depicts contradictive causality dependence between the optimal Pareto allocation (representing welfare) and the institutions, which generate prices. Burness and others (1980) affirm that by including of subjective probabilities about equilibrium prices in objective functions of consumers and by using the last in determining the *ex ante* optimal allocation, the idea for optimality directly depends on one specific institution for allocation of recourses. When we accept this scenario, however, it would be impossible to compare the central planner's allocation and the competitive allocation. Therefore, although from the descriptive (predictive) point of view the expectations about prices can participate in the objective function, according to the welfare economic theory this is not acceptable. So assuming that future spot markets are active the outcomes for the welfare in the theory Arrow-Debreu are made on the base of "a flawed notion about *ex ante* optimal allocation" (such that incorporates in the states the expectations for the future prices) (Burness and others, 1980, p. 13).

The criticism to the revised version of the Arrow-Debreu theory sets the beginning of a new science direction, known as the sequence markets, which in most of its part, precludes the possibility for construction of a complete markets system. In relation to the last we can present more arguments.

Feiger (1976), for example, investigates the inconsistency of the system of complete markets and the speculative behavior, where the speculations are presented as a transfer of price risk, and are due to the different expectations of traders.

For Feiger speculations hare appeared, when markets are incomplete, i.e. when it was impossible to insure against future spot prices. Only if the contingent contracts signed in the first phase include all the future spot prices, it would be possible to construct a complete market system, in which speculations never take place (Feiger, 1980, p. 680).

Hirshleifer analyzes computing capabilities of traders who make decisions in the contingent claims model with multiple time intervals and concluded that unfortunately the inference of Feiger for depending of contingent contracts on future prices increases grotesquely the scale of the problem with decision making (Hirshleifer, 1976, pp. 695-696). Additions of Feiger change the model of complete markets in the following way. In the original model of Arrow such system requires S×C contingent claims, while the revised by Feiger model must contain S×C× $\widetilde{p}$ contingent contracts, where $\widetilde{p}$ presents the set of possible spot prices for the state of nature given. In the intertemporal economy with additional information signals the system of complete markets requires S×C× $\widetilde{p}$ ×M contracts, where M is a set of possible messages, which may be received. All the problems discussed in this part have appeared in larger forms in this complicated model.

The theories of sequence markets changed the interpretation of the role of information, and in this way the speculative behavior. The model of rational expectations of Jordan and Radner (1982), for example, presents the non-price information as a determinant of a market model of trader, while the general analysis of Krebs (2006) specifies the arriving at a fully transparent equilibrium with buying of private information. The speculative behavior acquires new dimensions in the context of bilateral contracting under asymmetric information, and later in connection with hold-up problem and incomplete contracting.

The impossibility for construction of complete markets system is proved most truly from the point of view of financial markets. To demonstrate the *ex ante* optimal state of competitive equilibrium in economy, Debreu accounts the specificity of production and assumes that in the complete system of markets the maximization of the value of firm stock is equal to the profit maximization. “Accepting prices as a given, j-producer maximizes profit, as maximizing the value of his shares. In this way he is influenced neither by his assignment of probabilities for occurrence different events, nor by his attitude to the risk. So the j-corporation chooses such producer plan, for which the value share is maximized” (Debreu, 1959, p. 100).

The share of the firm is a contract with unconditional contingent claims, as it entitles the owner the right to receive a proportion from the profit in each state of the nature. Therefore, the real financial system contains incomplete markets of contingent claims. Incomplete markets correspond to emerging of risks, which can’t be hedged with the well known financial instruments, and leads to achieving a suboptimal competitive equilibrium. In order to reflect this omission Diamond (1967) introduces the concept of constrained optimum. Diamond’s concept is consistent with the rational expectations and the corrected foresight of Hayek (1937). Diamond claims that Pareto optimum is defined in the frame of sets of allocations, which may be achieved through existing market structures.

To prove his thesis Diamond used a model with one good (commodity) and two time intervals. He assumes constant return to scale (the relation of the quantities produced in two different states of nature doesn’t depend on the scale),

as well as that each firm expects its market value to be correlated with production scale. Diamond compares competitive allocation, as a result of maximizing the market value of the firm, with the allocation of the government under restrictions like taxes, subsidies, and other forms of reallocation, that don't depend on the state of nature, and declares that in striving to maximize their market values the companies achieve constrained Pareto optimal allocation.

Expanding the model of Diamond, Hart (1975) affirms, that with more goods and time intervals the stock markets wouldn't achieve even restricted optimal allocation. Something more, the equilibrium couldn't exist. With given values of exogenous variables in the model of Hart there is a set of equilibriums on the stock markets and each of them could be Pareto dominated by another[14]. This result is familiar as structural inefficiency on the stock market.

In a similar way Stiglitz also thinks, that any equilibrium, even if it were a unique one, may be suboptimal. Through the Capital Assets Price Model (CAPM) Stiglitz (1972) proved the existence of structural inefficiency, which he called marginal inefficiency. According to Stiglitz (1982) the private markets distort the marginal assessment of profitability of investments, and it is true, even when a unique equilibrium exists. Stiglitz explains the structural inefficiency in the following way. In complete set of markets marginal rates of substitution between goods for any two states of nature for all individuals are equalized. In an incomplete set this couldn't happen, but it may be achieved by a more efficient allocation of risk (coming near to the equality between the marginal rates of substitutions) through the change of prices (respectively profits) of risky assets. Since the government (central planner) can do that through reallocation of investments and ownership over (shares of) different assets, and the market can't, consequently the central allocation is Pareto dominant (Stiglitz, 1982, p. 242).

Pursuant to Stiglitz's model, Loong and Zeckhauser (1983) showed, that individuals often undertake inefficient and too risky production decisions. The inefficiency is even bigger than in the case of constrained optimum of Diamond. They indicate the sources of this inefficiency: (1) Externalities of production technologies; (2); Different rates of substitutions of goods among different kinds of individuals; (3) The failure of producers (Loong and Zeckhauser, 1983, p. 173).

Except for the rational expectations the constrained optimum of Diamond is absolutely consistent with the complete contracting and is a main assumption in the first approach to dynamic disequilibrium. It is not an accident that Stiglitz (1989) directed his outcome to the information asymmetry and contract theory.[15]

Most of the critical analysis presented in this article, indeed modify the unique instrumentation of the Arrow-Debreu theory and participate in the construction of more

[14] According to Starret (1973) the reason for inefficient allocation in the model of Diamond is the accounting of transaction costs.

[15] For the entire contribution to the theory of markets and information asymmetry in 2001 Stiglitz received Nobel Prize in Economics.

realistic models of the processes of decision making, insurance against the risk, balancing of spot markets and long term trade agreements.

Generalizations and conclusions

Arrow and Debreu changed for ever the way in which the economic theory formulates the model of uncertainty. After more than half a century of criticism and additions their general framework continues to be the base of new theories of competitive markets under conditions of uncertainty and of long-term contracting.

In spite of the wide recognition of the methodological approach in the theory of Arrow and Debreu, the empirical results from application of the theory are too insignificant. The exchanges of ideas between Arrow and Nagatani enabled us to precise the real existing of Pareto optimal dynamic equilibrium in an uncertain economic environment: “In economy with contingent claims and two time intervals an *ex ante* Pareto optimal equilibrium can be achieved, as well as every competitive equilibrium, general saying through an appropriate allocation of resources”. In an economy with financial securities with two intervals, as well as in the both models of uncertain economy with many time intervals, Pareto optimal equilibrium is possible only in the context of flawed concept for incorporating of future prices in the states of nature (Burness et al., 1980, p. 13).

The optimal results in the models of Arrow and Debreu are deduced on the base of idealizing assumptions for preferences, production, information available on the market, and the size of the markets. Setting aside some of these assumptions, Grossmann, Starr, Harris and others changed the original models and reached to alternative optimal results. Radner, Nagatani and Feiger raised some fundamental questions about reliability of optimal results, even under restrictive conditions of the Arrow-Debreu economy. These authors have found inner contradictions in the interpretation of uncertainty, faced by the individuals in a contingent economy.

The popularity of some analysis in which the authors affirm that the interest to the trade disappeared after first time interval, although it is apparent, that such situation is possible only in a model with two intervals, is notable. These authors can hardly explain sources of uncertainty inherent for the economy Arrow-Debreu such as future spot prices, changing preferences, changing expectations, or even more, the opportunity for capitalizing the profit of other agents, who are with different expectations about states of nature.

From the publications of Starr (1973) and Harris (1978) the thesis is perceived, that with the appearance of speculations in a model with many time intervals, only when traders have same expectations, next trade phases are redundant. But even this possibility is restricted from exceptionally brave admission for fixed preferences.

According to Radner (1968), when the traders don’t dispose of identical information, the complete markets don’t exclude next trade phases. “The appearance of new markets generates incentives for trade, as the equilibrium prices on these markets are the carrier of additional information, which exceed the original information structure, *etc*. The functioning of spot markets suggests that

agents account not only the uncertainty about the economic environment but also this, connected to the strategies of other agents" (1968, p. 35, 55)

The debates about the revised version of theory Arrow-Debreu enriched the last outcome of Radner and grounded the main challenge in front of more realistic models of dynamically developed economy: There is a sequence of interrelated markets, one for each time period, and no one complete in the sense of theory Arrow-Debreu.

The model of restricted optimum of Diamond supplements this thesis and lies in the base of alternative models of rational expectations and complete contracting.

From the point of view of the concept of disequilibrium the criticism and additions to the theory Arrow-Debreu give an opportunity the following conclusions to be derived: (1); The intertemporal equilibrium presents the ideal equilibrium path, around which the equilibrium trajectories converge (2) Theories of sequence markets accept appearance of behavioral uncertainty and are the real alternatives of theory Arrow-Debreu in dynamic presenting of the markets; (3) The admission for restricted optimum of Diamond (1967), which is a projection of the corrected foresight of Hayek (1937), must substitute the admission for perfect foresight; (4) Long term contract are an important instrument for insurance of behavior uncertainty.

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