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YIELD CURVE MODELLING AND FORECASTING IN AN UNDEVELOPED FINANCIAL MARKET: THE CASE OF BULGARIA

An attempt is made to apply the popular Nelson-Siegel-Svensson parametric yield curve model in the case of an illiquid and undeveloped financial market. The findings suggest that the financial market's illiquidity does not seem to diminish or distort the ability of the yield curve model to be used as a tool for deriving a clear picture of the shape and the dynamic of the yield curve. The in-sample and out-of-sample performance of the model for fitting the yield curve is further explored and illustrates how the modelling approach can provide a consistent framework for projecting the yield curve conditional macroeconomic scenarios that could be widely used for policy simulations.

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The term structure of interest rates can be a valuable source of information for central banks and policymakers. It plays a dual role for decision makers: on the one hand, it provides instant feedback on monetary policy decisions and communications and on the other hand, it provides up-to-date information about market expectations, risk assessment of investors and their evolution in response to changes in economic conditions. Thus, the term structure of interest rates could serve as an early warning indicator of improvement or deterioration in macroeconomic conditions.

A major part of the empirical literature on yield curve modelling is devoted to the most developed financial markets, while empirical research employing the term structure of interest rates in illiquid and undeveloped financial markets is much more limited. In such an environment, where the government securities market is still immature, yield curve modelling presents a special challenge mainly due to the availability of market data in the context of a limited supply of debt securities often observed over a short time span. Hence, the interpretation of the estimated yield curves requires much more caution.

The main aim of the current study is to introduce the Dynamic Nelson-Siegel-Svensson (DNSS) model based on the zero-coupon yield curve. The paper represents an effort to set up this model in an undeveloped financial market, as is the case of Bulgaria. As the yield curve factors are not observed, the model is estimated by using the state-space method. The state equation evolves the yield curve factors over time using a VAR(1) model, and the observation equation translates the yield curve factors into fitted yields using a loading matrix. The analysis explores the in-sample and out-of-sample performance of the DNSS model for fitting the yield curve and tries to assess how well the integration of macroeconomic variables may explain the dynamic evolution

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of yields over time under different economic developments in inflation and industrial production. The macroeconomic variables included in the model are treated as “unspanned” and therefore not affecting the cross-sectional relationship between observed yields for different maturities at a given point in time. It is important to highlight that the aim of the proposed framework is not to provide superior yield curve forecasts and therefore these projections should not be assumed to outguess the market and serve for short- and medium-term investment decisions. Rather, the paper illustrates how the modelling approach can provide a consistent framework for projecting the yield curve conditional on more or less realistic macroeconomic scenarios that could be widely used for policy simulations, alternative scenarios, and sensitivity analyses.

Overall, the results from the in-sample and out-of-sample forecasts support two conclusions. Firstly, the DNSS model provides a good in-sample and out-of-sample fit, without being overly impressive. Secondly, under different macroeconomic scenarios, the yield curve generally seems to exhibit changes in its shape in line with the macroeconomic theory – flattened yield curve at a time of weak economic activity and upward sloping during economic expansion. These findings confirm that the examined yield curve model is a desirable tool when trying to form a clear picture in regard to the shape and the dynamic of the yield curve. The financial market’s illiquidity does not seem to diminish or distort this significantly.

Literature review

The term structure of interest rates presents the relationship between the yields-to-maturity of a set of bonds and their time-to-maturity. It is a descriptive measure of the cross-section of bond prices observed at a given point in time. An *affine* term structure model describes the stylized time-series properties of the term structure of interest rates. Any affine term structure model assumes that the term structure of interest rates is a linear function of a small set of common factors that provide uncertainty in the model. These factors, often called state variables, represent a random process that is restricted by the assumption of an absence of arbitrage in the underlying financial market. The decomposition of the yields into latent factors is based on one or another statistical technique[1] (e.g., Nelson and Siegel, 1987; Svensson, 1994; Kenz, Litterman and Scheinkman, 1994; Duffie and Kan, 1996). This provides a straightforward algorithm for simulating the term structure. When modelling the yield curve, many central banks and practitioners frequently apply the factor-based Nelson-Siegel model using three parameters (factors): level, slope and curvature. Among the preferred models is also the extended Nelson-Siegel-Svensson framework (Svensson, 1994) where a fourth parameter (factor) is added to increase model flexibility and improve

[1] Anderson et al. (1996, p. 25) categorize the existing estimation methodologies for the decomposition of observed bond yields/prices into zero-coupon and forward rate curves in two groups: the first one makes assumptions about changes in state variables and asset pricing methods using either equilibrium or arbitrage arguments; and the second one consists of models that are based on statistical technics where the zero-coupon yield curve is described by smoothing the data obtained from bond prices.

the in-sample fit. The Nelson-Siegel approach is further developed by Diebold and Li (2006), who introduce the Dynamic Nelson-Siegel model (DNS). The authors represent the parameters as time-varying factors, which allows the representation of yield curve movements by the dynamics of three factors.

Most of the cited research is reasonably related to developed financial markets. Recently, a growing strand of the literature has been dedicated to the yield curve modelling in illiquid and undeveloped financial markets. In the past decade, attempts to estimate zero-coupon rates and to model the yield curve have been made in the financial markets of India, Taiwan, Slovenia, Croatia, the Czech Republic, and Hungary, among others. In those studies, several yield curve modelling approaches are considered. The factor-based Nelson-Siegel and Nelson-Siegel-Svensson methodologies remain the most common frameworks for modelling the yield curve in illiquid and undeveloped financial markets as well due to their simplicity, which does not affect the models' ability to fit market data reasonably well (Table 1).

Table 1

Overview of the selected yield curve modelling research in undeveloped financial markets

Market	Research	Analysed models
Indian	Virmani (2006)	NelsonSiegel and Svensson
	Dutta et al. (2005)	Svensson B spline and smoothing spline
	Subramanian (2001)	NelsonSiegel and Svensson Cubic B spline and smoothing spline
Taiwanese	Chou et al. (2009)	NelsonSiegel and Svensson
Slovenian	Grum (2006)	NelsonSiegel and Svensson B spline and smoothing B spline Merrill Lynch exponential spline
Croatian	Zoricic, D. (2012)	NelsonSiegel and Svensson
Czech	Kucera, D. and Komarkova Z. (2019)	FamaBliss bootstrap method
	Hladíková and Radová (2012)	Nelson–Siege method
	Kladívko (2010)	Nelson and Siegel method
Hungarian	Reppa (2009)	NelsonSiegel method

The fit of the Nelson-Siegel model and its variations proved to be rather good regardless of the country of application. However, yield curve forecasts produced by these models draw no direct connection between the evolution of the term structure and any future dynamics in macro-economic variables. Yet, it is of importance not only for investors but also for policy makers to understand how the yield curve reacts to macroeconomic shocks.

In that sense, in their prominent paper, Ang and Piazzesi (2003) modify the standard Nelson-Siegel three-factor affine term structure model by adding two macroeconomic factors. They find that the included macroeconomic variables improve yield factors, accounting for up to 85% of the variation in interest rates. Inspired by these results, several authors among which are Hördahl et al. (2006), Ang, Piazzesi and Wei (2006), Evans and Marshall (2007), and Rudebusch and Williams (2008) further explore different approaches of joint modelling the term structure and the macroeconomic conditions and show that macro factors have considerable effects on the yield curve. Among the authors who integrate macroeconomic variables into the DNS framework are Diebold, Rudebusch, and Aruoba (2006) who find strong evidence of the effects of macro variables on the future movement in the yield curve and rather weak evidence of a reverse influence[2].

Most of the analyses within the latter approach have focused on the relation between the yield-curve latent factors and monetary policy, inflation and real activity (for example Diebold, Rudebusch and Aruoba, 2006; Carriero, Favero and Kaminska, 2006; Dewachter and Lyrio, 2006; Hordahl, Tristani and Vestin, 2006; Rudebusch and Wu, 2008; Hoffmaister, Roldós and Tuladhar, 2010). It is very likely that such an approach relates closely to the vast literature on the power of the yield curve slope (and possibly the curvature) to predict fluctuations in real economic activity and inflation. In the current study, following the approach of several central banks and financial market practitioners, the Nelson-Siegel-Svensson model is employed in an attempt to model the Bulgarian yield curve and to produce (un)conditional out-of-sample forecasts. The paper further examines whether the shape and position of the estimated yield curve in an undeveloped financial market are in line with the macroeconomic theory under different macroeconomic scenarios.

The model and estimation technique

Diebold and Li (2006) introduce a dynamic model for the yield curve by factorization of the Nelson-Siegel (NS) spot rate representation (Nelson and Siegel, 1987), as follows:

$$y_n(\tau) = \beta_{0n} + \beta_{1n}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{2n}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \quad (1)$$

In the dynamic Nelson-Siegel model (DNS), $y_n(\tau)$ represents the zero-coupon yield curve function observed at time *n* for yield to maturity τ, and $\beta_{0n}, \beta_{1n}, \beta_{2n}$ are the time varying parameters[3] that capture the level (L), slope (S) and curvature (C) of the yield curve at each period of time, *n*. The DNS model fractions the yield curve into three dynamic latent factors $(\beta_{0n}, \beta_{1n}, \beta_{2n})$ and factor loadings $[1 \;\; \frac{1-e^{-\lambda\tau}}{\lambda\tau} \;\; \frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}]$. The latent factors shape yield dynamics at any time, *n*, while the factor loadings determine the cross section of yields for any maturity, τ. The factor β_0 (*L*), called the long-term factor, governs the long-term level of the yield curve. The loading on β_0 (*L*) is equal to 1 for all maturities. As maturity increases, the equation (1) implies that the yield curve converges to β_0. The loading on the factor $\beta_1(S)$, $(1-e^{-\lambda\tau})/\lambda\tau$, has a maximum equal to 1 at the shortest maturities and monotonically decays to zero as maturities increases. Thus, the yield curve converges to $\beta_0 + \beta_1$ when maturity

[2] For an overview of macro-finance literature, see Gurkaynak and Wright (2012).

[3] In the case of a cross-sectional environment, the βs are parameters, while in a time-series perspective, the βs are variables.

decreases to present time. The factor $\beta_1(S)$ is called a short-term factor as it is closely related to the yield curve slope, which, as explained in Diebold (2006), is the ten-year yield minus the three-month yield ($y_n(120)$- $y_n(3)$). The factor β_2 (C), has a loading that is null at the shortest maturity, increases until an intermediate maturity and then falls back to zero as maturities increase. In that sense, the factor β_2 (C), which is called medium-term factor, is related to the yield curve curvature and is defined as twice the two-year yield minus the sum of the ten-year and three-month (2y_n(24)- y_n(3)- $y_n(120)$). The parameter λ[4] and β_2 govern the possible presence of a hump in the yield curve. More specifically, λ determines at which maturity the hump is observable, while β_2 sets its magnitude and the direction. The three factors, β_0 , β_1 , β_2 described as long-term, short-term and medium-term factor, can be interpreted in a dynamic fashion as level, slope and curvature, as shown by Diebold and Li (2002).

In order to increase the flexibility and improve the fit of the Nelson-Siegel model, Svensson (1994, 1995) and Soderlind and Svensson (1996) add two additional terms. Thus, the respective Nelson-Siegel-Svensson (NSS) zero-coupon yield curve is:

$$y(\tau) = \beta_0 + \beta_1\left(\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau}\right) + \beta_2\left(\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau}\right) + \beta_3\left(\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau}\right) \quad (2)$$

The parameters, $\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2$, in the NSS model have the same interpretation as in the Nelson-Siegel model, where the two additional parameters, β_3 and λ_2, determine the magnitude and the position, respectively, of a possible second hump in the yield curve.

The Diebold and Li (2006) model (1), which is a variant of the Nelson-Siegel model and its variations, is formulated such that the level, slope, and curvature factors follow a vector autoregressive process of first order, or VAR(1), and as such the model immediately forms a state-space system. Thus, the Kalman filter[5] could be employed[6] to fit the yield curve at each point in time by estimating the underlying dynamics of the factors. The Nelson-Siegel form formulated as a state-space model is (Diebold and Li, 2006):

Observation (measurement) equation:

$$Y_n = H \cdot F_n + \varepsilon_n \quad (3)$$

State equation:

$$F_n = k + A \cdot F_{n-1} + \eta_n \quad (4)$$

The observation equation Y_n constitutes a vector of yields observed at time n for different maturities $\tau = (\tau_1, \tau_2 .. \tau_T)$. This equation represents exactly the affine relationship between market zero-coupon rates and the unobserved state variables.

[4] Diebold and Li (2006) keep the λ parameter constant at 0.069 over time in order to reduce the volatility of the factors, thus making the model more predictable.

[5] For more information in regard to the Kalman filter, see Harvey (1989).

[6] The internally built Matlab State-Space Model (SSM) toolbox and the Kalman filter are utilized in the estimation of the model. I am particularly indebted to Ken Nyholm for sharing his Matlab code for his guide "A Practitioners Guide to Yield Curve Modelling" (2019). A modified version of this code was utilized in this study.

The factor loading matrix H comprises the vector of the Nelson-Siegel (or Nelson-Siegel-Svensson) yield curve factors F_n. The yield curve factors F_n are identified by constraining the factor loadings (H) to follow the smooth pattern proposed by Deibold and Li (2006). In the four-factor Nelson-Siegel-Svensson model, the loading matrix H is, as follows:

$$H = \begin{pmatrix} 1 & \frac{1-e^{-\lambda_1\tau_1}}{\lambda_1\tau_1} & \frac{1-e^{-\lambda_1\tau_1}}{\lambda_1\tau_1} - e^{-\lambda_1\tau_1} & \frac{1-e^{-\lambda_2\tau_1}}{\lambda_2\tau_1} - e^{-\lambda_2\tau_1} \\ 1 & \frac{1-e^{-\lambda_1\tau_2}}{\lambda_1\tau_2} & \frac{1-e^{-\lambda_1\tau_2}}{\lambda_1\tau_2} - e^{-\lambda_1\tau_2} & \frac{1-e^{-\lambda_2\tau_2}}{\lambda_2\tau_2} - e^{-\lambda_2\tau_2} \\ & & \vdots & \\ 1 & \frac{1-e^{-\lambda_1\tau_T}}{\lambda_1\tau_T} & \frac{1-e^{-\lambda_1\tau_T}}{\lambda_1\tau_T} - e^{-\lambda_1\tau_T} & \frac{1-e^{-\lambda_2\tau_T}}{\lambda_2\tau_T} - e^{-\lambda_2\tau_T} \end{pmatrix}$$

In regard to the state equation (8), k is a vector of mean parameters and the matrix A collects autoregressive parameters. The orthogonal, Gaussian white noise processes η_t and ε_t are defined such that:

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & N \end{pmatrix}\right)$$

In the same fashion as Diebold and Li (2006), it is assumed that the state equation factor disturbances η_t are correlated, and therefore the corresponding covariance Q matrix is non-diagonal. However, in regard to the observation equation, with ε_t representing the idiosyncratic components, the model assumes that the matrix N is diagonal such that deviations of yields of various maturities from the yield curve are uncorrelated.

A yield curve model with macro factors

Theoretically, there is a strong connection between macroeconomic variables and the yield curve. Diebold and Li (2006) and Diebold (2006) find that a certain relationship exists between the yield curve factors (L_n, S_n and C_n) and some macroeconomic variables. For example, Diebold and Li (2006) argue that the level factor (L_n) is strongly correlated with inflation, the dynamics in the slope factor mirror output fluctuations, and the curvature factor is not related to any macroeconomic variable. To address the findings of Diebold and Li (2006), which are also confirmed by other studies, and to help explain better the dynamic evolution of the yield curve factors, the annual core inflation and the industrial production index are incorporated in the model (4). The general state-space representation of the DNS model allows the inclusion of these macroeconomic variables as exogenous variables along the lagged yield curve factors in the state equation. Including the macroeconomic variables, the state-space representation of the system is presented as follows, where only the state equation is modified:

Observation (measurement) equation:

$$Y_n = H \cdot F_n + \varepsilon_n \qquad (5)$$

State equation:

$$F_n = k + A \cdot F_{n-1} + Q \cdot M_n + \eta_n \qquad (6)$$

The observation equation (5) remains unchanged. On the other hand, the state equation (6) is changed to include the matrix Q which contains coefficients that account for the relationship between the yield curve factors and the selected macroeconomic variables at time n $M_n = [HICP, IPI]'$. The four unobservable Nelson-Siegel-Svensson yield curve factors at time *n* are collected in F_n^y. The joint evolution of the yield curve factors and the macroeconomic variables is described in an unrestricted VAR(1) process. In line with the above model specification, the level, slope and the first and second curvature factors (F_n) are affected contemporaneously or spanned by both the cross-section of yields and the macroeconomic variables.

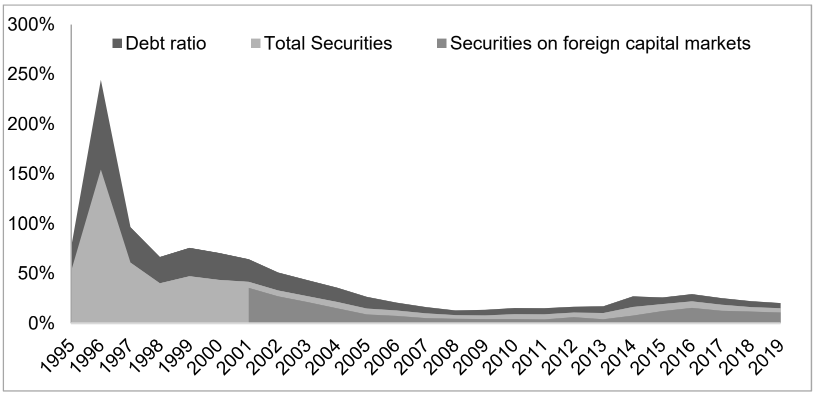
The NSS model's underlying state variables in (6), i.e. the factors, are estimated via the Kalman filter which is useful when the underlying state variables are not directly observable. The basic tool for dealing with the standard state-space model is the Kalman filer which recursively makes inferences about the unobserved values of the state variables by conditioning on the observed zero-coupon rates (observation equation). These recursive inferences are then used to construct and maximise a log-likelihood function to find an optimal parameter set for the system of equations. The Kalman smoother is then deployed to provide conditional expectations of the state variables given the maximum likelihood estimates of the parameters[7].

The Bulgarian Government Bond Market

In 2001 Bulgaria launches a series of long-term operations to reduce its Brady debt and to improve its position on the foreign capital markets (Figure 1).

Figure 1

Debt-to-GDP ratio and the composition of government securities*



*The debt-to-GDP dynamics and the decomposition of government debt to total government securities and securities issued on the foreign capital markets.

Source: ECB, section Government finance statistics; Ministry of Finance, monthly bulletins of general government debt; own calculations.

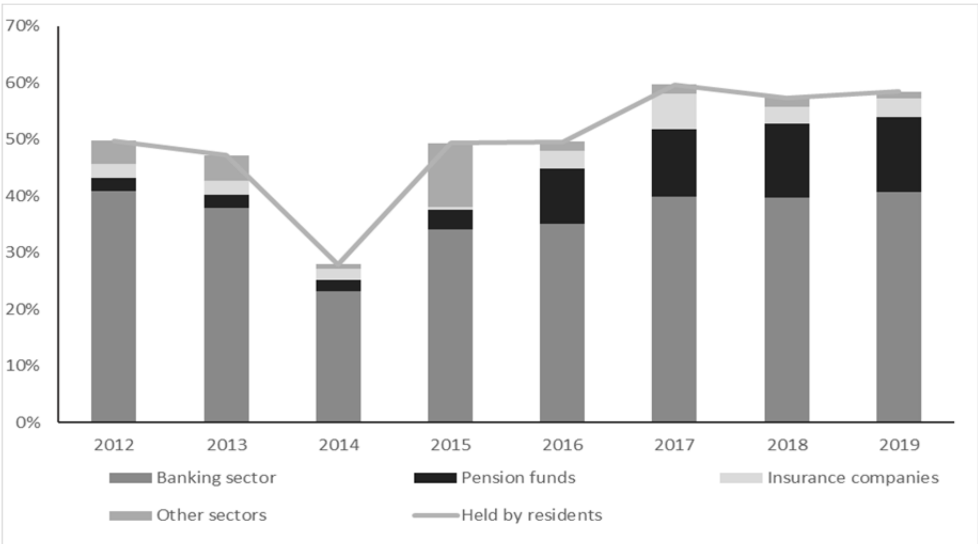
[7] Kim and Nelson (1960) provide a detailed explanation of state-space models and the Kalman filter.

The first Bulgarian issue of Eurobonds in November 2001 is accepted well by the investors with high demand from Germany, Britain, Italy, Bulgaria and Greece. In the following year, 2002, Bulgaria continues with the large-scale restructuring operations with the issuance of new global bonds all of which are with fixed interest rates. Proceeds from these issues are used to repurchase and exchange outstanding Brady bonds. Issuance of Global securities on the foreign capital markets is resumed only in 2012 with a 5-year security. During the years, the Bulgarian presence on the foreign capital markets is rather limited with issuance of government securities in a small range of maturities in line with the adopted Government Debt Management Strategy (10-year bonds in 2014; 7-, 12-and 20-year bonds in 2015; and 7- and 12-year bonds in 2016).

The investor clientele on the Bulgarian bonds issued on the foreign markets and held by Bulgarian residents is illustrated in Figure 2.

Figure 2

Bulgarian government securities (GS) issued on the international capital markets (ICM) and held by Bulgarian residents, sectoral break down* (2012-2019)



*The share of government bonds issued on the international capital markets, which is held by Bulgarian residents, and the corresponding breakdown of the same share across several economic sectors.

Source: Ministry of Finance, data from the monthly bulletin of general government debt; own calculations.

As seen in Figure 2, throughout the years, the share of Global government bonds held by Bulgarian residents is kept relatively stable at around 50%. A notable exception is 2014, when the total volume of Global bonds increases with the issuance of a 10.5-year Eurobond. In the years 2012-2019, large institutional investors seem to be the domestic banking sector followed by the institutional pension funds and pension insurance companies. In the period under consideration, Bulgarian banks keep their share relatively stable, suggesting that investors' preferences ("preferred habitat") for holding bonds with certain characteristics have remained more or less stable during the years. At the same time, the second largest institutional buyers of Bulgarian Global bonds are pension funds and pension insurance companies, which slowly rebalance their portfolio towards the Bulgarian Eurobonds throughout the years.

Turning to the domestic market, in the years after 2000, the secondary market of government securities is dominated by bonds with the term to maturity of 3, 5, 7 and 10 years. However, the volume of registered transactions is uneven during the different periods of the year. In the period 2000-2019 there is a tendency for most active trade on the days of acquiring new government securities and at the time of principal and interest repayments in connection with maturing government securities. Still, there are months without any trading activity for some maturity segments. Therefore, in its attempt to construct zero-coupon yield curves for the Bulgarian Eurobonds, the current study relies on the government bonds issued on the foreign capital market where daily trade data is available from Bloomberg.

Data used for calculating the yield curve

For the estimation of the dynamic Nelson-Siegel-Svensson model, the present study uses end of month zero-coupon bond yields for the following yield curve maturities: {12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 144, 180, 240, 360} months. The zero-coupon yields curve is calculated based on data for the Bulgarian coupon Eurobonds issued on the international capital markets obtained from Bloomberg. As data on trading activity for domestic bonds on the secondary market is relatively scarce, the utilization of Eurobonds seems to be the best data possible. Concerning the Eurobond sample population, only after the emission of Eurobonds in March 2015 do 4 or more data points become available continuously, which allows the calculation of the yield curve. However, the calculation of the zero-coupon yield curve presents a special challenge mainly due to the lack of available market data in several maturity spectrums in the context of a limited supply of debt securities observed in a short time span. The maturity structure of Eurobonds is based on the Government Debt Management Strategy and the redemption profile of government debt, which ensures no concentration of debt payments. Therefore, Eurobonds span not in all maturities and thus gaps emerge in several maturities. Consequently, in the current study, an interpolation and extrapolation technique are applied to close the maturity gap (see Table A 1 in the Appendix). To obtain end-of-month yields, the procedure described below is followed:

A daily zero-coupon yield curve is estimated for the period June 2015-June 2020, a total of 1035 trading days, following the widely used parametric method of Nelson-Siegel (1987) and extended by Svensson (1994). As suggested by Nymand-

Andersen (2018), the sample population starts three months after the newly emitted Eurobonds as in this period yields tend to exhibit higher volatility than other maturity classes. The same logic applies for bonds with residual maturities of less than three months. In the estimation of the Nelson-Siegel-Svensson parameters (β_i, τ_i) the Gradient-based algorithm is applied in the optimization process where the yields errors are minimized. As the current study is primarily interested in interest rates, it aims at minimizing the deviation between estimated and observed yields. The solution of the optimization problem is sensitive to the initial values of the starting parameters. To help the convergence of the optimization algorithm, the fact that the parameters have specific financial interpretation facilitates the choice of starting values "near" the solution. More specifically, following the adopted approach of Deutsche Bundesbank described in Nymand-Andersen (2018), the starting values are set at:

$\beta_0 = (y_t + y_{t-1} + y_{t-2)})/3$ (mean yield of the three bonds with the longest time to maturity)

$\beta_1 = (y_1 - \beta_0)$; where y_1 is the yield of the bond with the shortest time-to-maturity;

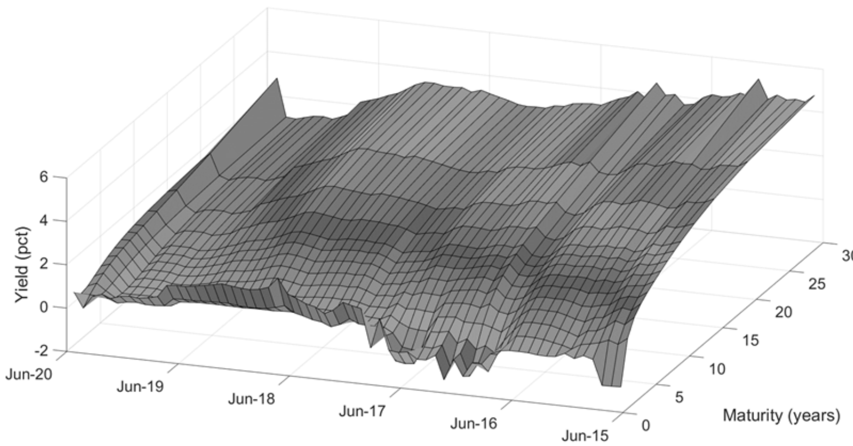
$\beta_2 = \beta_3 = -1; \ \tau_1 = \tau_2 = -1;$

In case the algorithm does not converge to a solution, parameters from the day before are used, which represents a common practice among practitioners. Finally, following the above-mentioned estimation approach, end-of-month zero-coupon bond yields for the horizon {12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 144, 180, 240, 360} months are selected.

There is a common agreement in the literature that the yield curve offers a useful set of information for monetary policy purposes and provides some information about the expected path of future short-term rates and the outlook for economic activity and inflation. Generally, a positive slope (10Y-1Y) of the yield curve is associated with investors' anticipation of future increase in real economic activity. Figure 3 depicts an upward-sloping yield curve for the Bulgarian economy in the period June 2015-June 2020, featuring higher long-term interest rates rather than short-term interest rates, especially in the period June 2015-June 2017. At that period, inflation is relatively stable in the negative territory and IPI exhibits mostly positive growth. In the second half of 2017, the short-term bond yields increase, signalling higher future inflationary expectations form market participants. A few months later, the inflation rate becomes positive and industrial production index contracts while the sovereign yield curve tends to flatten. In the second half of 2019 up until February 2020, the yield curve is at its flattest level. In March 2020, as the severity of the Covid-19 crisis emerged, the exceptional degree of uncertainty about the economic outlook causes an increase in bond yields across the whole yield curve. The macroeconomic data for the Bulgarian economy – core HICP index and industrial production index – observed at a monthly frequency are provided by Eurostat. The two macroeconomic variables are converted into year-on-year percentage change (see Figure 4).

Figure 3

Bulgarian yield curve*

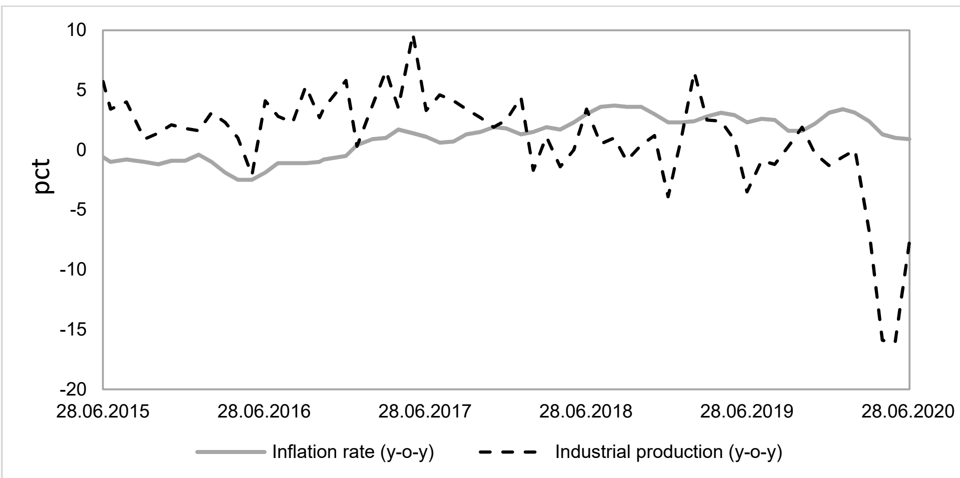


*The Bulgarian zero-coupon yield curve estimated based on the Eurobonds yields traded on the international capital markets is shown. End-of-month yield data are presented for the period June 2015-June 2020, for maturities from 1 to 30 years.

Source. Own calculations.

Figure 4

Industrial production and core inflation rate developments



Source. Eurostat.

Empirical Results

The main purpose of the model presented above is to provide a tool, which allows for the generation of historically consistent long-term yield projections, which could be relevant for strategic asset allocation decisions. Results produced by this framework could facilitate the discussion in investment committees. The presented methodology allows for the generation of scenarios for the future evolution of the yield curve conditional on more or less realistic realizations of macroeconomic variables. However, it is still interesting to analyse how well the DNSS model performs in an in-sample and out-of-sample forecasting exercise. Parameters of the Nelson-Siegel-Svensson model are estimated by utilizing the internally build Matlab State-Space Model (SSM) toolbox and the Kalman filter. As using only the in-sample goodness of fit as a sole criterion for judging term structure estimation can be misleading (Bliss, 1996), an out-of-sample exercise is also included in the analysis of the model's performance.

In-sample fit

The presented in-sample fit is based on the state-space representation of the DNSS model with the Kalman filter. Table 2 displays the in-sample fit of the thus specified model.

Table 2

In-sample fit of the Dynamic Neslon-Siegel-Svensson yield curve model*

Maturities in months	12	24	36	48	60	72	84	96	108	120	144	180	240	360
RMSE	28	24	23	12	3	8	12	12	11	8	2	12	19	10

*The yield curve parameters are estimated via the Klaman filter. The degree of the fit is assessed via the root mean squared error (RMSE) denominated in basis points.

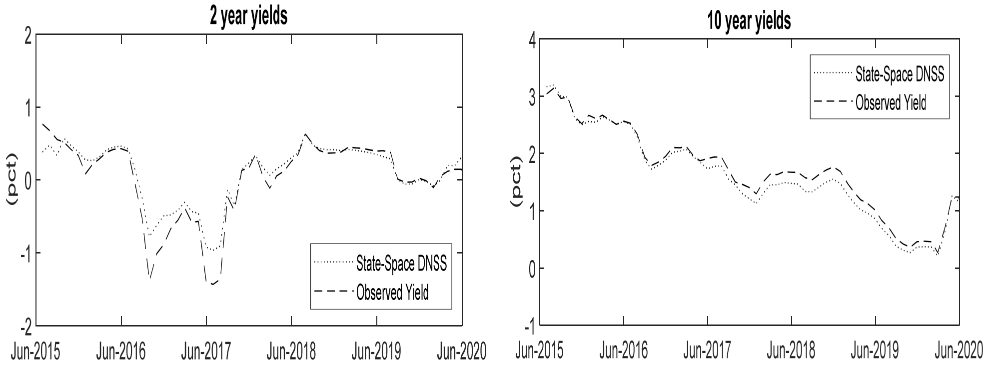
In order to provide a visual representation of the differences between the observed and the model-fitted yields, Figure 5 shows the actual and the fitted 2- and 10-year bond segments of the Bulgarian yield curve. A general observation can be made that the model fits the data reasonably well, without being overly impressive. However, the in-sample performance is different for the various maturities. The worst fitted maturities of the DNSS model are the short-term bonds (12, 24 and 36 months) in contrast with the mid- and long-term bonds, i.e. bonds ranging between 5 and 15 years, for which the Kalman filter provides a better in-sample fit. As was to be expected, it is easier to fit yields in-sample (i.e. the error decreases) as longer Eurobonds are included in the estimation sample. Indeed, high values of the RMSEs[8] in the short end of the yield curve are in line with our expectations, as the Eurobond sample used to estimate the zero-coupon yields shows more concentration of Eurobonds in the mid- and long-term maturity segments and a much lower representation of Eurobonds in the shorter maturities. Furthermore, the estimated errors also represent the inability of the estimated parameters to fit the model properly. This suggests

[8] The RMSE is calculated as $\left[mean\left[(y_{t(i)} - \hat{y}_{t(i)})^2\right]\right]^{\left(\frac{1}{2}\right)}$, where $y_{t(i)}$ and $\hat{y}_{t(i)}$ are the time series for the i'th maturity point.

that the errors may indicate the presence of idiosyncratic variation in yields, which is not captured by the model.

Figure 5

Observed and model-fitted yields



*The fitted yields are estimated via state-space representation, as described in equation 6.

The number of yield curve factors that are assumed to drive the bond yield movements is crucial in determining the in-sample performance of the model. In the current study, the choice of the four factor DSS yield curve framework is based on the share of the variation which the model aims to capture through the optimal number of selected factors (principal components).

Table 3 presents the total cumulative fraction explained by the first four extracted principle components (PCs)/factors[9].

Table 3

Cumulative variability explained by the extracted yield curve factors*

Number of PC/factors	Cumulative proportion of variance including this PC (in %)
1st	80.8
2nd	92.6
3rd	99.1
4th	99.7

*The cumulative fraction of variability explained by the principal components extracted from the Bulgarian yield curve data is presented. The data covers the period from June 2015 to June 2020 and is observed monthly.

Given that the first four principal components capture 99.7% of the historical variability of the Bulgarian yield data, a sensible choice is to include four factors, i.e. to apply the DNSS model, in the analysis of the empirical factor structure underlying yields.

[9] The steps followed to generate the principal components (PCs) are described in Moody's Analytics (2014).

Out-of-sample forecast

Regardless of the in-sample fit performance of the model, it is still interesting to analyse how well it performs in an out-of-sample forecasting exercise for producing unconditional and conditional forecasts of the yield curve. To explore the relative forecasting performance of the DNSS model, the latter is estimated recursively, and short-term forecasts are produced using an expanding data sample from April 2019 to June 2020. The model is first estimated on a sub-sample covering the period from June 2015 to April 2019 and produces forecasts for the {12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 144, 180, 240, 360} months segment of the yield curve for the horizon of 1-month to 6 months ahead. Then, one observation is added to the data sample and the model is re-estimated after which a new set of forecasts are generated again for the horizon of 1-month to 6 months ahead. This process is repeated until the full sample is covered.

The RMSE of the DNSS and the Random Walk models forecast are calculated for each period between April 2019 and June 2020, denominated in bases points. Forecasts for s-months ahead are generated for each maturity segment and the RMSE are calculated for the 2-, 5-, 7-, 9- and 10-year yield curve segments (Table 4).

Table 4

Root mean square errors of the DNSS model and Random Walk model forecasts, in basis points

Maturity\Horizon	Forecast horizon					
	1-month	2 months	3 months	4 months	5 months	6 months
Model						
24M	16	26	35	35	32	31
60M	46	70	83	94	102	105
84M	47	67	77	84	89	93
108M	46	56	57	60	62	65
120M	48	53	49	50	51	52
Random Walk						
24M	16	27	36	36	33	32
60M	46	73	87	98	107	109
84M	47	68	78	86	92	97
108M	46	57	58	63	67	72
120M	48	54	52	54	56	58

It is clearly visible from Table 4 that the out-of-sample RMSE of the DNSS model forecasts are significantly higher than their in-sample counterparts. Furthermore, the estimated RMSE of the model forecasts across all maturity segments indicates that the precision of the model forecast significantly deteriorates mainly in the short- and medium-term maturity segments as the forecast-horizon is increased. On the other hand, it is interesting to notice that the RSMEs of the 10-year maturity segments remain fairly constant across the forecasting horizon, at around 50 basis points. To assess the out-of-sample performance of the model, a simple Random Walk model is estimated, which is commonly accepted as the benchmark model when comparing out-of-sample yield curve forecasts. The Random Walk model is defined as:

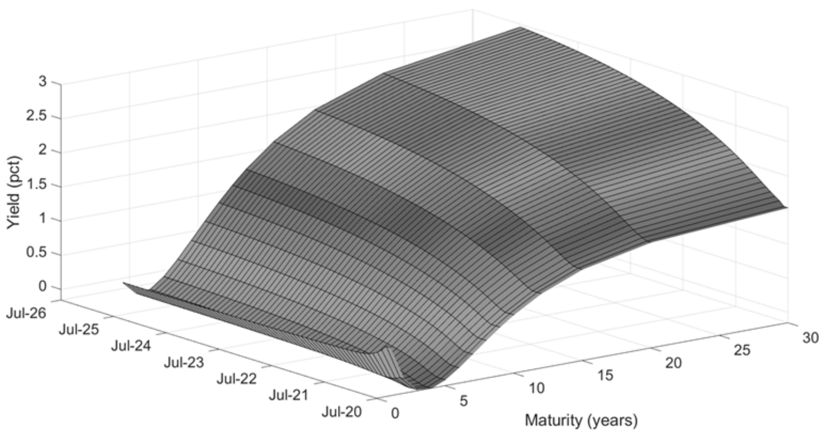
$$\hat{y}_{t+h|t}^{(\tau)} = y_t^{(\tau)}$$

The Random Walk for interest rates implies a simple no-change forecast of yields (y) in a given maturity segment (τ) from one period to the next period. Hence, in this model, the h-month ahead forecast is simply the τ-maturity bond yield in the period t. Overall, Table 4 shows that the DNSS model seems to perform slightly better, outperforming the Random Walk in forecasting the movements of out-of-sample yields.

Having assessed the out-of-sample performance of the DNSS model, Figure 6 provides a graphical illustration of the unconditional yield curve forecast for the next 60 months. In the next 5 years, the yield curve is projected to remain upward sloping, becoming steeper at the end of the projection horizon. Short-term bond yields stabilize close to the zero bound while the long end of the yield curve reaches levels close to 3%. However, the out-of-sample forecasts must be cautiously interpreted as the presented simulations rely on zero-coupon yields curve derived from the Eurobond portfolio with scarcity of maturity segments and relatively short time series. Despite the data limitations, the presented simulation can still be considered a valuable tool for forecasting expected ranges of future interest rates, but it is definitely not aimed at producing forecasts for tactical investment decisions aimed at outperforming a given benchmark strategy.

Figure 6

Unconditional yield curve forecast*



*The yield curve factors are obtained via the Kalman filter.

Conditional projections

It is well-known in the literature that there is a link between the macro economy and the shape and location of yield curves and their time series evolution. The current DNSS model provides a powerful framework for the long-term yield curve forecasting conditional on more or less realistic realizations of macroeconomic variables. In terms of long-term asset allocation decisions, it is relevant to know what the yield curve would look like if one or other macroeconomic environment materializes and how the yield curve converges to such scenario-based economic outcomes. The current framework provides a methodology allowing the generation of a visual representation of the shape and the evolution of the yield curve under different macroeconomic scenarios, which facilitates the discussion in investment committees where long-term investment strategies are adopted. Thus, the framework provides a common language for traders, economists and senior management.

To illustrate the conditional yield curve projections, two hypothetical macroeconomic scenarios are analysed. Yield curves are projected over a horizon of 60 months where the yield curve forecasts are made conditional on two macroeconomic scenarios. These scenarios try to describe a future pessimistic or optimistic economic environment.

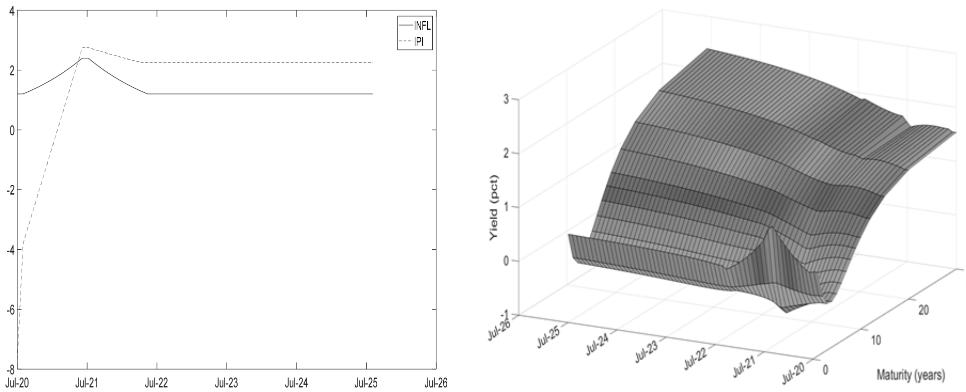
In the next twelve months, the optimistic economic scenario foresees a steady increase in the core inflation rate of up to 2.5% followed by a gradual decline down to 1.2% afterwards (Figure 7). Throughout the rest of the forecasting horizon, the inflation rate stabilizes around 1.2%. Meanwhile, in the first 12 months of the projection horizon, industrial production rebounds from its low crises levels and returns to a steady growth path reaching 2.75%. In the next 12 months, the IPI growth path faces a gradual downward adjustment before a stabilization of 2.25% is reached and kept throughout the rest of the projection horizon.

The hypothetical pessimistic scenario (Figure 8) assumes that in the next two months the IPI growth rate will gradually climb out of the deep valley into which it plunged in June 2020, but that it would then sharply fall again down to -7.5%. Some slow recovery is expected in the second half of 2021 and in the first half of 2022 where the IPI growth will stabilize at 1.2% throughout the rest of the projecting horizon. At the same time, the rate of the inflation is expected to gradually decline reaching almost zero in the first 12 months and to then slowly rise and stabilize at 1.2% until the end of the forecasting horizon.

Figure 7 presents the evolution of the median yield curve conditional in the hypothetical optimistic economic scenario. The short-term end of the yield curve closely follows the inflation movements represented in the optimistic economic scenario. The steady increase in the HICP rate in the first 10 months results in a spike in the 1-year yield levels of close to 1.5%. The increase in inflation seems to induce more significant changes on the “slope” factor, implying that the interest rate response is more immediate and stronger in the short end of the yield curve than in the long end of the curve. Meanwhile, the long-term segments initially increase following the stable recovery in the IPI indicator, however, the effect on the long end of the yield curve is more muted. In July 2021, when inflation and IPI show first signs of a slower growth rate, the yield curve becomes “hump-shaped”, i.e. medium-term interest rates show more significant increase then the long-term interest rates, which, in line with the literature, indicates lower economic activity expectations. Later, when inflation and IPI turn to a constant growth rate, the short and the long end of the maturity spectrum stabilize.

Figure 7

An optimistic economic scenario*

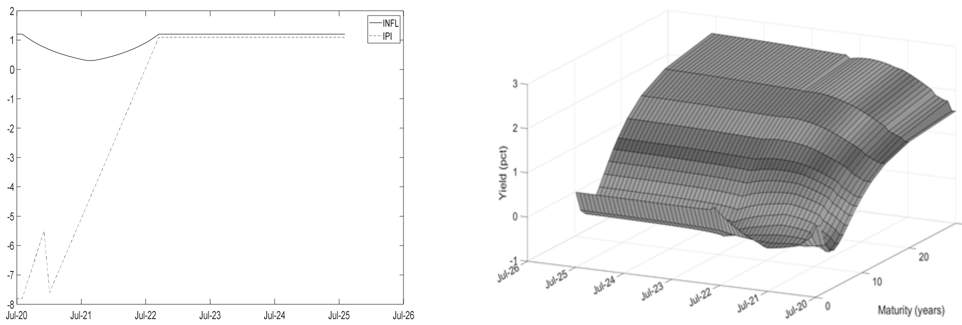


*The LHS panel shows the evolution of the macroeconomic variables for a hypothetical optimistic economic scenario. The RHS panel shows the corresponding development in the yield curve when the hypothetical macro scenario is based on an optimistic economic scenario. The yield curve factors of the model are obtained via the Kalman filter as projections that are calculated conditional on the macroeconomic developments.

Figure 8 shows the evolution of the median yield curve conditional on the hypothetical pessimistic economic scenario. The short end of the yield curve perfectly mirrors all developments in the inflation. More specifically, the 1-year interest rates first decline, turning into a negative territory following the decrease in the inflation, and then the short end of the curve gradually increases influenced by the slow acceleration in the inflation. Meanwhile, up to the time when the IPI sees a second decline, the yield curve is flatter with 10-year yields preserving low levels of around 1%. Later, when IPI growth rate appears to show signs of recovery from the deep slump envisaged by the pessimistic scenario in combination with the slow recovery in the inflation, the long-term yields start increasing. Higher long-term yields are observed up to the point where the inflation and the IPI growth rate stabilize throughout the forecasting horizon and thus preserve the same shape and position of the yield curve.

Figure 8

A pessimistic economic scenario*



*The LHS panel shows the evolution of the macroeconomic variables for a hypothetical pessimistic economic scenario. The RHS panel shows the corresponding development in the yield curve when the hypothetical macro scenario is based on a pessimistic economic scenario. The yield curve factors of the model are obtained via the Kalman filter as projections that are calculated conditional on the macroeconomic developments.

Conclusion

The present paper attempts to model and forecast the government yield curve in the undeveloped financial market of Bulgaria based on the Nelson-Siegel-Svensson framework. The model captures the evolution of yields in the time-series dimension as well as in the maturity dimension. The model is estimated by a state-space method using the Kalman filter and facilitates the projection of the Bulgarian yield curve. In general, the in-sample and out-of-sample fits of the model are satisfactory. Besides the latent factors, two additional macroeconomic variables are incorporated as exogenous variables. This allows the generation of conditional yield curve scenarios, linking expectations on future key macroeconomic variables to the shape and location of the yield curve.

The evidence in the present study suggests that the evolution of the estimated yield curves, albeit based on a rather short historical sample owing to data limitations at the country level, is in line with the macroeconomic theory. An upward-sloping yield curve is associated with periods of economic expansion, while flattening of the treasury yield curve signals weaker economic activity. The out-of-sample results show that the presented model provides a general framework that can link expectations on future key macroeconomic variables to the shape and location of the yield curve.

References:

Analytics, M. (2014). Principal Component Analysis for Yield Curve Modelling, https://www.moodysanalytics.com/-/media/whitepaper/2014/2014-29-08-pca-for-yield-curve-modelling.pdf

Anderson et al. (1996). *Estimating and Interpreting the Yield Curve*. Wiley.

Ang, A.,Piazzesi, M. and Wei, M. (2006). What Does the Yield Curve Tell us about GDP growth. *Journal of Econometrics*, Vol. 131, pp. 359-403.

Bliss, R. (1996). *Testing Term Structure Estimation Methods*. Working Paper 96-12a. Federal Reserve Bank of Atlanta.

Carriero, A., Favero, C. and Kaminska, I. (2006). Financial factors, macroeconomic Information and the Expectations Theory of the Term Structure of Interest Rates. *Journal of Econometrics*, Vol. 131, N 1-2, pp. 339-358.

Chou, J., Su, Y., Tang, H. and Chen, C. (2009). Fitting the term structure of interest rates in illiquid market: Taiwan experience. *Investment Management and Financial Innovations* 6 (1), pp. 101-116.

Dewachter, H. and Lyrio, M. (2006). Macro factors and the term structure of interest rates. *Journal of Money, Credit, and Banking*, Vol. 38, N 1, February, pp. 119-140.

Diebold, F. X. (2006). The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach. *Journal of Econometrics*, 131, pp. 309-338.

Diebold, F. X., Rudebusch, G. D. and Aruoba, S. B. (2006). The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach. *Journal of Econometrics*, 131, pp. 309–338.

Duffie, D., Kan, R. (1996). A Yield-Factor Model of Interest Rates. *Mathematical Finance*, 6, pp. 379-406.

Dutta, G. S. (2005). Term Structure Estimation in Illiquid Government Bond Markets: An Empirical Analysis for India. *Journal of Emerging Market Finance,* 4 (5), pp. 63-80.

Evans, C. L. & Marshall, D. A. (2007). Economic determinants of the nominal treasury yield curve. *Journal of Monetary Economics*, 54, pp. 1986-2003.

Grum, A. (2006). *The Development of the Slovenian government debt market and estimation of the yield curve: Financial Stability Report*. Expert Papers, Banka Slovenije, Ljubljana.

Gürkaynak R., Wright, J. (2012). Macroeconomics and the Term Structure. *Journal of Economic Literature*, Vol. 50, N 2, June, pp. 331-367, https://www.aeaweb.org/articles?id=10.1257/jel.50.2.331

Harvey, A. C. (1989*). Forecasting Structural Time Series Models and the Kalman Filter*. Cambridge University Press.

Hladíková H, Radová, J. (2012). Term Structure Modelling by Using Nelson-Siegel Model. *European Financial and Accounting Journal*, 72(2), pp. 36-55.

Hoffmaister, A., Roldós, J. and Tuladhar, A. (2010). *Yield Curve Dynamics and Spillovers in Central and Eastern European Countries*. International Monetary Fund Working Paper N WP/10/51, February.

Hordahl, P., Tristani, O. and Vestin, D. (2006). A joint econometric model of macroeconomic and term structure dynamics. *Journal of Econometrics*, Elsevier Vol. 131, N 1-2, pp. 405-444.

Kladívko, K. (2010). The Czech Treasury Yield Curve from 1999 to the Present. *Journal of Economics and Finance*, 60(4), pp. 307-335.

Knez, P., Litterman, R. and Scheinkman, J. (1994). Exploration into Factors Explaining Money Market Returns. *Journal of Finance*, 49, pp. 1861-1882.

Kučera, A., Dvořák, M., Komárková, Z. (2019). The Czech Government Yield Curve Decomposition at the Lower Bound. *Czech Journal of Economics and Finance*, N 1, http://journal.fsv.cuni.cz/storage/1427_02_36_kucera_final_issue_1_2019_.pdf69.

Minassian, G. (2005). Brady's odyssey of Bulgaria. Bulgarian National Bank, Discussion Paper DP/51/2005 (*in Bulgarian*).

Nelson, C. R., Siegel, A. F. (1987). Parsimonious Modelling of Yield Curves. *Journal of Business*, 60(4), pp. 473-489.

Nyholm, K. (2019). *A Practitioners Guide to Yield Curve Modelling*., https://www.cambridge.org/core/elements/practitioners-guide-to-discretetime-yield-curve-modelling/6087FE5133D5F93D9BF5B955510BA643

Nymand-Andersen, P. (2018). *Yield curve modelling and a conceptual framework for estimating yield curves: evidence from the European Central Bank's yield curves*. European Central Bank, Statistics Paper Series, N 27.

Reppa, Z. (2009). *A joint macroeconomic-yield curve model for Hungary*. MNB Working Papers 2009/1. Central Bank of Hungary.

Rudebusch, G. D. and Williams, J. C. (2008). *Forecasting Recessions: The Puzzle of the Enduring Power of the Yield Curve*. Federal Reserve Bank of San Francisco Working Paper No. 2007-16.

Rudebusch, G. D. and Wu, T. (2008). A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy. *Economic Journal*, Vol. 118, pp. 1-21.

Soderlind, P., Svensson, L. E. O. (1996). *New techniques to extract market expectations from financial instruments*. Working Paper WP 5877. National Bureau of Economic Research.

Subramanian, V. (2001). Term Structure Estimation in Illiquid Markets. *Journal of Fixed Income* 11 (1), pp. 77-86.

Svensson L. E. O. (1994). *Estimating and Interpreting Forward Interest Rates: Sweden 1992-1994*. NBER Working Paper N 4871.

Svensson, L. E. O. (1995). Estimating forward interest rates with the Extended Nelson-Siegel method. *Sveriges Riksbank Quarterly Review*.

Virmani, V. (2006). *Assessing NSE's Daily Zero Coupon Yield Curve Estimates: A Comparison with few Competing Alternatives*. Working Paper No 2006-05-05. Indian Institute of Management.

Wan, E. A., and Merwe, R. V. D. (2001). The unscented Kalman. In: *Kalman Filtering, Chapter 7*, pp. 221-280. Wiley.

Zoricic, D. (2012). Yield curve modeling possibilities on the Croatian financial market. PhD thesis. University of Zagreb, Croatia Faculty of Economics and Business.

Appendix

Table 1

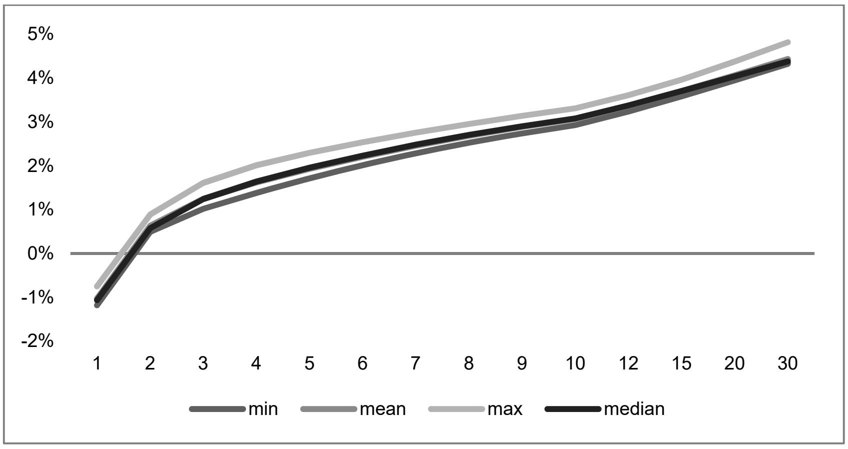
Eurobonds included in the estimation of the zero-coupon yield curve*

Maturity in years	ISIN	Issuance date	Redemption date	Interest Coupon (in %)	Nominal Value (in EUR)
2	XS0802005289	09-07-2015	09-07-2017	4.250%	950 000 000
7	XS1208855616	19-03-2015	26-03-2022	2.000%	1 250 000 000
7	XS1382693452	14-03-2016	21-03-2023	1.875%	1 144 000 000
10	XS1083844503	26-06-2014	03-09-2024	2.950%	1 493 000 000
12	XS1208855889	19-03-2015	26-03-2027	2.625%	1 000 000 000
12	XS1382696398	14-03-2016	21-03-2028	3.000%	850 000 000
20	XS1208856341	19-03-2015	26-03-2035	3.125%	900 000 000

*The characteristics reported for each Bulgarian Eurobond used in the estimation of zero-coupon yield curves.

Figure 1

Descriptive statistics, estimated yield curves*



*The Figure presents a set of descriptive statistics for the estimated end-of-month zero-coupon yields based on the Bulgarian Eurobonds traded on the international capital markets. The data covers the period June 2015-June 2020, for maturities from 1 to 30 years.

Source. Own calculations.

14.10.2020