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INDIVIDUAL DECISIONS AND COLLECTIVE CHOICES IN THE HISTORY OF ECONOMIC THOUGHT

This paper highlights the difficulties encountered in building individual decisions. In addition, possible incompatibilities can be found between individual choices and collective choices. Therefore, focus will be placed on how to transfer an analysis of rational individual choice in the context of collective choices. Voting is usually the most used expression of individual wills to arrive at the formulation of a collective decision. The model used highlights the paradox of electoral participation and hence the benefit of 'no vote'. Along the path, the difficulties encountered when one tries to transform individual preferences into collective choices through certain procedures will be examined. Therefore, other pathways should be identified in order to describe a number of decision-making procedures in the most appropriate possible way.

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The 'game theory' can be considered as a chapter in that series of theoretical elaborations which refers to the 'decision theory', having as its object the problem of choice. In this regard, it is possible to detect the tendency towards deepening in the direction of 'classical theory'.

These studies, which focus their attention on the analytic elements that are the subject of the theory, highlight the difficulties that, for a modeling use, imply the simplification imposed by the version of the classical economy. The object of a reflection in this direction is the representation of the players' preferences, with respect to which the ordering of possible outcomes in a single hierarchy for each player would constitute an excessive simplification and distortion of empirical phenomenology (Sen, 1977). The utility theory has been accused of being overly structured; whereas Sen argues that it has too little structure.

We can highlight the distinction between 'personal' preferences and 'ethical' preferences introduced by Harsanyi (1955), but more articulated and detailed studies appear on the so-called 'hypergames' (Fraser and Hipel, 1979; Bennet, 1977) in which the game analysed by the interpreter results from the overlapping of the different games that represent the definition of the conflicting situation by the various players.

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Another problem is the transformation of one game into another with different stakes (Shelling, 1960), through changes in player preferences and support for new possible strategies.

Another trend, detectable in the literature on games, is typical of a series of studies that propose a re-elaboration of classical game theory in the direction of less severity in the requirements to be imposed on elements embedded in the analysis. With respect to the problem of representation of player preferences, these studies aim to abandon the cardinality requirement imposed by classical utility theory in favor of an ordinal representation of preferences. Or, again, to take a representation of a player's preferences over the possible outcomes, which sets each outcome as a reference and classifies the others into two systems depending on whether or not they are preferred to that one (Howard, 1977). Howard rejects the postulate that is at the heart of the classic theory of co-operative games: that of the binding nature of the agreements. Renouncing this postulate means dropping the demarcation between co-operative and non-cooperative games and incorporating in theory the problem of co-operation between players.

The trends described above are related to the problem of the representation of situations of conflicting interaction. But the interest of the theorists is, above all, aimed at defining the criteria of rationality on which to define strategies and outcomes as rational or non-rational; that is, at the definition of the concept of game solution. In this regard, it is appropriate to avoid underestimating the problem of the definition of rationality criteria and to focus instead on a descriptive theory that does not depend on a rigorous definition of 'rational decision'.

One cannot imagine eliminating from the game theory the enucleation and formalization component of rationality models, by imagining to save the structure of representation of complex decision-making situations.

The complexity of a situation represented in a gaming model emerges from the tension among multiple rationality criteria or, in any case, from the incompatibility between multiple rationality criteria among which one is not in a position to choose. Not surprisingly, Rapoport (1970) suggests a redefinition of the concept of rational decision, making it possible to clearly highlight the differences between individual and collective rationality. The contribution of game theory to the readability of complex situations depends, in a significant way, on the ability to expound the rationality criteria attributed or attributable to the actors by clarifying the compatibility conditions of different criteria.

As already pointed out, the game theory focuses on the problem of choice. This term refers to situations in which a decision maker has the ability to identify, within a bounded range, the goals to be attained and the strategies to be taken in order to achieve them. Decision is the process that results in choices. What is meant by this work is the very important difference between decision theory and game theory, precisely because the first one deals with the problems in which a single decision maker is present, whereas the feature of game theory is to consider the actor as a decision maker and all his opposing partners as endowed with intentionality, equally rational and conscious. In addition, the problems that arise when trying to turn individual decisions into collective choices will be highlighted.

The framework of the theory: case studies

Let us assume that there are two decision makers-players: the first one, who we will call A, chooses from a number of actions $a2...a4...an$; the second one, who we will call B, chooses between $b1, b2 ... bn$. Let us imagine that these actions form a finite set, even if it is a completely arbitrary and provisional assumption. To this pair (aj, bj) of actions is associated a consequence (Cij) and the following assumptions are formulated: 1) each of the two players knows his own possible actions and those of his opponent; 2) each of the two players knows the consequences of all pairs of actions; 3) each has a transitory preference on the consequences; 4) each of the two players knows the preferences of the other.

A very simple case is the one in which the preference orders of the two players are exactly the opposite. Then we will be in the presence of a zero-sum game. This is, of course, an exemplification to the extent that one can suppose that the two players' utility can be added. It should be assumed that one can make interpersonal utility comparisons and use common measure units. Exemplifying assumptions only in the event that the outcome of the game is represented by sums of money and such sums are sufficiently small in relation to the respective economic situations, which in turn are the same. Another case involving one of the principles of minimum rationality that engages players in examining situations of uncertainty may in some cases lead to unique solutions and is the principle of strict dominance. If a player, whatever his opponent's choice, stands for a decision that he considers the most advantageous, he may discard the latter decision without regret as it is strictly dominated by the former. The elimination of strictly dominated decisions does not allow, however, to come to a single decision couple. For example, in Table 1:

Table 1

	$b1$	$b2$	$b3$
$a1$	6	5	8
$a2$	3	9	0

No decision can be discarded either for A or for B, in the name of a strict dominance. A, for example, can decide to choose the action that, despite the best choices of B, assures him the highest gain, that is, to choose the line of Table 1 whose minimum is higher. In our example, it is line 3 that assures him a profit of 6, whatever the decision of B may be. This minimum gain is called the security level of A. In turn, B will choose the column where the maximum value is weaker; in our example, it is column 1 (Table 1) that assures him to lose at most 6, whatever the choice of A may be. Obviously, the whole reasoning would lose value if each player, by interpreting the possible reasoning of his opponent, could profit by modifying his own decision. For example, if B, knowing the reasoning of A bringing it to action 3, could use this information for another choice of action.

Such a pair of actions $a3b1$ is called 'in equilibrium' and it is characterized on the Table for the resulting utility, which is the maximum of its column and the minimum of its line and is often referred to as a 'pass'.

The utility of the pass is called the value of the game. The conjunction of the cautious actions of each of the contenders is not necessarily a pass. It is a pass if the two players' levels of cautious safety are equal. For example (Table 2):

Table 2

	$b1$	$b2$	$b3$	$b4$
$a1$	1	2	4	-8
$a2$	3	1	5	8
$a3$	7	2	6	0
$a4$	4	3	-1	-4

Depending on whether one chooses $a1, a2, a3$ and the opponent, through guessing, chooses the worst action for A, the player A gets $-8,1,0,-4$, respectively, so that his safety strategy is $a2$ which assures him 1. If the player B chooses $b1, b2, b3, b4$ and his 'soothsayer' opponent should, in turn, always choose the worst action for B, the latter would get $-7,-3,-6$ or -8, respectively, so that his cautious safety strategy $b2$ assures him -3. For completeness, it should be said that such coupled actions (i, j) are defined as Nash's equilibrium (1951), where each action is the best response to other actions. In this case, each of the two opponents has no interest in unilaterally changing the decision-making framework:

$$Ci^x y^x = \underset{i}{\underline{\max - Ciy}}$$

$$-Ci^x y^x = \underset{j}{\underline{\max - Ci^x}}\, j$$

Which is equivalent to:

$$Ci^x y^x = \underset{j}{\underline{\min Ci^x}}\, j$$

One finds the definition of pass with a minimum in the line and a maximum in the column. Returning to the previous example, the conjunction of the above-mentioned actions is not necessarily an equilibrium since the specific caution levels may be different.

To avoid excessive optimism about the notion of equilibrium, we can also assume that equilibrium does not exist. For example, the following values in Table 3:

Table 3

4	2
3	5

Or, again, that there are at least two equilibriums, as in the following example (Table 4), corresponding to the utilities equal to 2:

Table 4

2	4	2
1	5	0

However, as can be stated, the value of pass is the same. The property is general, as can be seen by mentally isolating the lines and columns that contain two passes (Table 5) C and C‘:

Table 5

C	A
B	C'

In fact, C‘, the minimum of its line, is less than B, and B is less than the maximum C of its column. $C' \le C$ always. Likewise, C', the maximum of its column, is higher than A, which is itself higher than the minimum C of its line, so that $C \le C'$. Equality $C = C'$ is the result of two inequalities. Finally, even in the case of a single equilibrium, the corresponding strategies are not an absolute prescription.

These examples are needed that the only important condition is that a player’s decision cannot be guessed and anticipated by his opponent. Let us assume, for example, Table 6:

Table 6

	B_1	B_2
A_1	0	2
A_2	3	1

If A assumes that B will choose option 1, it will be in his interest to make decision 2 (utility 3 versus 0). But then, if B thinks that A will make decision 2, it will be in his interest to make decision 2 (loss 1 rather than 3). Therefore, if A thinks that B will make decision 2, it will be in his interest to decide option 1 (utility 2 versus 1). In turn, if B thinks that A will choose option 1, it will be in his interest to make decision 1 (loss 0 instead of 3), and so on indefinitely. So, when opponents are perfectly informed and rational, the only imaginable solution is to take the risk. This does not mean leaving everything to chance, but rather relying on a precise process of probabilities linked to the various possible decisions. This process is called a 'mixed strategy', and it is a decision-making process which can deliver effective solutions. In a zero-sum game there is nothing to negotiate since the gain of the one is equal to the loss of the other. The art of diplomacy consists in trying to turn a zero-sum game into a negotiable game, allowing each of the contenders to save face. If, conversely, the game is not zero-sum, the utility table that comes from each pair of actions is more complex, because we need to separately indicate the gains of A and those of B, by trying to lead the two players to an acceptable solution for both through rational negotiation.

In fact, any negotiation leads to weighing up, by mutual agreement, the utilities of the two opponents. If one moves in a Pareto context, where the interpersonal utility comparison is devoid of meaning, one cannot have a solution. Thus, every individual who claims to have found a solution has made an interpersonal utility comparison. Given two solutions, there are modifications to each of the utility scales that transform one of the two possible agreement points into another.

Analysis of Nash's theorem

For completeness of the analysis, we also refer to Nash's theorem (1951), which proved an elegant result, solving the following requirements: (1) *symmetry*, if the roles of the antagonists are exchanged, solutions are also exchanged; (2) *consistency*, the solution lies along the negotiation segment of negotiable games; (3) *stability*, a modification (of V form = au +b, with a> 0) of the utility scales of either antagonist does not modify the solution; (4) *independence*, if one widens the set of possible decisions, the solution either does not change or places itself on one of the added points.

Nash's result is, as follows: if one gives a pair of utilities (x^1, y^1), the previous conditions determine only one solution in the negotiating segment. Of course, the solution found depends on (x^1, y^1); the point that represents this pair is called a 'status quo point' and many proposals have been put forward in favor of its choice. We cite two: 1) Shapley's choice (1953), where one takes the safety point; 2) Nash's choice (1953), where, as in Shapley's choice, one separates into two zero-sum games, but this time each one chooses the strategy that would be the most advantageous to his opponent (one takes the power of influence into account, that is, the loss that each one is capable of inflicting on the other).

Other assumptions consist in separating two stages of decision-making: in the first, one tries to maximize the sum of the two players' utilities and in the second, one negotiates the split of the loot. But these proposals merely conceal a presumption of interpersonal comparability of utilities. In addition, in some more complex cases, one is faced with great difficulties. For example, Table 7:

Table 7

	B_1	B_2
A_1	(3,3)	(0,4)
A_2	(4,0)	(1,1)

In this case, negotiation is more complex since none of the four decision pairs are excluded a priori. The game of A, in turn, is (Table 8):

Table 8

	B_1	B_2
A_1	3	0
A_2	4	1

And there is not even a chance to look for a mixed strategy. Whatever the choice of B may be, it is in the interest of A to choose A_2 since his utility is greater (4 instead of 3, 1 instead of 0).

Similarly, the game of B is (Table 9):

Table 9

	B_1	B_2
A_1	3	4
A_2	0	1

Even in this case, there is no possibility of looking for a mixed strategy. Indeed, whatever the choice of A may be, it is in the interest of B to choose B_2, because his utility is greater (4 instead of 3, 1 instead of 0). Ultimately, each one is led to decision number 2, whereas through negotiating he may agree to choose decision (A_1, B_1), which could give 3 to both players. The difficulty lies in the fact that, after negotiation, it is in the interest of each one to betray the agreement, whether or not the other respects it, since the betrayer would gain:

- against a fair opponent, 4 instead of 3;
- against an unfair opponent, 1 instead of 0.

This situation, however, characterizes the prisoner's dilemma (Tucker 1983), the best known of the decision-making dilemmas. The starting point is a table of choices very similar to the previous ones used for pairs of values (Table 10):

Table 10

	c	d
c	(x,y)	(z,t)
d	(u,v)	(w,s)

The prisoner's dilemma features some inequality relationships that appear in the table among the various utilities that appear in relationships. In this specific case, the actions are marked by letters: c for co-operative actions and d for non-cooperative actions. There are many examples which consider two arrested individuals, questioned for a crime. If only one collaborates, telling his truth, the other will be condemned and the one who co-operates will be released and acquitted. If they both confess, they will stand trial with an unfavorable outcome. If, on the contrary, both are silent, they will have to be released.

If the utilities associated with different behaviors are set, we infer a utility of 50 for trial, 0 for conviction, 200 for acquittal, and 10 for release while awaiting judgment (Table 11):

Table 11

	c	d
c (is silent)	(50,50)	(0,200)
d (confesses)	(200,0)	(10,10)

Whatever the attitude of the other arrested individual (it is in the interest of either to confess), this would lead to the result (10.10), while co-operation would lead to the result (50.50). Among the four possible outcomes (10,10) is the only one which is not Pareto-optimal. In fact, the outcome (50.50) would be better for both. However, in line with Nash's equilibrium, this is the foreseeable outcome of equilibrium since it is not in the interest of either player to discard it unilaterally. Therefore, what we have defined as zero degree of rationality, namely the application of the principle of strict dominance (Burns and Buckley 1974), leads to this decision pair as the only 'rationally' possible one. Rapoport (1965) has outlined this decision-making process through the following inequalities: $S < P < R < T$, where R is the reward for mutual co-operation, T is the temptation to choose accusing, P is the punishment for mutual accusations, S is the reward of the betrayed co-operator.

To these inequalities, Rapoport adds the condition $2\,R > S + T$ to avoid the tacit collusion assumption that would make the defect rational with ½ probability compared to co-operation.

In the case of the prisoner's dilemma, the distinction between the descriptive path and the normative path appears clear. At the regulatory level, the decision of double defection (non-cooperative confession) is the only rational since it is a Nash equilibrium. The S, T, R, P values are of little importance, because only the order of preference (utility) is important.

With a low T, mutual cooperation perpetuates with a ratio $R + tR + t^2R + t^3R...$ a series whose formula allows to express the sum $R/1-t$.

By reckoning that there is no winning strategy, against an opponent who uses defection as a systematic attitude, the best strategy is systematic defection; whereas, against an opponent who co-operates until the first defection, the best strategy is systematic co-operation.

Nash's equilibrium allows to make some remarks. The perpetual traitor's strategy is in itself a Nash equilibrium. If, in fact, either player betrays systematically, the other player is losing if he a priori discards the same strategy, because he will receive P instead of S for the choice to which he will have co-operated without any change for the moves to follow. Conversely, the permanent co-operator's strategy is not in itself a Nash equilibrium. If a player co-operates steadily, it is in his opponent's interest to betray, as at each betrayal he will receive T instead of R.

Therefore, not by chance, other strategies have been assumed, such as the one called 'tit for tat' (Axelrod and Hamilton, 1981), whichentails to co-operate for the first choice and then act depending on how the opponent acted at the previous move. This is a strategy that does not automatically lead to any standard concept of equilibrium. Not even to an assumption of privilege, either for betrayal or for co-operation. The outcome will depend on the discount rate applied to the different choices in succession. It may also happen that any temptation to change behavior is discouraged and therefore the 'tit for tat' strategy consolidates into a Nash equilibrium. It is obvious that there may be other solutions to the prisoner's dilemma, as well as to many other situations in the economic context, for which expected decisions (expectations) are relevant to defining equilibrium states and are related to the intervention of an external condition to the players involved.

It is understood that, in the absence of a regulator, egoisms involve poverty and insecurity, as well as that co-operation cannot develop among individuals on the basis of utilitarian rationality.

The state allows to solve the dilemma according to the principle that: I find it useful to submit to the law, since I can benefit from the advantages of the fact that others also submit to it. But at the same time, it is a paradox, as it leads rationally to eliminate strictly dominant decisions.

From individual decisions to collective choices

In the previous paragraphs we have analyzed the difficulties encountered in building individual decisions. Now, on the other hand, we will deal with the problems that arise when trying to turn individual decisions into collective choices.

Our analysis starts from the aggregation technique, which allows us to reach the collective decision. The focus of this study, therefore, is to identify a sort of urn where individual preferences will flow into, with a view to developing collective choices. Our focus is the social actor and his decision on a social issue as being the result of a rational calculation that allows him to avoid the uncertainty and risks of the future (Morselli, 2015).

The collective decision will, of course, have consequences on an individual level and situations may arise for which some would have preferred different decisions (Lagerspetz, 2014; 2016). Participation in a society implies that some of these situations have been accepted a priori and that some rule has been recognised to address all the situations in which decisions are not unanimous. Some rules, for example, refer to dictatorship and imposition, situations in which individual preferences are subordinate to the decisions of the dominant individual or group. We can consider the majority rule, but it can represent different and even contradictory forms. In fact, every individual, when taken individually, may have a will different from the general will. Thus, we have an issue of individual rights in relation to the collective decision. This means that it is not always possible to combine individual and collective choices. Gibbard (1974) and Sen (1970) have highlighted possible incompatibilities between individual rights and collective choices.

Voting is usually the form of expression most used by an individual to express his will in the process of coming to a collective decision. But we can also find other forms of expression. For example, individual wills dictated by fear or anger can strongly determine the behaviours of crowds, such as panic outbreaks or riots, for which preference aggregation certainly does not obey rational patterns. However, at least in the political sphere, rationality has its limits, and the paradox of voting is an obvious example of that.

The model

The first model to explain the voting decisions/abstentions of individuals was proposed by Downs (1957), through rational voting theory, which extended the structure of Hotelling's spatial competition among companies (1929)[1].

If an individual behaves rationally, we must assume that he acts if the expected benefits outweigh the costs of the action(s). If we apply the same rule to political voting, we will have an apparent paradox. Voting represents a cost: the time it takes to get to the polling station, the time needed to acquire information about the candidate and on his campaign programme. Whereas we assume that voting brings a benefit that depends on whether the voter would benefit from the victory of his candidate and the likelihood that his expression of vote may be decisive for the victory of the candidate. We could hence devise a formula: $U = PB - c$, where c represents the opportunity cost of the effort to go to vote, B represents the benefit expected from the success of the candidate selected and P represents the probability that one's vote will prove decisive.

It is clear that, apart from exceptional cases, the comparison of the costs and benefits should lead one not to vote since, no matter how weak the costs and how great the benefits, the chance that one's vote will prove decisive is rather low in an electoral college of normal size.

The model used highlights the paradox of voter turnout. Furthermore, the model appears to be in contradiction with a phenomenon that, although rare, is now occurring rather frequently, namely the blank vote, based on which $P = 0$ and therefore U have a negative value for any value assigned to B and c.

[1] Smithies (1941) improved Hotelling's spatial competition model by introducing an elastic demand at each point, so both enterprises or both parties, moving away from the extremes, lose their consumers or voters who suffer the greater costs of transport or ideological distance.

As for the blank vote, two explanations are possible: (1) either the elector behaves irrationally, or in other words, based on other criteria that aim to maximise his personal usefulness; (2) or the model is insufficient because it does not consider other different benefits. This is the reasoning that Tullock (1993) follows when he talks of a certain "mental credit" D. The voter's behaviour is therefore defined by the new function $U = PB + D - c$, where D incorporates several subjective factors: (1) the satisfaction of performing a civic duty; (2) being recognised by other voters or the possibility of discussing politics; (3) gaining recognition for the elector's political activism.

The idea of considering the total benefit expected by the voter (and therefore of including D) comes from Riker and Ordeshook (1968) (also followers of the rational approach of the voting theory). According to Llavador (2000) and Leppel (2009), the paradox of electoral participation does not come from the weakness of rational voting theory, but from the failure to consider alienation as an alternative cause of abstention to indifference (the first study comes from Brody and Page, 1973). When the party is far from the ideal politics of the voter, the voter has no interest in voting since no party meets his or her interests. Even if the two parties elaborate different proposals that can justify the cost of voting, a voter can abstain for alienation.

However, the paradox of electoral participation is relevant even after changing the utility function, as shown by Ferejohn and Fiorina's theoretical analysis (1974).

In order not to get trapped in ideology, the behavioural patterns cannot go without formalisation, which means factoring in an analysis of the intensity of preferences.

Collective decision and the intensity of preferences

In the definition of the rule concerning the collective decision, are we to take into account the differences between the intensity of individual preferences? Does an indifferent majority have the right to impose its will on a passionate minority? In some situations, indifference leads to abstention, but apart from this case, the differences in intensity are hardly measurable or comparable between individuals, even if the individual behaviours may represent acceptable indicators. However, according to Dahl (1956), it seems inevitable to conclude that benefits and costs are distributed in a completely arbitrary way and it is impossible to form any general principle from their distribution. It therefore seems that political democracy is almost immune to any rationalisation and formalisation modelling, except for decision-making processes that involve small electoral colleges. In this regard, Dahl refers to the so-called politarchy, which is characterised by a set of more or less satisfactory conditions:

●*During the vote*: (*a*) each member expresses his preference; (*b*) during the count, voting expressions are evaluated in the same way; (*c*) the candidate with the most votes wins;

●*Before the vote*: (*a*) each member chooses the candidate he/she prefers; (*b*) everyone has the same information about the candidates;

●*After the vote*: (*a*) all the least favourite choices (candidates) are eliminated; (*b*) collective decisions become enforceable;

●*In the periods between one vote and the next*: all decisions are influenced by previous electoral outcomes and if decisions need to be made, they must conform to the outcome of the previous election.

Dahl's claims show the gap between the ideal functioning of a democracy and its daily practice.

Moving on to an analysis of the cases where voters are called upon to pronounce themselves on more than two issues, the scenario obviously becomes more complicated, and the multiple needs of common sense become increasingly contradictory. Two approaches can help us in the analytical path, namely, the Borda method (1781) and the Condorcet method (1785). Neither of these methods proved satisfactory because they do not consider the intensity of preferences. Even when applying the Borda scores to the methods, the results of which depend on the comparability of the candidates assessed two by two by the voters, the results show no improvement.

Now let us consider an electoral body of n electors, among which p rank the two candidates a and b in the order $a > b$ and the remaining $n - p$ voters in the order $b > a$. We apply the Borda score to the method (Diss and Gehrlein, 2012). If we apply the normalised benefits, which consist in giving one point to the first or the last (in our case with two contenders, the second), candidate a is ranked first by p voters and second by $n - p$, so the result is linked to the value of p . Candidate b is ranked first by $n - p$ voters and second by p , and his advantage is $n - p$. Thus, the winner is a if $p > n - p$, or even if $2p > n$, if we want to say that a got more than half of the votes.

In the end, the only result we see is that the candidate who won is the one with the most voters. Therefore, the Borda method, when applied to two candidates, expresses only the majoritarian method. This result, already demonstrated in 1781, was mathematically taken up by May's theorem (1952). In fact, he states that majority voting is the only democratic alternative, the only procedure for choosing between two candidates.

We can now ask whether the majoritarian method can be used to rank several candidates by comparing them two at a time. When two candidates are competing, each voter can vote for one or the other or abstain, and the candidate who gets the most votes is the one chosen. The different analytical methods use a proprietary grid of these techniques in which point to the fact that no voter can be sure, through the method chosen, that his choice coincides with the choice of the electorate; or that each candidate can win. These properties are present in the Borda method well as in those which will we now analyse, but when considered together they provide interesting elements for reflection:

1) *properties of Bentham*: each voter is free to express his vote and there is no external property that prevents him from making certain choices;

2) *anonymous method*: no voter has the chance to influence the outcome in any relevant way;

3) *neutral method*: no candidate is favoured by the system;

4) *monotonous method*: if a candidate wins, he will never move from the status of elected to that of loser.

From the properties to the formal path

Now we will try to transform the four properties mentioned into a formalised path. We have two candidates x and y, and define for each voter and i an indicator of preference D_i:

$D_i = 1$ if the voter prefers x to y;

$D_i = -1$ if the voter prefers y to x;

$D_i = 0$ if the voter is indifferent to x and y.

A rule of collective choice for our group n of voters is, therefore, a function f that to numbers $D_1, D_2.........D_n$ associates number D with the same conventions:

$D = 1$ if the group prefers x to y;

$D = -1$ if the group prefers y to x;

$D = 0$ if the group is indifferent to x and y.

Let us now rephrase the four properties that we mentioned earlier using this formula.

1) *Property of Bentham*. It indicates that we can calculate D for any value of numbers $D_1........D_n$, or that function f is defined on the Cartesian product $(-1,0,1)$ n converted into an integer.

2) *Anonymous method*. If we change numbers $D_1, D_2.........D_n$ in any way, the value of f $(D_1, D_2..........D_n)$ is unaffected. In this case function f is symmetric.

3) *Neutral method*. If each of the numbers $D_1, D_2.........D_n$ is replaced by its opposite, number D resulting from the calculation would also be replaced by its opposite. In this case function f is an odd function.

4) *Monotonous method*. If, after an example of numbers $D_1, D_2.........D_n$, the value of f is 0 or 1, and if one of the two numbers $D_1, D_2.........D_n$ varies

(from -1 to 0 or 1, or from 0 to 1), then the new value of $f\left(D_1, D_2..........D_n\right)$ becomes 1.

These four properties characterise the majority vote and no other decision rule contains them all together. For the anonymity property, function $f\left(D_1, D_2.........D_n\right)$ depends on the number of voters who prefer one candidate to another, namely by number 1, which we identify as $N(1)$, and by number -1, which we identify as $N(-1)$, and which appear in the sequence $D_1, D_2.........D_n$.

Analysing now, the symmetry property, it follows that if $N(1) = N(-1) \rightarrow D = 0$. In fact, it is impossible to think that $D = 1$ because otherwise, by changing each of the numbers D_i into its opposite, you would change D into its opposite, which would become -1. For the same reason, we cannot imagine $D = -1$ and therefore it remains $D = 0$.

This reasoning formalises the intuition that if a method is anonymous and neutral, in the case of equal votes for each candidate, the candidates are ranked on a par. Assuming now that $N(1) = N(-1) + 1$, the monotonous method indicates that D, which takes the value 0 in the event of a tie, is now 1 and as a result D still applies 1 if $N(1) = N(-1) + 1, N(1) = N(-1) + 2$, and so on. All this defines exactly the majority vote shown by Borda (1781) and then taken up by May (1952).

Other methods have tried to offer completeness and logic to collective choices, such as the ones developed by Coopeland's (1951) or Dodgson's (1958); all methods that strive to make Borda and Condorcet compatible. Arrow (1951) concludes that a set of conditions, as elementary and common sense as they may be, are irreconcilable.

Faced with these problems, the risk is to rely on dictatorial decision or the randomness of random chance, or to the procedure of the random dictator. The latter assumption concerns the choice of randomly drawing a voter and considering his decision as the collective choice. Gibbard (1974) showed that this paradoxical procedure is the only one that under some conditions proves Pareto-optimal, non-manipulable and non-dictatorial in the traditional sense. In fact, it eliminates the wasted vote both in favour of a candidate who would have no chance of being elected, and of a candidate who is sure to be elected; in addition, it avoids the problem of minorities being regularly discriminated against.

Conclusions

We have seen the difficulties encountered when one tries to transform individual preferences into collective choices through certain procedures. Other paths, therefore, should be taken, not only from a regulatory point of view, but simply to describe some decision-making procedures more appropriately. Rawls (1972) and his acceptance of the interpersonal utility comparison may be considered.

Now let us imagine that a majority is realized to prefer a to b, b to c, and c to a. On the other hand, a cycle can involve more than three elements and it is no longer satisfactory that one method forces the preference of a_1 to a_2, a_2 to a_3 and a_{n-1} to a_n and therefore a_n to a_1. This concept of cycle is related to that of the Condorcet winner. Under certain conditions, a Condorcet winner is a candidate chosen by the majority of the voters. Then it is possible to prove that if there is no Condorcet winner, there are at least three candidates who form a cycle. In this context, the issue of comparability, or the separability between narrow preferences and weak preferences, will only be considered as an attempt to overcome the impasse.

Therefore, rationality and paradoxes surround the decision, and not by chance many have evoked the limits of rationality. The reference is mainly to Elster (1983) and Simon (1972). Elster has focused his attention on so-called impossible decisions, i.e. decisions for which rationality conflicts with will. There are also decisions for which contradiction is contained in the same term 'decision', as well as situations where weakness of will prevents any rational decision. Whereas Simon argues that one of the factors behind the alleged failures of human decision-making processes, compared to the assumptions of orthodox theory, concerns the contents of limited rationality.

In fact, the discovery of the existence of social preferences, understood as positive and/or negative predispositions towards the social and economic conditions of others, complicates in a decisive way the theory of economic rationality. The latter binds the decision only to reasons of individual utility without any interest in the plight of others. The models of strategic interaction are also to be changed radically by them. Insights and emotions frequently violate all the principles of rationality, but certainly do not eliminate them (Morselli, 2018). One has the sensation of a cognitive duplicity where rational logic and emotions are forced to cohabit. What determines the prevalence of intuition over reasoning or vice-versa? It is conceivable that the context with the factors conditioning it assumes a decisive role. But because the contexts cannot all be summarized in a theoretical model, what follows is the awareness of the extreme complexity and non-linearity of the phenomena that are often the result of the interaction of different economic agents. What arises is an extreme difficulty to develop models with a comprehensive predictive capacity and the holistic impossibility of explaining economic phenomena, abolishing the role of individual economic action and its cognitive genesis.

In addition to that, criticism to unlimited rationality does not rely only on the awareness of the reduced computational capacity and of calculating the conscious and intentional part of the human mind. In reality, such a limiting condition pairs with the influence of intuitive, emotional, affective, tacit factors that characterize the intuitive mind (as opposed to the conscious reasoning mind). Therefore, the choices and decisions of *homo oeconomicus* are moving on a strongly connected cognitive duplicity, with the prevalence of one or the other of the components that is heavily dependent, typically on the situations and contexts; but also on the different attitude compared to emotional categories such as regret (Loomes and Sugden, 1982) or disappointment (Gul, 1991). This leads to the necessity of building the decisional context, one that incorporates information from the "environment" and of the mental and behavioural model of the individual actor. The conclusion that follows is that the "decision" has nothing axiomatic to it, being the final act of a previous and complex process that involves objective and subjective conditions (as, moreover, already generally contained in Simon's concept of bounded rationality, 1972). It is on this path that we allow an analysis of decisions to shift the focus from the decision in itself to the representation of the alternatives by opening the way for a series of empirical studies on the construction of strategies on *problem solving* and learning.

The results outlined above do not seem to be reassuring, especially considering the fact that many phenomena today can be linked to individual decisions, but many more are related to collective choices, which only by chance we can imagine as the sum of choices and decisions of individuals although they appear this way, at least atfirst glance.

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